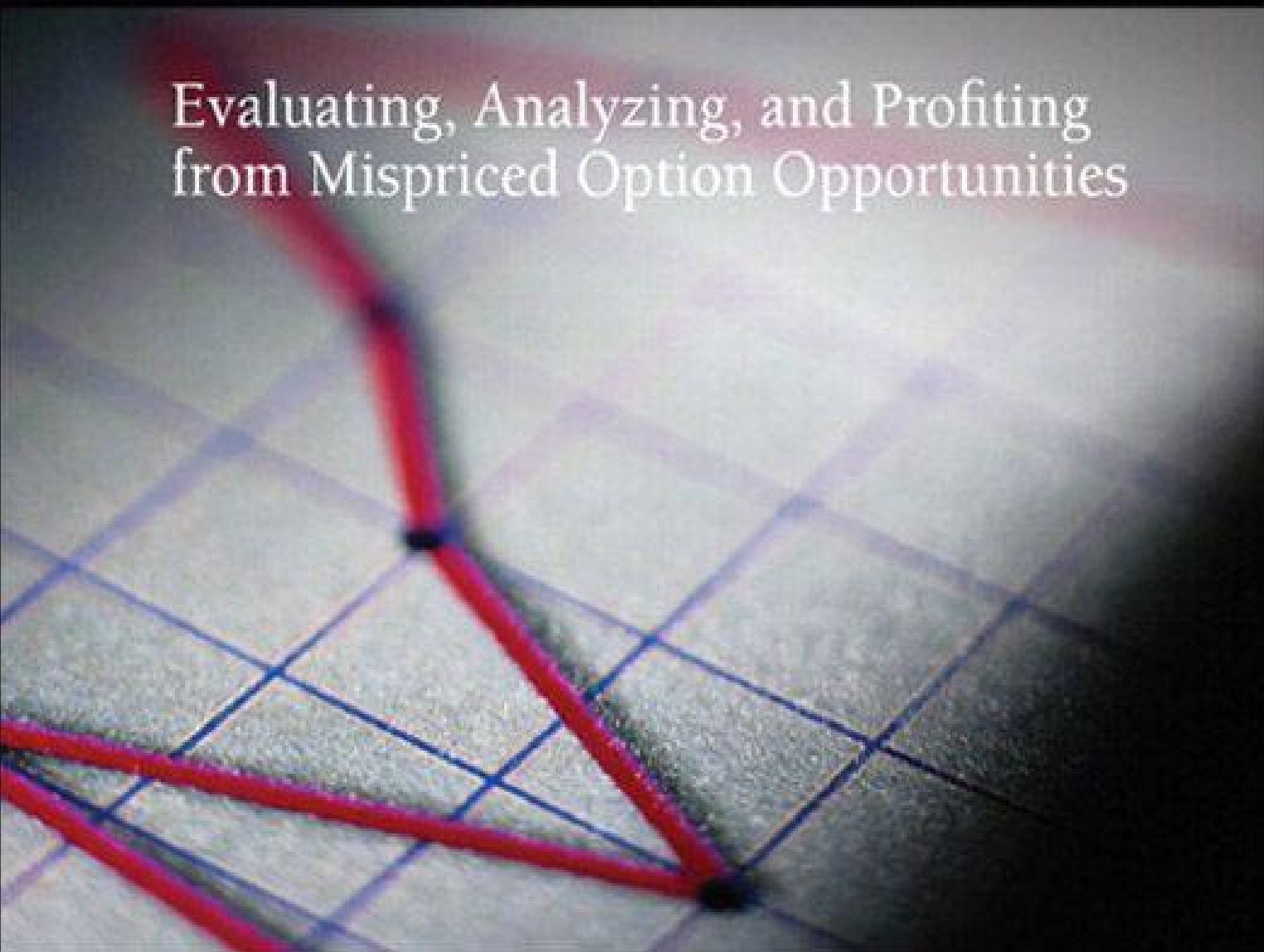
VISIT…

Sergey Izraylevich ■ Vadim Tsudikman

SYSTEMATIC OPTIONS TRADING

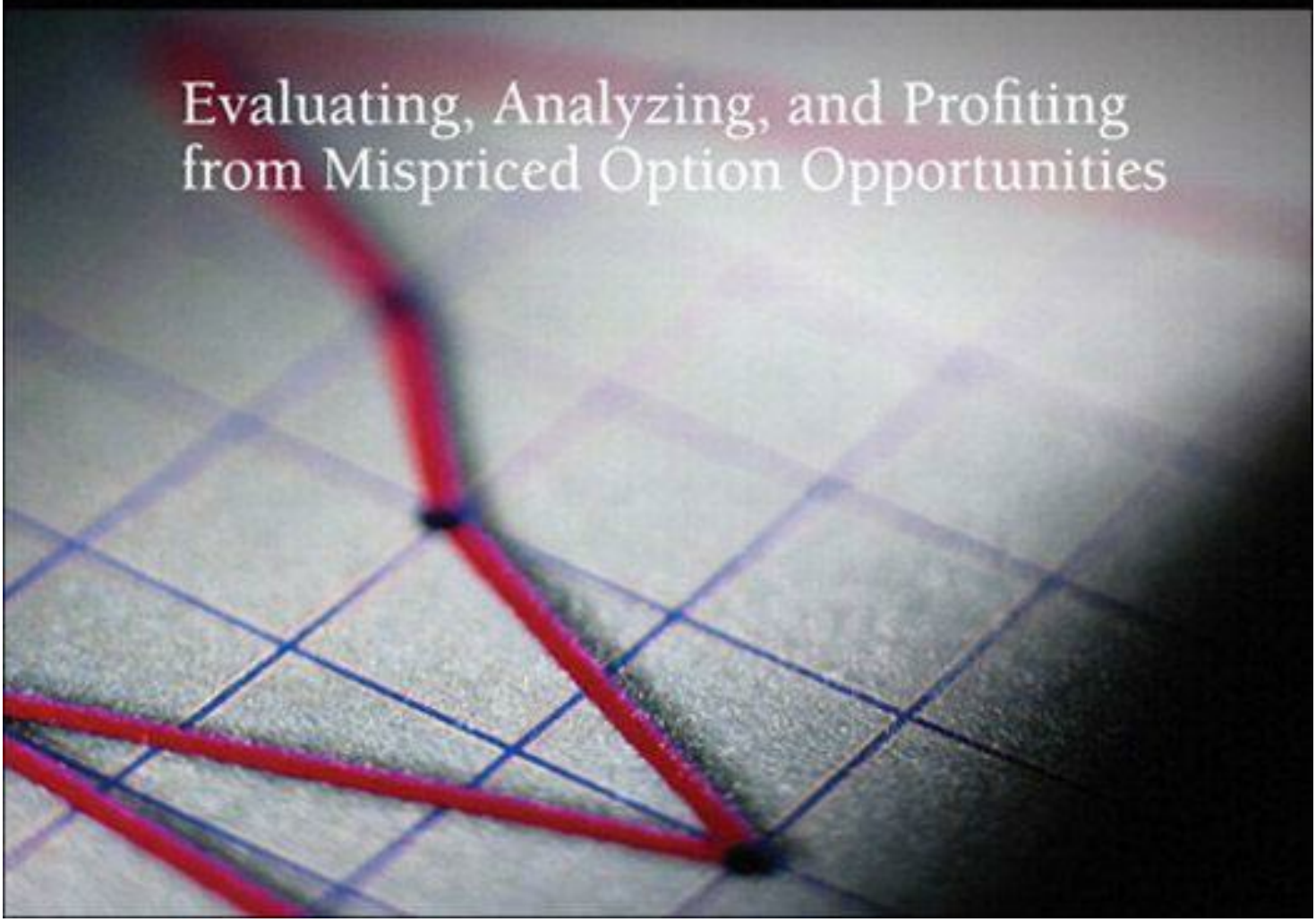
Evaluating, Analyzing, and Profiting from Mispriced Option Opportunities



Sergey Izraylevich ■ Vadim Tsudikman

SYSTEMATIC OPTIONS TRADING

Evaluating, Analyzing, and Profiting from Mispriced Option Opportunities



Systematic Options Trading

Evaluating, Analyzing, and Profiting from Mispriced Option Opportunities

Sergey Izraylevich and Vadim Tsudikman

Vice President, Publisher: Tim Moore
Associate Publisher and Director of Marketing: Amy Neidlinger
Executive Editor: Jim Boyd
Editorial Assistant: Pamela Boland
Operations Manager: Gina Kanouse
Senior Marketing Manager: Julie Phifer
Publicity Manager: Laura Czaja
Assistant Marketing Manager: Megan Colvin
Cover Designer: Gary Adair
Managing Editor: Kristy Hart
Project Editors: Jovana San Nicolas-Shirley and Andy Beaster
Copy Editor: Apostrophe Editing Services
Proofreader: Williams Woods Publishing Services

Indexer: Word Wise Publishing Srevices
Senior Compositor: Gloria Schurick
Manufacturing Buyer: Dan Uhrig

© 2011 by Pearson Education, Inc.
Publishing as FT Press
Upper Saddle River, New Jersey 07458

This book is sold with the understanding that neither the authors nor the publisher is engaged in rendering legal, accounting, or other professional services or advice by publishing this book. Each individual situation is unique. Thus, if legal or financial advice or other expert assistance is required in a specific situation, the services of a competent professional should be sought to

ensure that the situation has been evaluated carefully and appropriately. The author and the publisher disclaim any liability, loss, or risk resulting directly or indirectly, from the use or application of any of the contents of this book.

FT Press offers excellent discounts on this book when ordered in quantity for bulk purchases or special sales. For more information, please contact U.S. Corporate and Government Sales, 1-800-382-3419, corpsales@pearsontechgroup.com. For sales outside the U.S., please contact International Sales at international@pearson.com.

Company and product names mentioned herein are the trademarks or registered trademarks of their respective owners.

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

Printed in the United States of America

First Printing September 2010

ISBN-10: 0-13-708549-4

ISBN-13: 978-0-13-708549-1

Pearson Education LTD.
Pearson Education Australia PTY, Limited.
Pearson Education Singapore, Pte. Ltd.

Pearson Education North Asia, Ltd.
Pearson Education Canada, Ltd.
Pearson Educatión de Mexico, S.A. de C.V.
Pearson Education—Japan
Pearson Education Malaysia, Pte. Ltd.
Library of Congress Cataloging-in-Publication Data

Izraylevich, Sergey, 1966-
Systematic options trading : evaluating, analyzing, and profiting from mispriced option opportunities / Sergey Izraylevich and Vadim Tsudikman.
p. cm.
ISBN-13: 978-0-13-708549-1 (hardback : alk. paper)
ISBN-10: 0-13-708549-4
1. Stock options. 2. Options (Finance)

3. Investment analysis. 4. Portfolio management. I. Tsudikman, Vadim, 1965- II. Title.
HG6042.I97 2011
332.63'2283—dc22
2010006153

This book is dedicated to Professor Uri Gerson, Hebrew University of Jerusalem.

Contents

Introduction

PART I Criteria as the Basis of a Systematic Approach

chapter 1 General Presentation and Review of Criteria Properties

1.1 The Main Tool for Solving the Selection Problem

1.2 Formal Definition

1.3 Philosophy of Criteria Creation

1.4 Mission Fulfilled by Criteria

1.5 Forecast as a Key Element of the Criterion

1.6 Classification of Criteria

1.6.1 Universal Criteria

1.6.2 Specific (Nonuniversal) Criteria

chapter 2 Review of the Main Criteria

2.1 Criteria Based on Lognormal Distribution

2.1.1 Description of Lognormal Distribution

2.1.2 Expected Profit on the Basis of Lognormal Distribution

2.1.3 Profit Probability on the Basis of Lognormal Distribution

2.2 Criteria Based on Empirical Distribution

2.2.1 Description of Empirical Distribution

2.2.2 Expected Profit on the Basis of Empirical Distribution

2.2.3 Profit Probability on the Basis of Empirical Distribution

2.2.4 Simplified Calculation Algorithm

2.2.5 Modifications of Empirical Distribution

2.3 Criteria Based on the Ratio of

Expected Profit to Loss

2.3.1 Basic Concept and Criteria Calculation Method

2.3.2 Criteria Calculation Example

2.4 Criteria Based on Expert Distribution

2.4.1 Basic Concept and Criteria Calculation Method

2.4.2 Set of Standard Distributions

2.4.3 Combining Separate Standard Distributions into a Unified Probability Density Function

2.4.4 Criteria Calculation on the Basis of the Unified Probability Density Function

2.4.5 Construction and Valuation of Complex Strategies Based on the Unified Probability Density Function

2.5 Specific (Nonuniversal) Criteria

2.5.1 Break-Even Range

2.5.2 IV/HV Ratio

2.5.3 Relative Frequency Criterion

2.5.4 The Ratio of Normalized Time Value to the Coefficient of Absolute Price Changes

Distribution

chapter 3 Evaluation of Criteria Effectiveness

3.1 Introduction

3.2 Methods of Criteria Effectiveness Evaluation

3.2.1 Correlation Between a Criterion and Profit as the Main Effectiveness Indicator

3.2.2 Transformation of the Criteria Effectiveness Indicator

3.2.3 The Dynamics of Transformed Effectiveness Indicators

3.2.4 Selection of the

Averaging Period

3.3 Peculiarities of Criteria Effectiveness Evaluation

3.3.1 Number of Combinations Used in the Analysis

3.3.2 Expressing Profit

3.3.3 Expressing Effectiveness Indicators

3.4 Review of Criteria Effectiveness Indicators

3.4.1 Correlation Between Criterion and Profit Values

3.4.2 Correlation Between a Criterion and Profit Indexes

3.4.3 Correlation Between the

Sharpe Ratios of Criterion and Profit

3.4.4 Areas Ratio

3.4.5 Other Effectiveness Indicators

3.5 Summary

PART II The Main Areas of Criteria Application

chapter 4 Selection of Option Combinations

4.1 Introduction

4.2 Analysis of Criteria Effectiveness in the Selection of Option Combinations

4.3 Factors That Affect Option Combinations Selection

4.3.1 Absolute Values of the Criterion

4.3.2 Strategy and Underlying Assets

4.3.3 Simultaneous Analysis of Factors Affecting Combinations Selection

4.4 Multistrategy, Long-Term Evaluation of Criteria Effectiveness

4.4.1 Methods

4.4.2 Results

4.5 Summary

chapter 5 Selection of Option Strategies

5.1 Introduction

5.2 Evaluation of Criterion Effectiveness by Ranking Analysis

5.2.1 Methods of Ranking Analysis

5.2.2 Results of Ranking Analysis

5.2.3 Generalized Ranking Analysis and Introduction of the Threshold Parameter

5.2.4 Results of Generalized Ranking Analysis

5.2.5 Maximum Obtainable Values of the Criterion Effectiveness Coefficient

5.3 Traditional Methods of Evaluating the Criterion Effectiveness

5.4 Synthetic Approach to Criterion Effectiveness Analysis

5.5 The Model for Optimizing the Threshold Parameter

5.6 Summary

chapter 6 Selection of Underlying Assets

6.1 Introduction

6.2 Analysis of Criteria

Effectiveness in Selection of Underlying Assets

6.3 Multistrategy, Long-Term Evaluation of Criteria Effectiveness

6.3.1 Methods

6.3.2 Results

6.4 The Optimization Model for the Number of Underlying Assets

6.4.1 Utility Indicators

6.4.2 Utility Functions

6.4.3 Convolution of Utility Functions and Deriving Optima for Different Strategies and Criteria

6.5 Summary

PART III Multicriteria Analysis

chapter 7 Basic Concepts of Multicriteria Selection as Applied to Options

7.1 Introduction

7.2 The Pareto Set

7.2.1 The Algorithm of Forming the Pareto Set

7.2.2 Widening the Pareto Set and the "Layer" Notion

7.3 Convolution

7.4 Comparative Analysis of Multicriteria and Monocriterion

Selection Effectiveness

7.5 Comparative Analysis of Two Multicriteria Selection Methods: Pareto Versus Convolution

7.6 Summary

chapter 8 The Impact of Criteria Correlation on Multicriteria Selection

8.1 Introduction

8.2 Evaluation of Criteria Interrelationship

8.3 Criteria Correlation and Profitability of Pareto Selection

8.4 Criteria Correlation and Profitability of Selection Using the Convolution Method

8.5 Summary

Conclusion

Bibliography

appendix Basic Notions

Index

Acknowledgments

The authors would like to express their gratitude to the team at High Technology Invest Inc. Special thanks are due to Arsen Balishyan, Ph.D., and Vladislav Leonov, Ph.D., for their skillful assistance in research and manuscript preparation. We are also indebted to Mikhail Kolkovsky, Sergey Anikeev, and Eugen Masherov, Ph.D., for their useful comments and continued help at all stages of this project.

About the Authors

Sergey Izraylevich, Ph.D., chairman of the board of High Technology Invest Inc., began his career as a lecturer at The Hebrew University of Jerusalem and Tel-Hay Academic College. He received numerous awards for academic excellence, including Golda Meir's prize and the Max Shlomiok honor award of distinction. Sergey has traded options for more than 10 years and engages in creating automated systems for the algorithmic trading of options. He is the author of numerous articles

published in highly rated, peer-reviewed scientific journals. Sergey is a columnist for *Futures* magazine.

Vadim Tsudikman, president of High Technology Invest Inc., is a financial consultant and investment advisor specializing in derivatives valuation, hedging, and capital allocation in extreme market environments. With 12 years of options trading experience, he develops complex trading systems based on multicriteria analyses and genetic optimization algorithms. In co-authorship with Sergey Izraylevich, Vadim contributes articles to *Futures* magazine on the cutting-edge issues related to options pricing, volatility, and risk management.

Introduction

What the Book Is About and Who Should Read It

This book discusses the procedures of multidimensional search, selection, and utilization of potential trading opportunities existing in the options market. It contains no magic rules promising quick and guaranteed enrichment. Instead, you find comprehensive research aimed at

discovery and practical application of statistical regularities and probabilistic characteristics of option trading. The aim of our systematic approach is not the creation of ever-winning strategies. Rather we strive to implement a realistic idea—developing a system of algorithms and rules that provide you with statistical advantage over the average market participant. The trading system based on consistent application of the principles discussed in this book enables you to create and maintain positions with high (higher than the market average) expected profits and lower forecast risks.

The substantial part of this book is devoted to the problem of selection.

Statistical edge and probabilistic advantages depend on our ability to select the best variants from a great number of available alternatives. The options market is incredibly broad and diverse, whereas promising trading opportunities are rare and hard to identify. To avoid missing the chance to discover these scarce "pearls," an ample quantity of alternatives should be thoroughly estimated and analyzed. Continuous analyses of large data sets covering the entire options market is the only way to identify sparse trading opportunities that can be described as "the best of the available ones." Therefore, the issues related to the development, optimization, and practical

application of selection criteria are discussed broadly and examined in depth throughout the book.

This book is intended for you—traders, investors, portfolio managers, theoreticians, and economists—with different grounding level in options and mathematics.

If you are familiar with the basics of statistics and probability theory and have mastered the fundamentals of option trading, you can now proceed to reading this book. For those of you who have no previous experience with options but are familiar with the first two disciplines, we recommend you to start with the appendix in which we list

the main definitions and explain the notions and terms that are necessary to understand the contents of the book.

Those who are not familiar with probability theory and statistics have two options. You can start reading without delving into proofs and arguments, focusing rather on patterns and regularities described in the text and on conclusions resulting from them. In this case you must rely on the results presented by the authors and fully trust the validity of their judgments and conclusions. An alternative way, which is to dig into the basics of statistics and probability theory, can enable you to examine the material of the book critically. Even superficial knowledge of

the basics of these subjects provides an opportunity to form your own opinion on many important issues of option trading. The first way can take less of your time and effort, whereas the second one allows for getting the most out of this book.

Introduction

History of options dates back thousands of years. Social and commercial relations governed by rules similar to option terms came into existence at the dawn of human society. Various records are found in ancient documents and archeological sources dating back to the

ages of Pentateuch. In Genesis Jacob purchased an option to marry Laban's daughter Rachel in exchange for 7 years of labor. His prospective father-in-law, however, reneged, perhaps making this the first precedent of option default. Laban required Jacob to marry his older daughter Leah. Jacob obeyed his will, but because he loved Rachel, he purchased another option requiring 7 more years of labor. He exercised the second option on the expiration date and finally married his sweetheart.

Before the early 1970s the options market was poorly organized. Most transactions were executed over the counter, often through the mediation of banks or other financial institutions.

Essential terms of trade were not standardized, and in each case they were established through negotiations of the parties concerned. There was no formal and objective pricing mechanism that could be used as the starting point to determine the option premium. The watershed point happened in 1973, when two events brought about a fundamental change in the financial world. This was the year when Fischer Black and Myron Scholes published their famous option pricing model (Black & Scholes, 1973) and the Chicago Board Options Exchange (CBOE) began trading standardized option contracts. The first event provided traders with a formalized algorithm of option pricing. Despite

numerous drawbacks, this pricing model had one indisputable advantage: It enabled the comparison of market prices with a benchmark value. The second event initiated the development of an organized options market. This process is still underway today involving a growing number of investors and financial flows in option trading.

At the dawn of the new millennium, an important milestone in the development of the world derivatives market was passed. For the first time the volume of exchange traded options (having less than 30 years of history) exceeded the volume of futures traded since 1848. Since then, options have been continuously dominating among other

derivatives.

The undisputed leader in option trading is the U.S. market that absorbs more than two-thirds of the world trading volume. An important peculiarity of the U.S. options market is the competition of many exchanges offering the same product (that is, options for the same underlying asset). Although CBOE surpasses other exchanges with regard to the volume of traded contracts (approximately 30% of the total option trading volume), none of them controls more than one-third of the market. Such a competitive environment contributes to liquidity growth, spread shortening, and commission declining that attracts new market participants.

The prospects of options market development are beyond any doubt. Every year brings in additional financial flows; new trading strategies evolve; and option-based advanced structured products become more and more popular. As time goes by, the influence of large institutional investors will strengthen. In the last several years hedge funds became one of the dominating market drivers, and analysts forecast further inflow of their capital into the option trading. At the same time activity of individual investors on the derivatives markets is expected to become more intense.

The area of options application is

extremely wide. Mutual funds, banks, and investment companies use options as an instrument to regulate their investment risks. Buying Put options prevents financial institutions from liquidating long positions when they anticipate the underlying asset plunging. On the other hand, when market growth is forecast, buying Call options limits potential losses (if the forecast fails) to the premium paid for options. Buying options also creates considerable leverage adding to the investment potential and increasing the effectiveness of asset management.

Producers of various goods and consumers of raw materials use options to hedge the risks of market price

fluctuations. For example, by purchasing a Put option, an oil producer ensures that its future output will be sold at a price not lower than the option strike price. This is the way the company can be secured against a possible fall in the price of its production. On the other hand, an oil-refining company can buy a Call option for oil, thereby ensuring that its raw material will be purchased at a price not higher than the option strike price. Thus, the oil consumer can be secured against the price growth. International companies can hedge currency risks of their export/import operations by purchasing corresponding currency options.

Use of options to manage risks is called

hedging. Another area of applying options, often opposed to hedging, includes a class of speculative strategies aimed at earning profits by creating various structures composed of long and short options.

Speculative option strategies give investors broad opportunities incomparable with possibilities provided by other financial instruments in respect to their flexibility and potential profitability. The main feature of options distinguishing them from the majority of other financial instruments is the nonlinearity of their payoff function (which is the relationship between profit and the future underlying asset price). This feature enables the creation of

option combinations possessing almost any desired profit profile that makes options an indispensable instrument in achieving various goals for many financial market participants.

This book is intended for investors who strive to make profits using speculative option trading. *The principles described here can be applied to all option strategies*. Although some of them are frequently used to demonstrate the techniques of discovering the trading opportunities, whereas other strategies are not even mentioned, this selectivity is merely due to our wish to keep the text within reasonable limits.

The systematic approach presented in

this book is based on universal principles that can be applied to options on any type of underlying assets: stocks, futures, currencies, interest rates, and commodities. The same is true regarding different markets: Despite certain national specificities in legislation and regulation terms, options markets of all countries are suited for implementing the systematic approach.

To illustrate the different aspects of the systematic approach and to demonstrate its potential effectiveness in exploring the opportunities of option trading, we use historical data from U.S. exchanges. *The research described in this book is based on a database containing 7 years of price history of 2,500 stocks and*

their options.

Options: What Is Known and What Is Not

In the 1980s, when the first option exchange and the first pricing model emerged, options markets began to develop so fast that the existing theoretical background could not satisfy increasing practical needs. The facilities required to store and to process information incoming from trading floors were not yet established. As a result, statistical data processing and theoretical developments could not

satisfy growing demands of market professionals.

However, as time goes by the stream of information grows and the scope of theoretical research widens. Every year brings more and more professional publications on the subject. Options are thoroughly studied at universities, becoming one of the most popular topics of economical, mathematical, and interdisciplinary research. Option exchanges arrange seminars popularizing basic knowledge among beginners and organize advanced-level courses intended for market professionals.

A significant bulk of knowledge on options has been accumulated. These

attainments are systematized and published in popular and professional sources. The literature on options can be divided into two main categories.

The first category includes theoretical research on the basis of financial mathematics. A substantial number of scientific articles and books are devoted to the development of advanced option pricing models. They apply probability theory and discuss various complicated issues, including volatility abnormalities, nonlinearities, and interrelationships between parameters.

A strict and extensive mathematical background of option theory is given by Peter James (James, 2003). This book

represents the basis for researchers entering the world of options, though its complexity makes it comprehensible only to specialists with deep knowledge of mathematics. The basics of derivatives theory are perfectly described by John Hull (Hull, 2008). This is a textbook that covers all essential issues from basic terminology to complicated problems of financial mathematics. Option pricing is widely discussed in Espen Haug's book (Haug, 2006). It can be used as a universal handbook covering up-to-date progress in price modeling (see also Achdou and Pironneau, 2005, Rouah and Vainberg, 2007). Mathematical fundamentals of derivatives theory (not only options) are

widely covered by Salih Neftci (Neftci, 2000). Although the majority of theoretical works have not yet been implemented, some of the mathematical models are widely used by option exchanges, brokers, market-makers, and traders. The ability to apply theoretical attainments becomes increasingly essential and publications dedicated to this issue gain considerable practical value (Reehl, 2007).

Various aspects of volatility modeling and their implications on derivatives pricing were reviewed by Jim Gatheral and Nassim Taleb (Gatheral and Taleb, 2006). The authors examine all main properties of stochastic, local, and implied volatilities and describe many

classical and advanced mathematical models. A special emphasis is placed on the dynamic properties of the volatility surface and its relationship to options valuation. The discussion of volatility derivatives, barrier end exotic options is of particular interest. Besides this work, the theoretical problems of volatility modeling and forecasting were comprehensively treated by Ser-Huang Poon and Riccardo Rebonato (Poon, 2005, and Rebonato, 2004).

The second category of publications is based on practical option trading and summarizes the experience accumulated by market practitioners. It discusses strategies based on combining different options and describes methods of

building desirable profit profiles on the basis of option positions structuring (Banks and Siegel, 2007, Cohen, 2005, Cohen, 2009, Courtney, 2008, Vine, 2005). Strong emphasis is placed on methods of deriving arbitrage profit.

Lawrence McMillan is a widely known author of popular books on options. His publications (McMillan, 2002; Lehman and McMillan, 2003) include a detailed description of different option strategies and are extremely useful. The author highlights a multitude of versatile techniques indispensable for any option trader. Plentiful examples based on real market data, simple language, and broad coverage—those are the distinguishing features of his books. You can find not

only an encyclopedic review of option strategies in McMillan's books, but also a comprehensive description of delta-neutral hedging, arbitrage, and other specific techniques.

An excellent example of a handbook covering most aspects of option trading is the work by Michael Thomsett (Thomsett, 2009). An option trader can find there a detailed description of many useful strategies. The problems of return calculation and risk evaluation are also discussed in detail. The author gives much attention to technical aspects of option trading—information sources, taxation, accounting for dividends, and so on.

Books in which authors do not limit themselves to mere review of option strategies but discuss serious theoretical and practical issues without involving complicated mathematics are also helpful.

Sheldon Natenberg (Natenberg, 1994, 2007) describes the key elements of option theory in a popular and yet precise language. He discusses the peculiarities of implied volatility behavior and investigates the characteristics of the Greeks and specifics of their application as the instruments of risk analysis. Without superfluous mathematics, the author investigates such important phenomena as volatility smiles and skews.

Comprehension of complicated theoretical issues is facilitated by intelligible charts and tables.

The book by Allen Baird (Baird, 1992) targets the same audience. Being a fairly comprehensive introduction to option theory and practice, it spares the reader the wilds of complicated mathematics. Accurate description of risk management basics is among the main vantages of the book. The section devoted to the most typical mistakes made by trading beginners also deserves a special mention.

Certain books are dedicated to specific option strategies. For example, the idea of volatility trading is popularly

described in the work of Kevin Connolly (Connolly, 1997). Without resorting to complicated mathematics, the author applies dynamic hedging to combinations consisting of options and their underlying assets.

The book by Nassim Taleb (Taleb, 1997) also discusses various aspects of dynamic hedging and peculiarities of delta-neutral volatility trading strategies. This is the work written by a professional with years of experience in risk management. Although containing some inevitable portion of mathematics, it is still comprehensible to the majority of readers. In most cases the author uses diagrams and tables instead of formulae.

The book by Leonard Yates (Yates, 2003) belongs to the same category. The author discusses interesting ideas and gives ground for original trading strategies based, in particular, on negative correlation between VIX and S&P indices. The strategy is tested using historical data and the results indicate its potential applicability.

Many particular features of options trading were recently covered in impressive depth. These include pricing and risks associated with exotic options (De Weert, 2008), application of foreign exchange (Wystup, 2007), and commodity options (Garner & Brittain, 2007), trading at expiration (Augen, 2009), intraday trading (Augen, 2009),

protective strategies based on Put options, and so on.

Our knowledge on options goes beyond the literature dedicated to this narrow topic. Theory and practice of option trading apply various elaborations originating from different areas of finance, statistics, probability theory, and applied mathematics. For example, creating their classical option pricing model, Black and Scholes used the well-known lognormal distribution that was widely discussed and cited in statistical and mathematical literature. Later other authors created their own pricing models using other known distributions.

The potential benefit of adopting ideas

from adjacent scientific fields is far from being exhausted. For example, in classical option pricing theory the assumption of randomness of underlying asset price changes is the most questionable issue. Basically, it follows from applying lognormal distribution and means that the underlying asset price moves according to geometrical Brownian motion laws. The work of Edgar Peters (Peters, 1996) represents an interesting example of a more sophisticated approach to the description of price behavior. It applies chaos mathematics, fractal theory, and nonlinear dynamics to account for asset price fluctuations. Peters claims that these models describe price behavior

more accurately than standard probability distribution functions. Therefore, their application opens the gates for more accurate option price modeling. There is a lot of work to be done here, and new research of physicists and mathematicians will surely contribute to elaborating option theory.

The up-to-date achievements in the sphere of options theory can be summarized as follows. There is an adequate, albeit with certain drawbacks, option pricing model. Numerous versions of the basic model, eliminating some of its drawbacks and making the estimations more accurate, are also available. The basic principles of

creating option pricing models, based on assumptions about the main underlying asset characteristics, are reliably established. Basic option risk indicators ("the Greeks") are grasped. We know their features, interrelationships, and applicability in different situations. Various aspects of implied volatility behavior, including its dynamics, specific relationships with different parameters, and numerous anomalies, are profoundly investigated. We also possess an extensive set of advanced option strategies allowing construction of almost any desired payoff profile.

Despite this impressive progress, some important aspects still remain beyond theoretical and practical studies. Next

we summarize issues still requiring additional investigation.

The main topic of theoretical research (though directly related to investment practice) is the determination of the fair option value. The term *fair value* stands for the price that implies zero profit for both option sellers and buyers. This requires creation of realistic option pricing models (Katz and McCormick, 2005). It is common knowledge that apart from parameters that are objectively defined (current underlying asset price, strike, risk-free interest rate, and so on), the option price is determined by the forecast of underlying asset price dynamics. In the classical model this forecast is expressed by a

probability density function of lognormal distribution that is specified by two parameters: variance derived from historical volatility and mean value that is usually considered to be equal to the current price. This form of forecast has a number of drawbacks, though attempts to use other probability distributions gave only local improvements and added new drawbacks. Hence the main gap in option theory can be defined as the absence of alternative methods for creating probability forecasts of the future underlying asset price.

If the price is assumed to be a continuous value, then the forecast can take the form of probability density function. The construction of such

functions should be the principal topic of future research. We consider attempts to create one universal function for all cases to be unproductive. It should rather be a set of rules and algorithms for generating a whole class of density functions, each of which will be appropriate in certain conditions. The development of effective algorithms generating appropriate probability density functions will minimize the difference between modeled prices and fair values of options.

Apart from developing high-quality probabilistic forecasts, further research should target the development of optimization algorithms for parameters used in option pricing models. Even in

the Black-Scholes model—which is relatively simple and contains only a few parameters—the outcome strongly depends on the variance value. Historical volatility, which is usually used to derive variance, depends on the length of the historical period used for its calculation. The value of this parameter can change the modeled option price considerably. As models become more complicated, the number of parameters increases and their combined influence becomes more pronounced.

Another essential drawback of option theory consists in the insufficient development of specific risk indicators. (Some alternative indicators are

described in Izraylevich and Tsudikman, 2009d, 2010.) The majority of works on this issue are based on calculating the Greeks that are derivatives of the option price with respect to the underlying asset price (delta), volatility (vega), time (theta), and the interest rate (rho). (Derivatives of higher orders are also used.) Derivatives are calculated analytically using formulae of option pricing models. This implies that risk indicators obtained in this way inherit all the drawbacks of initial models. Such an approach to expressing option risks seems to be rather lopsided. Just as options market prices rarely match with the modeled ones, the Greeks calculated analytically almost never coincide with

real changes in option values. We believe that future research of option risk management should focus on three main issues.

The first one relates directly to the problem of improvements in pricing models. The modeling formulae should be modified to include not only high-quality probabilistic forecasts (previously mentioned) but also to enable calculation of useful indicators (derivatives or any other coefficients) that accurately reflect corresponding risks.

The second issue represents the empirical study of option price increments in response to changes of

underlying asset price, volatility, time to expiration, and risk-free interest rate. The patterns established in the course of these investigations can then be used (i) as independent risk indicators, (ii) for adjustment of the Greeks derived analytically, and (iii) to calibrate option pricing models.

The third issue corresponds to the estimation of risks of an option portfolio as a whole entity. Some risk indicators, such as theta and rho, are additive. Hence the dependence of the portfolio on time decay or interest rate change can be easily expressed as the sum of thetas or rhos of all options included in the portfolio. On the contrary, delta and vega are nonadditive. Therefore, if the

portfolio consists of options on several underlying assets, summing separate deltas and vegas is meaningless. One of the possible ways to solve this problem is to present the delta of each option as a derivative with respect to some index (such as S&P500 or NASDAQ) rather than with respect to the price of a corresponding underlying asset. (This issue is discussed in Izraylevich and Tsudikman, 2009b.) In the same way vegas of separate options can be expressed as derivatives with respect to volatility index (such as VIX or VXN) rather than with respect to volatility of a separate underlying asset. These procedures produce additive deltas and vegas that enable calculation of risk

indicators (by summation of additive deltas and vegas) characterizing the whole portfolio. Other ways to estimate risks of a complex portfolio should also be examined. Research in this field will certainly bring useful practical results.

In this review we outlined what is already known about options and how much still lies ahead of us. We defined the main lines of future research that are, in our point of view, of special interest. Some of the gaps in option knowledge are partially filled in this book.

The Concept of the Systematic Approach

This section introduces the basic concept of the systematic approach including its philosophy, objectives, and methodology. Here we overview the essence of operations required for consecutive execution of valuation, comparative analyses, and selection procedures. We strongly recommend you get acquainted with this material as it represents an all-embracing description of the general framework for systematic options trading.

The Goals and Objectives

One of the main issues in the option

trading is the problem of selecting the best variants among many available alternatives. The choice is wide and the objects to examine and assess are compound structures. Although continuous functioning in such complicated environment hampers the investment process significantly, it provides at the same time a broad spectrum of promising trading opportunities.

In the literature and in multiple services offered by brokerage firms and Internet sources, the problem of choice is generally solved through application of different market scanners and rankers. A typical scanner screens the market for underlying assets that currently have

extreme characteristics, such as divergence between historical and implied volatilities, daily volatility fluctuations, changes in trading volume, and so on. Afterward a ranker orders underlying assets according to the suitability for a particular option strategy. Then suitable combinations should be designed for all chosen underlying assets. Because a great number of combinations can be constructed for a given underlying asset within a given strategy, it is usually advised to use combinations' payoff charts (the functional relationship between the price of the underlying asset and a combination's profit estimated for a certain future date) as a basis for their

comparison and decision making. However, in most cases visual analysis is quite unfeasible if a large quantity of option strategies and underlying assets have to be compared simultaneously.

We regard the choice of suitable underlying assets for the *a priori* defined strategy as a differential approach. It is a forced measure resulting from the imperfection of analytical tools limited to simple scanning and visual analysis of payoff functions. Differential selection deprives the investor of the potential to utilize the whole spectrum of various trading opportunities provided by the market completely and effectively.

What we oppose to a differential

approach is an integral systematic approach based on the strictly formalized assessment criteria, universal procedures of multicriteria analysis, and well-structured selection algorithms. The systematic approach enables simultaneous processing of a considerable number of option strategies and underlying assets. Without such an integral system, the investor has little or even no chance to make prompt selection decisions and to adapt successfully to changing market conditions.

The main goal of the systematic approach is to create a complex portfolio containing a potentially unlimited number of option combinations corresponding to a variety of strategies

and underlying assets. Its application ensures that all trading opportunities appearing at any particular moment will be thoroughly estimated and none of the variants worth considering will be omitted. The systematic approach is absolutely indispensable for turning option trading into a long-term continuous process of income generation with controllable parameters of risk and profitability.

Valuation, Comparative Analysis, and Selection

The systematic approach is realized

through consecutive execution of the following procedures: valuation, comparative analyses, and selection. These procedures are applied to the multitude of option combinations. The combination represents a complex structure consisting of any number of long and/or short individual options corresponding to certain underlying assets. Each option combination can be characterized by the shape of its payoff function. When referring to the *option trading strategy*, we will imply a certain definitive shape of the payoff function that is inherent to all combinations belonging to the same strategy and that is qualitatively distinguishable from payoff functions of

combinations not belonging to this strategy. The set of option combinations available at any given moment in time for valuation, analyses, and selection of promising trading opportunities will be referred to as the *initial set*.

Valuation

Option combinations are valuated through the application of strictly formalized criteria developed specially for this purpose and expressing potential profitability and risk of the assessed variants in different ways. Criteria represent mathematical constructions with different degrees of complexity and

one or many parameters. Optimization of parameters is performed either by means of statistical analyses of historical time series or by expert forecasts. Because parameters optimized on historical data are inclined to suffer from the disadvantage of curve fitting, close attention should be paid to the validity of statistical patterns used to determine their optimal values. Expert forecasts also have significant drawbacks because they reflect opinions of particular specialists and thus represent rather subjective estimates. A systematic approach, applying both statistical analyses and expert forecasts, allows diminishing their drawbacks while amplifying advantages of these two

parameterization methods.

Development of sophisticated criteria capable of valuating option combinations adequately, and optimization of their parameters, are the crucial issues that determine the practical success of systematic approach. The first part of this book discusses the basic principles of criteria construction and parameterization; the main criteria are described and analyzed in detail.

After being valuated by criteria, every combination receives a numerical characteristic reflecting its investment attractiveness. Option combinations can be valuated by one or several criteria. In

the latter case the number of characteristics attributed to each combination is equal to the number of criteria.

Comparative Analysis

Following the completion of the valuation stage, the characteristics attributed to combinations need to be analyzed. During the analysis every combination is compared with all the others according to their characteristics. As a result, all variants constituting the initial set are ordered according to their quality indicators.

If the valuation was based on several criteria, then the analysis generates several orderings. In this case the same combination can have different positions in different orderings. For example, a combination can be the best one according to its expected profit, but at the same time it can be at the end of the list in an ordering obtained by the application of some risk-related criterion. Subsequently all orderings can be either used separately or combined into a unified one.

The unified ordering can be either complete or partial. Usually partial ordering appears when a complete one is unachievable. This may happen if some items turn out to be incomparable

by certain criteria or if they are valued differently according to different criteria. In such cases the entire set of alternatives is divided into groups, and these groups are consequently ordered as joint entities. Different methods appropriate for execution of such procedures are discussed in Chapter 7, "Basic Concepts of Multicriteria Selection as Applied to Option Combinations."

Selection

At the next stage the results of the comparative analysis are used to select a

limited number of combinations possessing superior quality characteristics. This procedure needs to be arranged thoroughly because it leads to the irreversible decision as to which combinations will enter the portfolio and which ones will be rejected.

You need to consider three main principles when choosing combinations suitable for inclusion into the portfolio.

- The number of combinations selected should be large enough to maintain diversification of the portfolio above some minimum level. Like in the classic portfolio theory, it minimizes specific risks related to individual underlying

assets.

- Criteria values of selected combinations should exceed the values of the rejected ones. The minimal threshold for this excess should be established for each particular situation.
- The relative superiority of some combinations over others (resulting from the comparison of their criteria values) should not be considered as the sufficient reason for selection. The absolute criteria values must also be taken into account. For example, between two combinations with an expected return of –$2 and –

$10, the first variant is preferable and, in principle, can be selected as the one with relatively better characteristics. However, the absolute value of the expected return corresponding to the first combination is negative and hence this combination, just as the second one, cannot be selected to enter the portfolio.

In practice, however, these principles contradict one another. Thus, following the second and the third principles an investor endeavors to decrease the number of combinations selected. At the same time the principle of portfolio diversification induces the opposite tendency—to increase this number.

Thereby the structure of the resulting portfolio represents a compromise (trade-off) between all three principles.

The selection procedure represents a set of rules determining how to draw the line separating potentially profitable combinations from those that lack such potential. Consider a simple situation: The initial set consists of *N* combinations ordered according to the values of a certain criterion; the investor must select *N'* best variants out of *N* alternatives. This problem may be solved by creating one or several utility functions. The argument of such function is the number of selected combinations (numerical value of the place occupied by the last selected combination in the

ordering). The value of the utility function is an indicator reflecting the measure of utility arising from the selection of this particular number of combinations. In other words, the utility function may be defined as the relationship between an average return (the maximum drawdown, the Sharpe ratio, or any other characteristic reflecting the investor's satisfaction) and the number of combinations selected.

Analytical methods are not applicable to the majority of utility functions because no formulae establish the relations between the value and the argument of these functions. Hence the values of utility functions are usually derived empirically from historical time series

using different statistical techniques.

If several utility functions are used simultaneously, they need to be combined into one unified function. Such unification is possible because all utility functions have the same argument (the number of selected combinations). The main requirement for the methods used to combine different utility functions is the unambiguity of the outcome that must be consistently interpretable. It means that the resultant function should be unimodal with a single evident maximum corresponding to the optimal number of combinations to be selected. Statistics offers several methods to combine empirical functions; the most popular are multiplicative and additive

convolutions. We have developed an additional method—a minimax convolution (see Chapter 5, "Selection of Option Strategies") that in most cases brings more reliable and unambiguous results.

Sequence of Operations, Notion of a "Matrix" and Its Reduction

The initial set consists of a huge number of option combinations that must be processed during the execution of valuation, analyses, and selection procedures. Suppose that at any time

moment there are m_i options traded for every underlying asset i. Assuming that any option can either be absent or present in the combination (in the latter case it can be either long or short) and that the proportion of different options is the same in all combinations, the number of possible combinations for one underlying asset is determined as 3^{m_i}. Accordingly, the total number of combinations for n underlying assets can be estimated as follows:

$$combinations = \sum_{i=1}^{n} 3^{m_i}$$

Even if only 1,000 underlying assets are available for trading, and on average

only 20 different options are traded for every underlying asset, then the procedures of valuation, analyses, and selection will cover more than 300 billion combinations! Moreover, the possibility to use unequal proportions of different options within one combination —which is quite realistic—generates a truly enormous number of variants to process. This number is so gigantic that computational procedures become unrealizable even for the most advanced computer hardware. Therefore, the initial set needs to be decreased to some reasonable quantities that are possible to work with on personal computers. You can achieve this decrease through the creation of combination-generating

algorithms that produce only potentially appropriate combinations instead of generating all possible variants. (Their appropriateness is determined by specific requirements and limitations of the particular investor.)

First, the investor must decide what strategies to use and then generate combinations corresponding exclusively to these strategies. This can significantly decrease the number of variants in the initial set. Then additional reasonable limitations should be applied within every strategy. For example, the following limits can be used for the *short strangle strategy*: The strike of the Call option must be greater than the strike of the Put option; the difference

between Call and Put strikes must not exceed 25% of the underlying asset's price; the ratio of Put options to Call options must be between 0.8 and 1.2. Such limits, on the one hand, are well founded and, on the other hand, they do not prevent an investor from taking full advantage of the majority of opportunities appearing in the options market. At the same time, these limitations reduce the initial set to such an extent that makes it processible for personal computers.

Further facilitation of computational procedures can be achieved if selection is realized as a series of consecutive subselections. We propose to adhere strictly to the following sequence of

operations. Before initiating any selection procedure, a range of potentially suitable underlying assets and trading strategies should be determined. At the same time the algorithms used to generate option combinations must be established. After that the initial set of variants available for trading can be represented as a three-dimensional space ({underlying assets × strategies × combinations}) on which the consecutive subselection procedures are executed.

If the algorithms used to generate option combinations allow creating only one combination for every underlying asset within every strategy (that is, one single combination corresponds to each

{underlying assets × strategies}), then the three-dimensional space of the initial set turns into a two-dimensional space. The two-dimensional initial set can be visualized as a table with lines corresponding to underlying assets and columns—to strategies. Each cell of this table contains one option combination relating to a given underlying asset and to a certain strategy. Such a table can further be referred to as a *two-dimensional matrix*.

If several option combinations are created for every underlying asset within every strategy, then each cell in the table will contain more than one combination for any {underlying assets × strategies}. In this case the two-dimensional matrix

becomes a three-dimensional one. (Its elements form the initial set.)

The procedure of selection can be viewed as a *reduction of the three-dimensional matrix*. We propose to realize it as a sequence of three consecutive operations (each of which can be regarded as a subselection).

The first operation represents selection of one or several best option combinations corresponding to a specific underlying asset and a given strategy. Suppose that the initial set consists of 30,000 elements (1,000 underlying assets, 5 strategies, and 6 combinations for every {underlying assets × strategies}). If during the first

subselection procedure only one combination is chosen out of the 6 possibilities, then the initial set of 30,000 items decreases to 5,000 and the three-dimensional matrix becomes two-dimensional. Chapter 4, "Selection of Option Combinations," discusses the methodology of this operation.

The second operation consists of choosing one or several superior option trading strategies for every underlying asset. In our example this leads to a further decrease of the initial set. If only one strategy is chosen, the initial set declines to 1,000 combinations. The result of this operation is the ultimate reduction of the matrix because after the execution of this subselection a unique

list of underlying assets corresponds to every strategy. Therefore, the remaining part of the initial set cannot anymore be presented as an entire table without gaps. This operation is discussed in Chapter 5.

The third operation is intended to select the best variants from the lists of underlying assets corresponding to every strategy. If this procedure selects approximately 10% of combinations out of those that were chosen during two previous operations (we assume this percentage as an average estimate though in practice it can vary substantially), then the initial set is finally reduced to just 100 variants. Chapter 6, "Selection of Underlying Assets," describes this

operation in detail.

This sequence of operations represents only one possible way to perform the procedures of valuation, analyses, and selection. Some other, more complicated approaches to the reduction of the initial set matrix can be developed. However, the scope of this book is limited to the preceding scheme because even such a relatively simple algorithm has more than enough particular features and specific peculiarities.

Overview of Trading Opportunities

This book covers various aspects of dealing with trading opportunities provided by options: from their detection and investigation to selection and deriving profits. This statement *a priori* assumes the existence of trading opportunities. Although this is quite obvious to the authors, it would be more appropriate to demonstrate the permanent presence of various trading opportunities in the options market. Besides, it is also useful to investigate their dynamics and structure.

The aim of this section is to perform the statistical investigation of trading opportunities existing at different moments in time for various underlying assets. An overview of trading

opportunities is also expedient because it provides you with analytical tools for evaluating the potential profitability of different options markets and underlying asset types.

What Is a Trading Opportunity?

A *trading opportunity* is the deviation of the market price of an option (or any option combination) from its fair value. The *fair value* is the price that implies zero profit for both the seller and the buyer of the option. This interpretation is related to the efficient market hypothesis

stating that any new information is immediately priced in and hence all traded assets are fairly valued, and extracting profit is impossible neither for sellers nor for buyers. It is common knowledge that the efficient market concept is an idealization unachievable at present days. Financial markets are ineffective; asset prices constantly fluctuate and deviate from their fair value thereby creating various trading opportunities.

Because we define the trading opportunity as the difference between the market price and the fair value, we need to establish the algorithms to estimate these variables.

At each moment *the market price* is characterized by three indicators: last, bid, and ask prices. The first indicator is of little importance because it deals with the past whereas we are interested in current trading opportunities. To discover them it is preferable to use bid and ask prices. If the investor prefers to be on the conservative side, the worst of these two prices should be used (that is, ask price—when buying options, and bid price—when selling them). This approach decreases the probability of a mistake but reduces the number of trading opportunities considerably. The less conservative investor can use the combination of bid and ask prices—their simple average or weighted average

with different weights for the best and the worst price. In this case the probability of erroneous inclusion of the option into the category of “trading opportunities” is higher, but the sample is more representative.

Estimating *the fair value* of any asset is an extremely difficult task, and options are not an exception. Their distinctive feature is that, in contrast to other financial instruments, options have expiration dates. This enables us to evaluate the accuracy of the fair value estimate in a reasonably short time period. The common way to estimate the fair value of options is the Black-Scholes formula and other similar models. However, they have a number of

significant drawbacks and cannot be used to obtain the fair values suitable for the estimation of trading opportunity. That is why we use another method to get more accurate fair value estimates.

Method for the Evaluation of Trading Opportunities

The quantitative expression of trading opportunities can be obtained by subtracting the fair value of an option from its current market price. For comparability of results the difference should be normalized by the strike price (allowing us to express the differences

between fair and market prices in percentage).

We assume the market price of an option to be equal to the average between the bid and the ask prices. The option fair value can be calculated using the method proposed by Ralph Vince (Vince, 1992):

$$\text{Option fair value} = \sum_{i=1}^{N} (p_i a_i),$$

where p_i is the probability of outcome i,

a_i is the profit or loss of outcome i, and

N is the number of possible outcomes.

To obtain the most accurate estimate of the fair value, Vince permits looking into

the future. Knowing the underlying asset price at the expiration date, we can find out the exact fair value of any option. In this case we have the only outcome (N=1) with probability $p_i = 1$. Accordingly, the fair value of the option is a_i.

For the Call option a_i is equal to the difference between the underlying asset price (UAP) at the expiration date and the strike price (SP) if UAP > SP, otherwise $a_i = 0$. For the Put option a_i is equal to the difference between the SP and the UAP at the expiration date if UAP < SP, otherwise $a_i = 0$. To be fully accurate, we need to discount this fair value by the risk-free interest rate

normalized by the time to expiration. However, within the framework of current research, this correction can be neglected.

Zero (or close to zero) difference between the market price and the fair value indicates absence of trading opportunities. The positive difference indicates that the option is overvalued and there is a trading opportunity to sell it. Similarly, the negative difference indicates that the option is undervalued and there is a trading strike price opportunity to buy it.

To avoid zero fair values we evaluated trading opportunities of simple option combinations (straddles) rather than of

separate options. The market prices and the fair value of combinations have been calculated for each of the 2,500 stocks and each date of the period from January 2, 2001, to August 16, 2007. Straddles were created using contracts with the nearest expiration date and strike prices closest to the current underlying price. Thus we obtained a table of 2,500 lines (according to the number of stocks) and 1,564 columns (the number of dates). Each cell of this table contains the value of the difference between the market price and the fair value of the combination corresponding to a certain date and to a given stock. In total, we calculated 3,910,000 values characterizing the presence or absence

of trading opportunities.

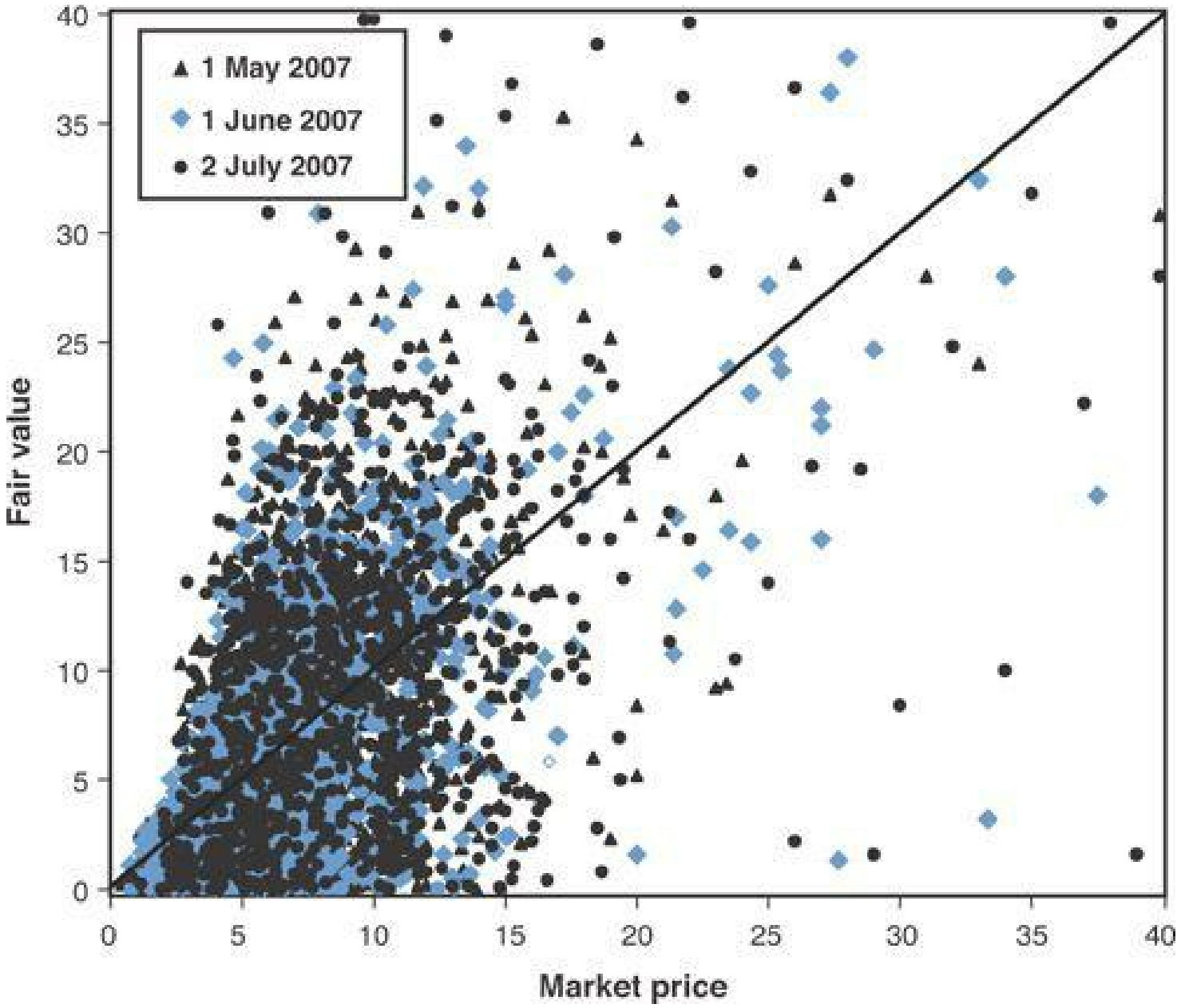
Structure and Dynamics of Trading Opportunities

To demonstrate the intraday structure of strike price trading opportunities existing in the options market, we arbitrarily chose several dates (May 1, June 1, July 2, 2007) and analyzed the deviations between the market and the fair values on these days. The quantity and quality of trading opportunities are vividly depicted in Figure I.1 with the market price of a combination on one axis and its fair value on another. Each

point on the figure relates to the combination corresponding to a certain underlying asset.

Figure I.1. Relation between market price and fair value of straddles observed as of May 1, June 1, and July 2, 2007, for 2,500 stocks. The line separates overvalued (points above it) and undervalued (points below it) combinations. All values are expressed as the

percentage of the strike price.



On the whole we can observe a firm

relation between the two indicators (R^2 = 0.32). Points at the line relate to combinations with no trading opportunities. Points to the north of the line represent overvalued combinations. Undervalued combinations are represented by points under the line. The scattering pattern of points in Figure I.1 indicates the presence of a considerable number of trading opportunities. Both overvalued and undervalued combinations are observed in large quantities. However, the extent to which they are over- or undervalued varies widely. Although many points are not situated exactly on the separating line, they are still very close to it, meaning that the trading opportunities in these

cases are negligible.

We propose the following heuristic rule to separate combinations with trading opportunities from others that do not possess any trading potential or only have an insignificant one. Combinations with a difference between the market price and the fair value of no more than one percent (that is, within the range of –1% to 1%) are considered as lacking trading opportunities. Combinations with a difference that is outside this range are considered as having trading opportunities. Combinations with a difference of >1% are overvalued; combinations with a difference of <–1% are undervalued. Based on this classification we can analyze data

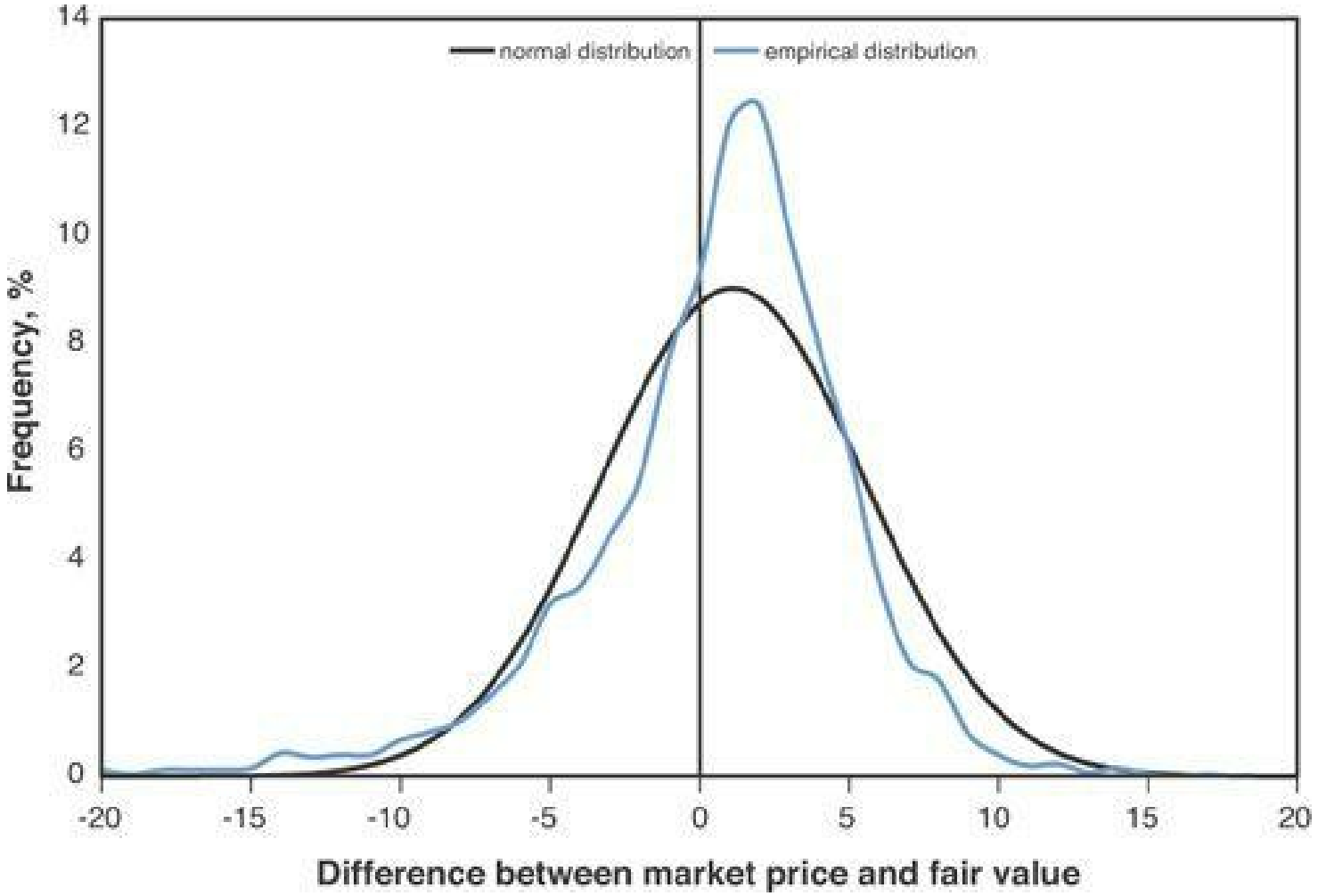
presented in Figure I.1. (Only one date is featured in the following discussion because there is no significant difference between different days.)

The best way to represent the structure of trading opportunities is to build a frequency distribution of the differences between market and fair prices (see Figure I.2). The distribution of trading opportunities existing as of July 2, 2007, is characterized by the skew toward positive values. This indicates that overvalued combinations prevailed over undervalued ones. Only 19% of combinations fall into the –1% to 1% interval that we consider to be the range with negligible trading opportunities. Fifty-two percent of combinations are

overvalued and 29% are undervalued. This means that more than 80% of stocks had the potential of realizing either short- or long-straddle strategies. Short positions could be opened for more than half of the combinations, whereas long straddles turned out to be profitable in slightly less than one-third of all cases.

Figure I.2. Two distributions of differences between market prices and fair values of straddles observed on July 2, 2007, for 2,500 stocks. Prices and

differences are expressed as the percentage of the strike price. Positive differences correspond to overvalued straddles, negative differences—to undervalued straddles.



The frequency distribution of differences between market and fair values deviates from normal distribution considerably (see Figure I.2). Small differences corresponding to the absence of trading opportunities are more frequent than it is expected under the normal distribution. Medium differences are observed less

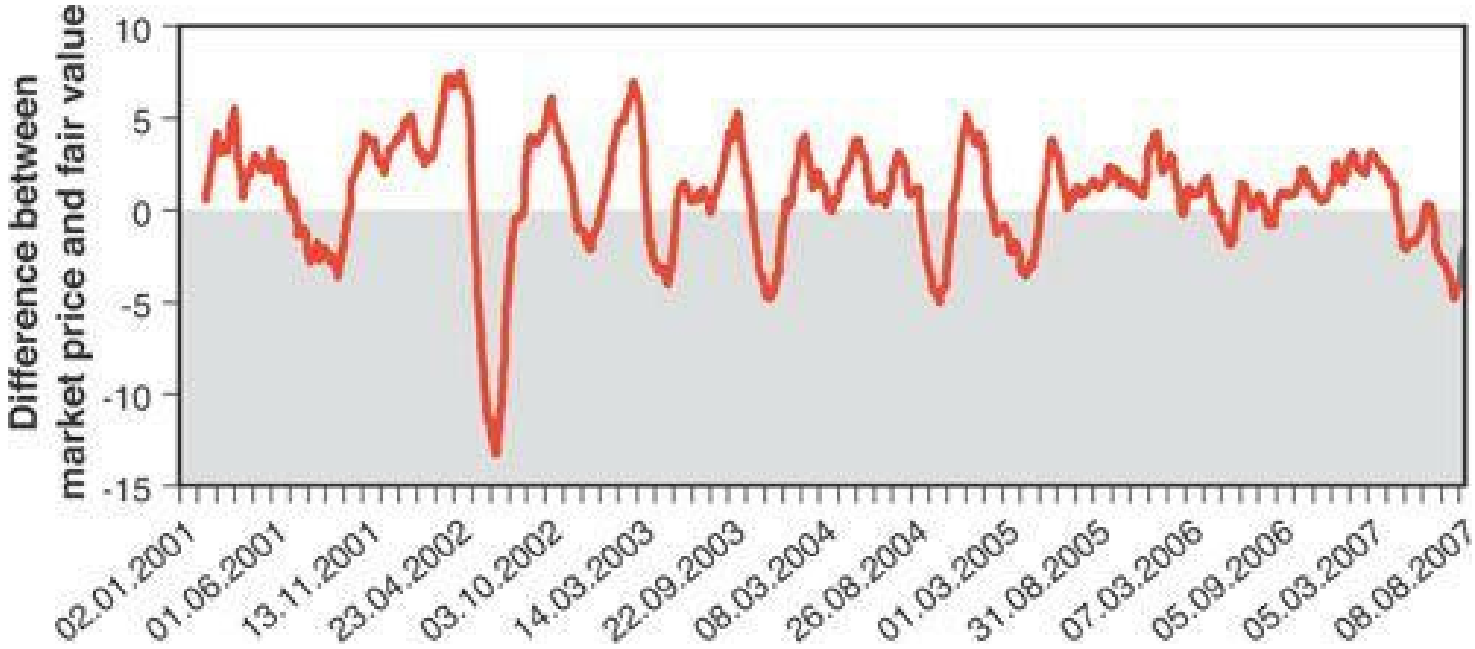
frequently than under normal distribution. Comparison of two distributions reveals asymmetry in distribution of trading opportunities. Moderately overvalued combinations are more frequent than under normal distribution whereas moderately undervalued combinations are less frequent. However, the situation with the distribution of big differences is the contrary—highly undervalued combinations (left tail of the distribution) are more frequent than highly overvalued ones (right tail of the distribution) (see Figure I.2).

So far we have been analyzing the distribution of trading opportunities between different underlying assets (to

be more exact, between combinations corresponding to these assets) within one day. At the next stage the time dynamics of trading opportunities will be considered for separate underlyings. We begin with one stock (AAPL will be used as an example) and calculate the difference between market prices and fair values of straddles for all dates within the period from January 2001 until August 2007. Figure I.3 shows these differences plotted against the corresponding dates. Visual analysis of these data reveals that undervalued periods alternate with overvalued periods. In general, we can say that the dynamics of this process is characterized by quasiperiodical cycles.

Although at first sight these cycles have similar periodicity, their detailed investigation suggests that trading opportunities can hardly be forecasted on their basis.

Figure I.3. Dynamics of the differences between the market price and fair value of straddles for AAPL stock. Prices and differences are expressed as the percentage of the strike price.



Using AAPL as an example we illustrated the dynamics of trading opportunities for just one underlying asset (Figure I.3). However, the analysis of other stocks (not presented here) gives similar results. The overwhelming majority of underlying assets shows similar behavior—more or less regular fluctuations between overvalued and undervalued areas. As in the AAPL case, there are periods when trading

opportunities are negligibly small.

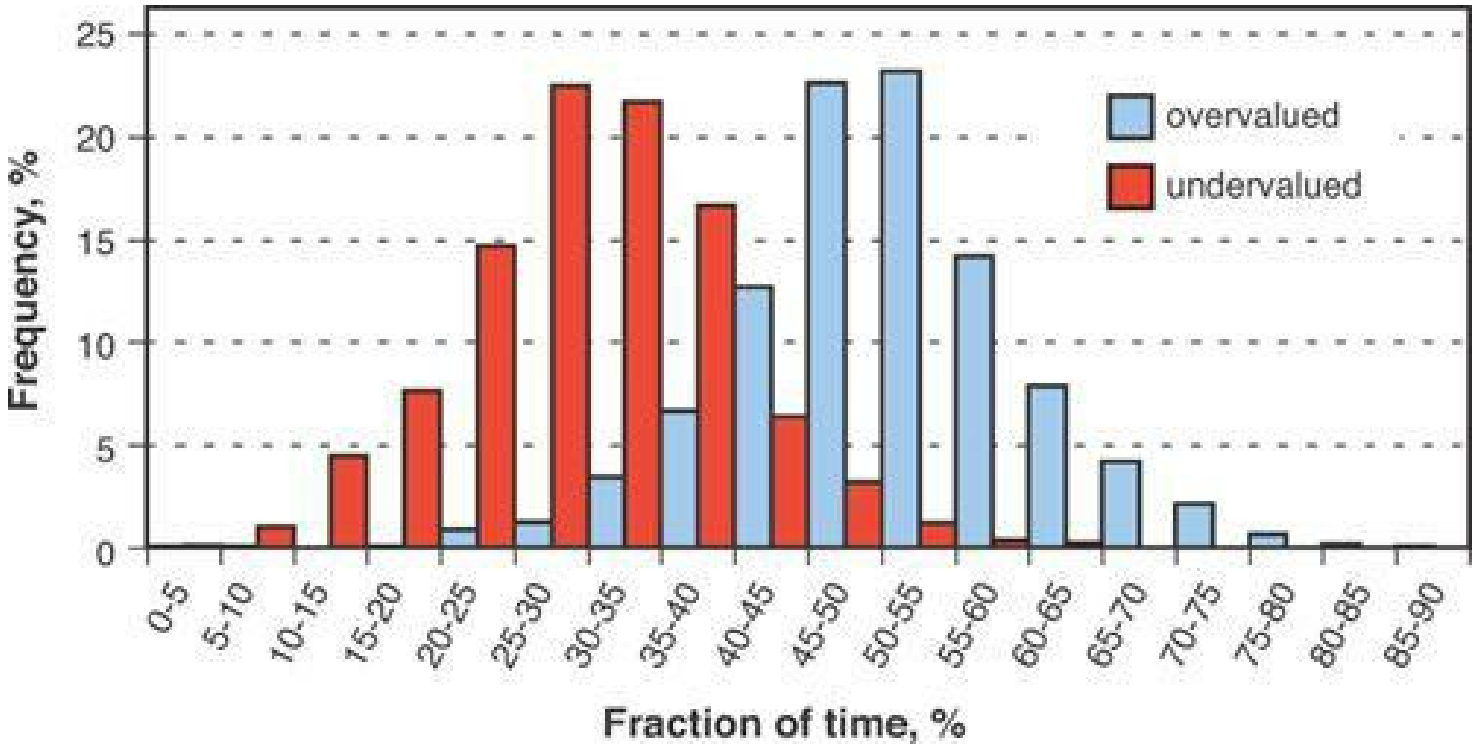
Previously we mentioned that the intensity of trading opportunities can vary widely. Although we agreed to consider the difference between the market and the fair values of more than 1% as indicating the presence of a trading opportunity, the profit potential may be quite low if the differences exceed 1% by just a slight margin. On the other hand, the difference of 5% and over has a strong profit potential. As it follows from Figure I.3 (and other research not presented here), medium trading opportunities (with a profit potential of approximately 2% to 4%) are prevailing in the market. However, it should not have a negative impact on our

evaluation of trading opportunities because these medium deviations of market prices from their fair values occur quite frequently.

The dynamics of trading opportunities analyzed by the example of AAPL indicates the approximate equality of periods when the options of a certain underlying asset are overvalued and when they are undervalued. Does such uniform distribution of trading opportunities (between over- and undervalued periods) reside in all stocks? Are there underlying assets with options that are undervalued or overvalued most of the time? To answer these questions we calculated the number of days when options were

overvalued and undervalued for each of the 2,500 stocks. We divided the obtained values by 1,564 (the total number of days) to express them as the percentage fraction of time. This data was used to build two distributions of time fractions: when options were overvalued and when they were undervalued (see Figure I.4).

Figure I.4. Distribution of time fractions when options were undervalued and overvalued.



Two distributions are shifted relative to each other: overvalued—toward the longer fractions of time, undervalued—toward the shorter fractions of time (see Figure I.4). This means that options are more often overvalued than undervalued. In general, we can conclude that most of the options are undervalued for no more than 25% to 40% of the time and are overvalued for 45% to 60% of the time

(see Figure I.4). Moreover, options relating to approximately 5% of stocks are overvalued for more than 70% of the time. On the other hand, options relating to only 3% of stocks are undervalued for just 50 % to 60% of the time. This means that options of some stocks are permanently overvalued, whereas constantly undervalued options are relatively rare.

As shown in Figure I.1, a multitude of trading opportunities consisting of both undervalued and overvalued options exists in the market simultaneously. To get a detailed notion of their dynamic structure, we can analyze the proportions of combinations possessing trading potential and those lacking it. This

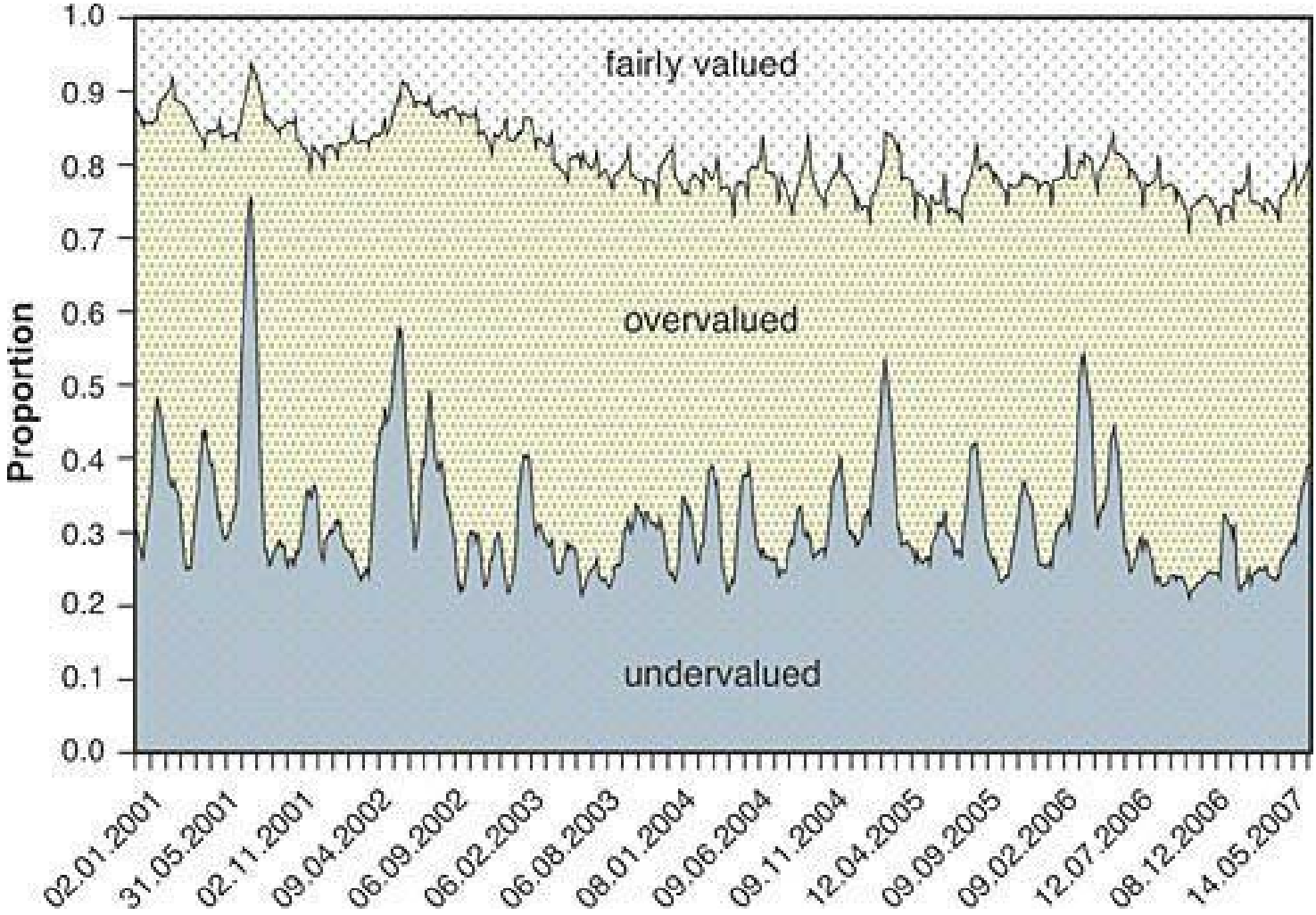
investigation demonstrates how the ratio of overvalued, undervalued, and fairly valued combinations changes in time. To clarify these issues we use 2,500 straddles (one for every underlying asset) for each of the 1,564 dates. For every combination we calculate the difference between the market price and its fair value. Based on this indicator, the straddles are classified into three categories in accordance with the heuristic rule previously proposed. Combinations with a difference of >1% are considered to be overvalued; those with a difference of <–1% are undervalued. Combinations with a difference between –1% and 1% are considered to be fairly valued (that is,

lacking any trading opportunities). For each date we calculate the proportion of combinations belonging to each of the three categories and observe the dynamics of their changes in time.

The proportion of fairly valued combinations is relatively stable in time —it fluctuates slightly within the range from 10% to 20% (see Figure I.5). Overvalued combinations prevail over undervalued ones. The former constitute approximately 50% to 60% throughout most of the time, whereas there are only 30% to 40% of undervalued combinations. (This corresponds with the conclusions drawn from the analysis in Figure I.5.) At the same time there are periods when the proportion of

undervalued combinations rises sharply (see Figure I.5).

Figure I.5. Dynamics of proportions of undervalued, overvalued, and fairly valued combinations.



The following conclusions can be drawn from this statistical investigation. Considerable trading opportunities consisting of overvalued and undervalued options are constantly available in the market. However, their relative ratios have complicated time dynamics that can hardly be forecasted

by discovering persistent cycles with regular periodicity. Consequently, an accurate prediction of future trading opportunities—whether most of the options will be overvalued or undervalued—seems to be unfeasible. Therefore, at every moment in time the investor should determine the trading potential of each separate combination and create corresponding strategies based on selling overvalued, buying undervalued, and excluding fairly valued options and their combinations.

Part I. Criteria as the Basis of a Systematic Approach

The first part of this book is dedicated entirely to the presentation of the basic tools required for systematic options trading—the diversified criteria intended for evaluation, comparison, and selection of options and their combinations.

In Chapter 1, “General Presentation and Review of Criteria Properties,” we consider the philosophy and main principles of criteria construction. The criteria are then classified by the extent of their universality and according to specific forecasting properties.

Chapter 2, “Review of the Main Criteria,” systematically overviews the main criteria required for realization of successful option trading. For each criterion we explain its statistical and probabilistic grounds and provide a comprehensive description of the computational algorithm. We also itemize all parameters relevant for every criterion and indicate the peculiarities of

their optimization. Special emphasis is placed on the conversion of forecasts provided by fundamental analysts into strictly formalized criteria. We describe the technique allowing combining different probability functions to create compound criteria benefiting from both expert and statistical approaches. We discuss the specific qualities of every criterion, including its applicability to different option strategies.

The forecasting quality of any criterion can vary significantly depending on the specific circumstances of its application and on prevailing market conditions. Chapter 3, "Evaluation of Criteria Effectiveness," is dedicated to evaluation of criteria effectiveness. We

describe different peculiarities of this analysis and indicate essential procedures which have to be executed to obtain unbiased estimation of criteria predictive power. The last section of this chapter presents seven indicators designed for evaluation of criteria effectiveness.

chapter 1. General Presentation and Review of Criteria Properties

1.1 The Main Tool for Solving the Selection Problem

At any given moment the market contains

a considerable number of trading alternatives existing in the form of option combinations that correspond to various strategies and different underlying assets. Within the concept of the systematic approach, we present this set (called “the initial set”) as a matrix with lines corresponding to underlying assets and columns—to option strategies. One matrix cell may contain several combinations corresponding to the same strategy and underlying asset. In this case the two-dimensional matrix turns into a three-dimensional one. What possible number of elements can be contained in this matrix at a certain point in time?

Consider a simplified situation: The

initial set is limited to combinations corresponding to 100 underlying assets (for example, stocks of S&P 100 Index) and to only four option strategies: short strangle/straddle and long strangle/straddle. We further assume that within every strategy the procedure of combinations creation is predefined (that is, the combinations generation algorithms are fixed). For simplicity sake, this procedure is assumed to be the same for all strategies. For instance, only option series with the nearest expiration date are used, and only contracts with strikes that differ from the current stock price by no more than 20% are accepted. The algorithm of combinations creation also complies

with the following restrictions: Within each combination the Put to Call ratio is 1:1, and the Put's strike does not exceed the Call's strike price.

Following the preceding rules, we limit the potential initial set considerably. However, even such an artificially reduced set contains a significant number of trading variants. We illustrate this using the AAPL stock as an example. On February 1, 2007, the stock price was $84.74; 20% of this equals $16.95, hence, only strikes within the range from $67.78 (84.74 – 16.95) to $101.69 (84.74 + 16.95) meet our requirements. This range contains six strikes from $70 to $100 with a $5 interval. Therefore, according to our algorithm, the AAPL

stock generates 21 combinations (6 straddles and 15 strangles) for the long strategy and the same number for the short strategy. The same procedure for YHOO (the price on February 1, 2007 was $28.35) gives 10 combinations.

Suppose that each of the 100 stocks belonging to the S&P 100 Index generates on average 10 combinations (the minimal value obtained in the preceding example) corresponding to two short strategies and the same amount corresponding to two long strategies. Then the three-dimensional matrix that is the initial set used to select the best trading alternatives contains 2,000 elements! Remember that we deliberately limited ourselves to just

100 stocks; only four option strategies were allowed; and a conservative algorithm of combinations creation was designed. If we consider other option strategies, additional underlying assets and more diverse combination creation algorithms, the number of elements included in the initial set would increase dramatically.

The U.S. market alone contains several thousand stocks with more or less actively traded options. Any reference book on the basics of option trading describes at least a dozen simple strategies. Taking into account that most optionable stocks have dozens of simultaneously traded option contracts (with different strikes and expiration

dates), we conclude that an average initial set matrix can contain millions of combinations! And every combination has it own payoff function, premium, margin requirements, and risk characteristics.

Although such diversity complicates the selection, it provides a chance to discover trading opportunities arising permanently in the option market (see the "Overview of Trading Opportunities" section in the Introduction). The method for revealing these situations is not trivial and requires combining many elements of the systematic approach.

How can we select a limited number of combinations possessing high potential

profit and low estimated risk from a large set of alternatives? Obviously, even a most experienced expert is unable to compare all variants without using formalized evaluation algorithms. Because the market situation is permanently changing, the decisions must be taken promptly in real-time mode. Selection of superior combinations is a complicated task that can be solved only by applying a system of strictly defined and thoroughly formalized criteria. Their qualified development, optimization, and application are indispensable conditions for the successful solution of the selection problem.

1.2 Formal Definition

The main task of the systematic approach consists in the selection of a subset meeting a certain criterion (or a number of criteria) out of the initial set of option combinations. As a result, the selected elements become preferable to the rejected variants.

To solve the selection problem, we need to define the set from which the preferred elements will be chosen and the criteria to which these elements have to correspond. The problem of defining the initial set was briefly discussed (when describing the concept of the

systematic approach). In this chapter we focus on criteria because their successful functioning is an indispensable condition for spotting favorable trading opportunities.

We postulate as a working definition that *a criterion is any numerical indicator suitable for evaluation and comparison of individual options and their combinations*. (Next we generalize and formalize this definition.) There are numerous areas of criteria application. They can be used to compare combinations for the same underlying asset to select the best one under the defined strategy, or to compare combinations belonging to different option strategies, or to compare

underlying assets as such to select the most suitable one for a certain strategy.

The procedure of criteria application implies the calculation of specially designed functionals for all the elements of the initial set. *An individual functional, which represents a unique combination of computational algorithms and mathematical functions, is called a criterion.* The argument of the criterion is an option combination, whereas its value is a real number. Criteria are used in this book as the main instrument of evaluation of the potential quality of option combinations.

In the next chapter we demonstrate how various principles of option

combinations evaluation can be used to derive criteria. To keep things simple we postulate the following general principle of criteria creation: The higher the criterion value, the better the profit potential of any given option combination. If this principle is observed, the selection problem is formally defined. There is a set of option combinations evaluated using a number of criteria. Based on these criteria values we have to select a subset of combinations with the best characteristics of future profit and risk.

1.3 Philosophy of Criteria Creation

The option price is largely determined by the uncertainty of future events. If we knew the future underlying asset price, the corresponding option price would be determined unambiguously by the difference between the future asset price and option strike (at least in the case of European options). The more uncertain the future, the more expensive the options become. Strictly speaking, the option price is determined by the distribution of probabilities assigned (by the person performing the option valuation) to all possible price outcomes of its underlying asset.

Let us assume that the stock is worth $50 today and the investor estimates it can

take any value from $0 to $100 (with a step of $1) in the future with equal probability for each possible value. Such probability distribution reflects a high uncertainty level and, hence, options must be quite expensive regardless of the strike value. Now assume that the investor assigns the highest probability (say, 60%) to the current price, 20% probability to the adjacent prices (49 and 51), and 0% probability to all other price values. This distribution indicates a high level of certainty. Now Call options with strikes from 52 to 100 will cost nothing. The price for Calls with lower strikes can be approximated by the following formula: 50 – strike + small premium for

uncertainty. In this case, the uncertainty fee (usually called *risk premium* or *time value*) will be low, because the author of probabilities distribution is sure that the underlying asset price will be within the narrow range between 49 and 51. There is always a strong relationship between the uncertainty level and the option value. This simplified example illustrates the essence of options valuation by a particular market participant.

This scheme represents the principle of option pricing by one particular investor; however, the whole market consists of a large number of participants. All of them evaluate options on the basis of different

probability distributions. Furthermore, even investors with a similar estimate of probabilities of future events can still have different notions about what risk premium corresponds to their uncertainty levels. If one imagines the relationship between the uncertainty and the option expensiveness as a line, its shape will characterize one single investor. The whole market is formed by averaging a large number of such relationships.

The average uncertainty level of all market participants can be derived from the market prices of options. We call it *common uncertainty*. Common uncertainty is frequently expressed by implied volatility that is derived from the Black-Scholes formula (or other

option pricing models) by substituting market prices of options there. Apart from this method of estimating common uncertainty, it is possible to develop other indicators that express it differently and are not burdened with option pricing models drawbacks.

We call the uncertainty level of one particular market participant (the investor) *individual uncertainty*. Although frequently derived from historical volatility, it can be expressed in a number of different ways. The drawback of the historical volatility method lies in the fact that the investor fully relies on past market events that do not necessarily correspond to future developments.

The interrelationship between the two types of uncertainty, the common and the individual one, represents the main philosophical principle on which all criteria are based. This interrelationship can be expressed directly or indirectly, but it is always contained in the algorithm of criteria calculation.

1.4 Mission Fulfilled by Criteria

In the section describing trading opportunities, we presented data evidencing frequent deviations of option market prices both toward being

overvalued and undervalued. The direction of these deviations tends to change in time: An option for a certain underlying asset can be overvalued or undervalued at different time intervals. Simply put, the trading success depends on our ability to identify such trends and to implement strategies in which overvalued options are sold and undervalued ones are bought. The main mission of the criteria is to identify the potential trading opportunities by revealing unfairly valued options and their combinations.

The simple classical principle—buy low (undervalued), sell high (overvalued)—is the basis of investing in any financial market instrument. If the

asset's fair value estimated by the investor is less than its current market price, the asset is considered to be undervalued and should be bought and vice versa. If the estimate is correct, the market price would approach the fair value in time and the investor would get his profit. Although investment in options is governed by the same basic rules, their valuation possesses some distinctive features peculiar only to these financial instruments.

The first peculiarity is the multitude of options for one underlying asset. At any given point in time, some of them can be overvalued and others can be undervalued. This enables us to create option combinations composed of

elements possessing a whole range of fair value estimates. The fair value of an option combination is a complex characteristic compounded of fair values of its components. This property is not typical for stocks, commodities, or any other nonderivative product because all of them can have only one simple, not composed, fair value.

The second peculiarity was already mentioned in the section describing trading opportunities. It arises from the fact that the life of an option is limited. For any other financial instrument, the deviation of its analytically derived fair value from the market price may hold for a long time. Even if the estimate was perfect, it can take a long time for the

market price to approach it, and the investor would have nothing to do but to wait for it. This may never happen. As a result, the investor may not have an opportunity to verify the accuracy of his valuation. On the contrary, options have a fixed expiration date. After this date the investor can state precisely whether his estimate was accurate.

The market evaluates options, which is reflected in their demand and supply prices. How can one judge on the fairness of these estimates? The basis for the judgment is criteria that compare the extent of the uncertainty regarding the future price of the underlying asset (common uncertainty reflected in options market prices) with the estimated

(individual) uncertainty. The mission fulfilled by criteria is to solve the problem of estimating the fairness of option market prices by comparing these two types of uncertainty. The computational algorithm of any criterion must express the values of both types of uncertainty numerically, reduce them to a unified scale, and compare them with each other. If the common uncertainty rate is substantially higher (lower) than the individual one, options are overvalued (undervalued). If the two uncertainties are estimated at a level close to each other, the options are considered to be fairly valued. The quality of a certain criterion is determined by its capability to reflect

the divergence of these two uncertainties.

1.5 Forecast as a Key Element of the Criterion

The principles of criteria creation are very diverse—from the simplest formulae to complicated algorithms with combinatorial constructions and time-consuming calculations. Every criterion processes the incoming information according to its internal algorithms. Regardless of their basic ideas and computational architectures, all criteria contain the common element expressing

the degree of our individual uncertainty concerning the future. The uncertainty is formulated by making a forecast of future events (which may be expressed either directly or indirectly). It means that the forecast, regardless of the form it may take, constitutes an integral part of any criterion.

A human being cannot foresee the future. Hence, we have to be satisfied with forecasts based on scenarios of future events. Within the framework of further research, we define future as a set of outcomes with a known structure (we know all events that can happen) and unknown realization. (We do not know the events that will happen.) The forecast is defined in the broad sense—

as predictive information about the future that is obtained in any manner and by any means including historical data analysis, empirically established patterns, trends, and relationships and theoretical models. (Remember that the forecast can be erroneous or contain little useful information.)

We distinguish between two fundamentally different approaches to the construction of forecasts.

The first approach, which is extensively used in many fields, is based on the analysis of past events frequency. The basic assumption here is that probability of future events can be derived from the frequencies of their past occurrences.

Within the framework of this approach, attempts to forecast the asset price are usually based on historical price data. Researchers focused quite a lot of effort on finding an adequate description of market price fluctuations in terms of probability. One of the commonly used methods is based on the analysis of past daily (weekly, monthly, and so on) price changes. The probabilities of future price changes are then obtained from their relative frequency observed in the past data. Another method is based on evaluation of basic price characteristics (such as mean, variance, kurtosis, skewness) and deriving the probabilities of future outcomes by the application of standard distribution functions. A great

number of mathematical methods help to produce probabilistic scenarios of future events on the basis of past data. If the outcomes are presented in the form of a discrete variable, the forecast represents the probability distribution function, in which every price is assigned its probability. If the variable is continuous, the forecast takes the form of a probability density function. In both cases, if all possible outcomes are taken into account, the sum of probabilities (or the integral of the probability density function) equals one. Guided by this approach we can make many useful forecasts taking the forms of density functions or probability distributions of the underlying asset price estimated for a

certain future date. Moreover, integrating different quantitative characteristics of options (for example, the payoff function) with respect to the probability density function, we obtain a whole class of option evaluation criteria—expected profit values, profit and loss ratios, profit probability, and many others.

The second approach to creating forecasts is based on the development of possible future scenarios by experts specializing in corresponding fields. Expert forecasts rely on diverse factors including objective and subjective ones. The latter include estimates based on their experience and professional insight or following from nonformalized

analysis of the situation. Despite the nonscientific nature of such an approach, it is not necessarily poor. Some of the intuitively discovered brilliant ideas have only found their theoretical and empirical grounds much later in time. Nevertheless, such a method of estimating the probabilities of future events remains limited to outstanding individuals and cannot be included in the paradigm of our systematic approach based exclusively on formalized rules. An objective approach to forecasting future outcomes by experts is based on applying commonly accepted laws, which predict the time evolution of the system depending on its current conditions. Unfortunately, in the field of

financial markets, such approach cannot be fully implemented because no well-established laws are capable of producing reliable predictions for future price movements.

Nevertheless, we can create criteria on the basis of expert forecasts. This can be done by expressing expert opinions numerically and combining them with standard probability distribution functions. We call such criteria *criteria on the basis of expert distribution*. This definition implies that these criteria are based not on past data (such as time series analysis) but on the fundamental analysis of a company, industry, or macroeconomic situation. Skilled experts can estimate probabilities of

different outcomes of forthcoming corporate events (quarterly earnings, sales reports, new products launch, and so on). These estimates are then used to build the price probability distribution for the expected event date.

The forecast can also be produced by creating a probabilistic scenario of future events on the basis of combining both previously mentioned approaches. For example, using historical prices we can build a probability density function and then modify it by adjusting one or more parameters. If these adjustments are based on fundamental estimates provided by an expert, the combination of two different approaches is evident and may be very productive.

1.6 Classification of Criteria

1.6.1 Universal Criteria

When using the term *universal criteria*, we refer to functionals allowing the comparison of option combinations related to different strategies and various underlying assets. For example, two completely different combinations, such as a Call butterfly for QQQQ and a bear Put-spread for IBM, can be easily compared using a universal criterion. An important advantage of such criteria is

their capability to compare and rank heterogonous option combinations.

Every option combination has its own specific and unique payoff function. It means that regardless of its belonging to a certain underlying asset and strategy, any combination possesses a universal characteristic expressing its quality in terms of estimated future profits and losses. The payoff function can be formalized as $PF(B,S,x,t,par)$, where B is a certain underlying asset, S is a certain option strategy, x is an underlying asset price, t is a point in time for which the payoff function is calculated, and par —values of other parameters.

Suppose that we evaluate combinations

consisting of options with the same expiration date. Formally, for every underlying asset and any future moment in time, we can build a probability density function with the underlying asset price as its argument and the probability of this price as its value. Assume that we obtained probability density functions corresponding to our underlying assets at the moment of expiration. We denote them by $f(B,x)$, where B is an underlying asset, and x is its price at the moment of expiration. Knowing $f(B,x)$ means that the following integral can be easily calculated:

(1.6.1)

$$I(B,S)=\int_0^{\infty} PF(B,S,x,T,par)f(B,x)dx \quad ,$$

Here T is the expiration moment. In probability theory integrals $I(B,S)$ represent the mathematical expectation of the combination profit. They are the typical examples of universal criteria. Some of these criteria are described in detail in Chapter 2, "Review of the Main Criteria."

For universal criteria it is natural to conclude that if the expected profit of "Call butterfly for QQQQ" is greater than that of the "bear Put-spread for IBM," then the first combination is preferable to the second one. If we

calculate $I(B,S)$ for each combination of the initial set matrix, we obtain a universal instrument allowing simultaneous comparison of all matrix elements regardless of the option strategy, underlying asset, or combinations creation algorithm. As was previously noted, the initial set contains millions of combinations to compare. This variety provides splendid material for searching promising trading opportunities whereas universal criteria offer necessary instruments for this quest.

Forecasting Universal Criteria

Many of the universal criteria are intended to forecast different characteristics of option combinations. Because future potential profit is our main interest, most universal criteria represent forecasts of combination profitability (expressed in various forms). Different estimates of relative profit value (ratio of the absolute profit to the initial investment) and yield (profit normalized by the time of the investment) form the whole class of forecasting universal criteria.

The forecast of the future underlying asset price is the most important point in creating universal forecasting criteria. The accuracy and the form of this forecast can be very different. Some

sources suggest considering price values outcomes and assigning probabilities to them (Macmillan, 2002). This is the special case of applying the probability density function. Such function was used in equation 1.6.1 to calculate the expected value of the combination's profit at the moment of expiration. Application of different instruments of probability theory and statistics gives rise to a wide class of universal forecasting criteria. Competent development and application of these criteria is a useful tool for the selection of superior option combinations.

Nonforecasting Universal Criteria

To create universal criteria we can also use various quantitative characteristics that are not directly related to the forecasting of profitability or yield. For example, the investor may be interested in estimates of various indicators that characterize the vulnerability of his position to different risks factors. In these cases special criteria can be created on the basis of Greeks or similar risk characteristics. A number of criteria can be constructed on the basis of different characteristics expressing variability of the underlying asset price. For example, the historical and implied volatilities may be combined in various ways to produce different indicators not related to profits of estimated

combinations. Another method of creating nonforecasting universal criteria is to combine average values of forecasting criteria with variability indicators. For example, the ratio of mathematical expectation to the standard error (commonly referred to as the Sharp ratio) is universal yet not forecasting criterion.

1.6.2 Specific (Nonuniversal) Criteria

Within the framework of the systematic approach, the most general way to present the criterion is by formalizing it as a function of two arguments: underlying asset *B* and strategy *S*.

Criteria expressed as such functionals were called *universal* because they enable comparison of any option combinations regardless of strategies and underlying assets.

However, there are situations when the structure and nature of the criterion are closely connected with the specificity of a certain option strategy. This can be illustrated by the example of the *short straddle strategy*. The payoff function of any combination created within this strategy has a pair of points where it equals zero. They are called break-even points, and the distance between them corresponds to the break-even range. Based on the ideology of selling straddles, we expect that the stock price

would not go out of the break-even range at the moment of expiration (or any other moment before the expiration). Hence, the relative width of this range is an important characteristic of combinations created within the short straddle strategy: The wider it is, the higher the probability to realize positive profit. The criterion based on the break-even range allows the selection of preferable combinations within a certain strategy or a class of closely related strategies with similar payoff functions (for example, short straddles and strangles). At the same time, such nonuniversal criterion is unsuitable to compare combinations created within different strategies (for example, short and long straddles).

Moreover, some specific criteria are absolutely unsuitable for certain option strategies. For example, bear and bull Put- and Call-spreads cannot be evaluated by the criterion based on a break-even range because the payoff functions of these strategies have the only break-even point. There is no need to say that the notion of range is meaningless with regard to the combination with a single break-even point.

chapter 2. Review of the Main Criteria

2.1 Criteria Based on Lognormal Distribution

2.1.1 Description of Lognormal Distribution

The effectiveness of option

combinations evaluation is mainly determined by the accuracy of the underlying asset prices forecast. One of the possible ways to produce a forecast is building the probability distribution of the price. When applying this method we do not try to predict the future price exactly but estimate the probabilities of many possible outcomes. For the discrete price scale, the distribution is established by assigning probabilities to all outcomes. In the continuous case probability, density functions are used. We begin to create criteria for evaluation of option combinations with applying one of such functions: logarithmic normal distribution.

Consider a problem of making a forecast

for a certain stock price in n days in the future. It is obvious that we cannot give a perfectly accurate estimate because the price depends on a huge number of interrelated factors unknown in advance. For example, the stock price that is $100 today can take any value in a month depending on the events that take place in the market. Of course, too low and too high values are unlikely whereas those close to the current price are more likely. Not trying to make an exact prediction, we assume that any value is possible and attempt to obtain the probability value for every price outcome. If the price is considered as a continuous real number, the probabilistic forecast of the future underlying asset

price can be expressed as a probability density function $f(B,x)$, where argument B denotes a certain underlying asset, and x is any possible price value.

Financial mathematics operates with series of underlying asset prices in consecutive time points t. For example, daily close prices $C(t)$, $t = 1, 2,...$ may be used to calculate relative changes

$$\frac{C(t_{i+1}) - C(t_i)}{C(t_i)}.$$

When price values are close, such relative changes (also called *yields*) are approximately equal to increments of natural logarithms of two adjacent prices:

$$\frac{C(t_{i+1})-C(t_i)}{C(t_i)} \approx \ln C(t_{i+1}) - \ln C(t_i) \ .$$

In case of a continuous price series, the approximate equality turns into the exact one:

$$dC(t)/C(t) = d\ln C(t) \ .$$

Price changes are affected by numerous factors. Their joint effect is unpredictable, which brings us to the key assumption: At any point of time *t* increments of price logarithms are random and independent of each other. The assumption of randomness implies that price changes cannot be described

by any deterministic model but rather resemble white noise or Brownian motion. The assumption of independence implies that the given yield is not affected by the preceding yields and does not exert any influence on the consecutive yields. The central limit theorem of the probability theory states that the average of a large number of independent samples has the distribution that is close to normal (also called Gaussian). Hence another key assumption is made: Increments of price logarithms are normally distributed. This implies that the price logarithm itself is also a normally distributed random variable. Accordingly, the price distribution is called a logarithmically

normal distribution or simply lognormal distribution.

Lognormal distribution is a standard price distribution model widely accepted by most economists. At the same time, its appropriateness for description of financial time series was frequently questioned, and this issue is widely debated on various scientific, theoretical, and practical occasions. Until now countless enthusiasts tried to find probability distributions that describe price dynamics more precisely. Literature is full of arguments and evidence stating that real prices have distributions, which differ substantially from the lognormal one. (“Fat tails,” excesses, and other “abnormalities” are

often used as arguments.) Nevertheless, classical option pricing theory is based on lognormal distribution. This distribution is widely used and generally accepted because it has three indisputable advantages.

First, the lognormal distribution density function has a simple formula with a few parameters. Second, and this is the most important fact, normal distribution of price logarithms' increments realizes the most general and hence the most robust approach exploiting the central limit theorem. Third, despite all criticism, numerous investigations proved that lognormal distribution quite adequately describes the distribution of real market prices.

Normal (Gaussian) distribution has two parameters: mean value a and variance σ^2. The following formula sets up a relationship between the price x and the probability density value $N(a,\sigma,x)$ corresponding to the future price outcome:

$$N(a,\sigma,x)=\frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-a)^2}{2\sigma^2}) \ .$$

Lognormal distribution of a random variable is determined by the parameters of normal distribution of its natural logarithm:

$$LogN(Mean,\sigma,x)=\frac{1}{\sigma x\sqrt{2\pi}}\exp(-\frac{(\ln(x/Mean)+\sigma^2/2)^2}{2\sigma^2}) \ ,$$

where *Mean* is the expected price value at the future time *t*. There is a simple relationship between the parameters of lognormal distribution and the corresponding normal distribution:

$$a = \ln(Mean) - \sigma^2 / 2 \ .$$

2.1.2 Expected Profit on the Basis of Lognormal Distribution

Criterion Calculation Method

Calculation of the expected profit is

based on the equation (1.6.1) proposed in Chapter 1, "General Presentation and Review of Criteria Properties," as generalization for the universal criterion. Using the concept of the classical probability theory, mathematical expectation of profit of an option combination at a certain point of time is determined as follows:

(2.1.1)

$$MeanLogNorm(B,S) = \int_0^{\infty} PF(B,S,x)LogN(Mean,\sigma,x)dx \ ,$$

where $PF(B,S,x)$ is a payoff function of combination S with the underlying asset B;

*LogN(Mean,*σ*,x)* is a probability density function of lognormal distribution;

Mean, σ, and *x* are the parameters of lognormal distribution.

The integral formalized by the equation (2.1.1) represents the criterion that we call "expected profit on the basis of lognormal distribution."

The payoff function *PF*(*B*,*S*,*x*) in equation 2.1.1 can be replaced by another function, for example, by the yield function expressed as the ratio of the payoff function to the investment amount normalized by the investment period. In this case the value calculated by equation 2.1.1 stands for another

criterion: the “expected yield on the basis of lognormal distribution.” Similarly, we can formulate other “expected value” criteria based on the integration of different functions by lognormal distribution. Following the basic principle of option combinations evaluation by criteria, we adhere to the rule that combinations with higher expected profit are preferable to other alternatives.

Criterion Parameters

As it follows from equation 2.1.1, to obtain the value of expected profit, we have to express the forecast of the future

stock price in the form of a probability density function. This requires determining the parameter values for the lognormal distribution. Application of this relatively simple distribution reduces the problem to the calculation of just two parameters: mathematical expectation of the underlying asset price at the point of time *t* and the variance of normal distribution of the underlying price logarithm.

The main advantage of option trading is the possibility to create market-neutral strategies that do not require predicting future price trend direction. Thus it is natural to assume that the mathematical expectation of the underlying asset price is equal to the current price as of the

criterion calculation date. It means that we do not make any assumptions about the growth or fall of the underlying asset but consider the current price as the most probable estimate for the forecast date. The alternative approach implies assigning an expert value to this parameter. For example, if a fundamental analyst forecasts price growth (or fall), then a mathematical expectation, which is higher (or lower) than the current stock price, may be used to calculate the criterion. In Chapter 1 we discussed the possibility of developing forecasts on the basis of expert estimates combined with a statistical analysis. The "expected profit on the basis of lognormal distribution" can be a typical example of

such combined criterion if lognormal distribution (built on the basis of historical prices) is combined with expert estimates of mathematical expectation of the underlying asset price.

The value of the second parameter—the variance σ^2 of normal distribution of the stock price logarithm—can be taken as equal to the square of stock historical volatility. Although this characteristic is of great importance in the option world, the consensus on the precise manner of its estimation is far from being achieved. Apart from the classical model, there is a variety of elaborated formulae intended to calculate more precise values of historical volatility (Garman & Klass,1980; Parkinson, 1980, and so

on). Depending on the chosen formula, the parameter value and, consequently, the criterion itself will change significantly. We dwell on the simplest case when historical volatility HV is calculated at a certain historical period as the mean-square deviation of logarithms of consecutive daily close prices ratios:

(2.1.2)

$$HV = \sqrt{\frac{1}{PN}\sum_{i=1}^{N} r_i^2} \qquad r_i = \ln\left(\frac{C(t_0 - i + 1)}{C(t_0 - i)}\right),$$

where t_0 is the current time;

P is the period of historical volatility calculation (expressed in fractions of a year);

N is the number of close prices ratios used (the number of days preceding the current date).

Equation 2.1.2 is a simplified version of historical volatility modeling. However, our experience shows that its accuracy is sufficient for the purpose of criteria computation. We refer to the number of days preceding the current date (N) as the length of historical volatility calculation period. Its proper choice is a key moment in calculating this parameter because, as can be seen from equation 2.1.2, the variance value is extremely

responsive to period length changes. Moreover, we can say that the true parameter that determines the practical effectiveness of the criterion is *N*. Rules of determining *N* are usually thoroughly protected intellectual property; they are discussed a lot but specific algorithms are always concealed. There is an opinion that most option trading specialists do not possess formalized algorithms for calculation of this parameter but rather rely on their intuition and experience. In any case, the existence of one unique and universal algorithm is most unlikely. There should be various rules for determination of *N*, and these rules should change depending on the option strategy in use, the forecast

horizon, and the algorithm of underlying assets selection.

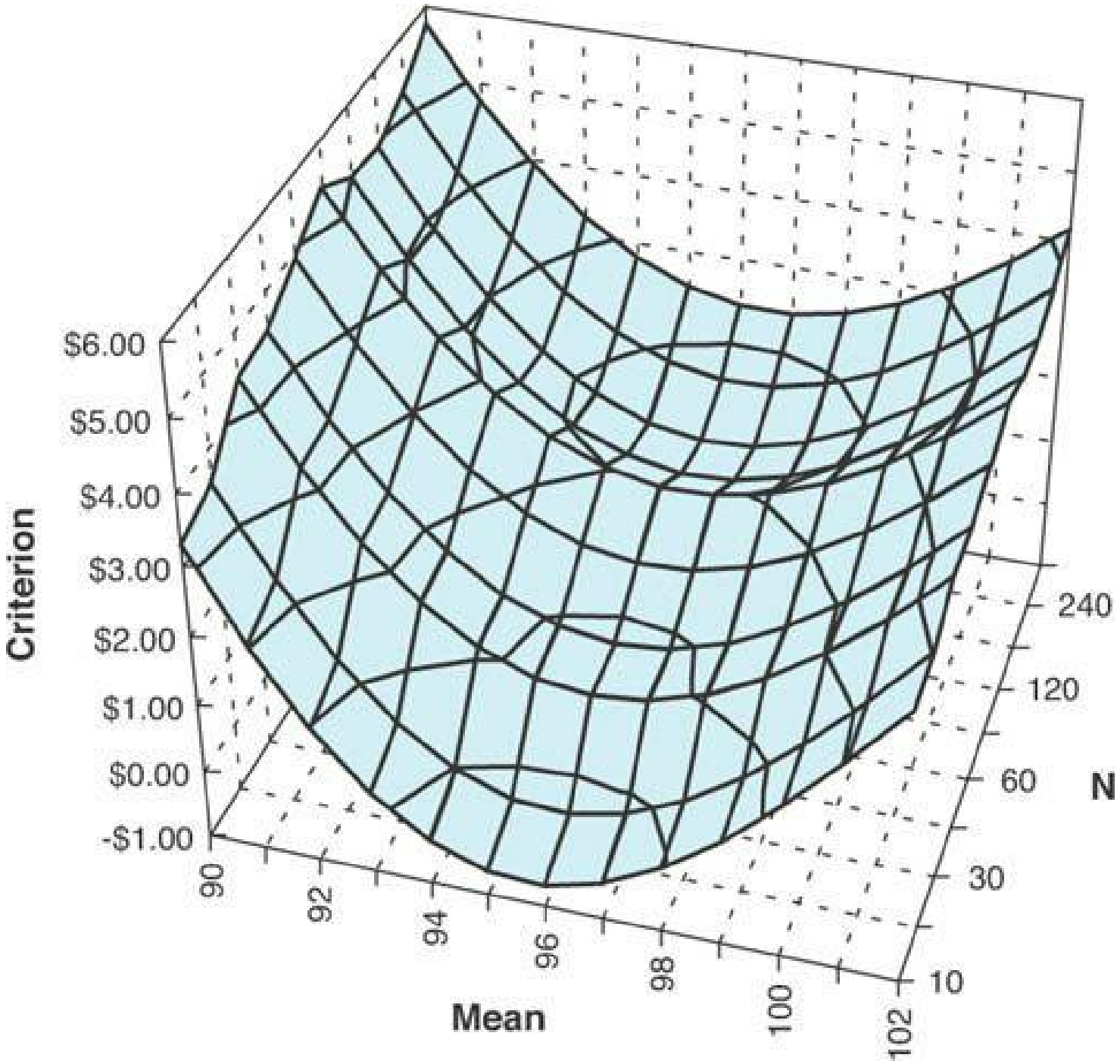
To illustrate the relationships between the "expected profit on the basis of lognormal distribution" criterion and its parameters, we constructed the option combination called *proportional long strangle* using the AAPL stock as the underlying asset. (A full combination description is given in the next example). Figure 2.1.1 demonstrates to what extent this criterion depends on values of lognormal distribution parameters (mathematical expectation of the underlying price *[Mean]* and the historical volatility calculation period length [*N*]). The criterion reaches its minimum when *Mean* is close to the

current stock price ($94.68). This is due to the fact that long strangle is unprofitable when the price is not changing considerably. Conversely, *Mean* values that are higher or lower than the current price lead to criterion value growth. The influence of the second parameter is no less obvious: when the period used for the calculation of historical volatility is short, the criterion values are low, but when the period length increases, the criterion also grows (Figure 2.1.1). This example shows that the criterion value may turn out to be either negative or positive (and even rather high) depending on values of its parameters. The form of this relationship depends strongly on

specific option strategy and underlying asset; furthermore, it can be changed in time even for the same {strategy × underlying asset} pair. Thus when selecting parameter values, it is necessary to analyze every situation thoroughly and to consider it in the context of the current time point.

Figure 2.1.1. Relationship between values of the "expected profit on the basis of lognormal distribution" criterion and two parameters of lognormal

distribution: mathematical expectation of the underlying price (Mean) and the length of period used for calculation of historical volatility (N). The option strategy is "proportional long strangle."



Criterion Calculation Example

The criterion value will be calculated for the "proportional long strangle" option strategy. On April 5, 2007, the close price of AAPL was $94.68. Options expiring on April 21, 2007, traded with the following ask prices: Call 90 — ask $5.2, Put 95 — ask $1.85. Consider the combination consisting of two long Put options and one long Call option.

To proceed with criteria calculation, we have to set up the parameters of lognormal distribution. We assume that mathematical expectation of the stock price at time t is equal to the current price $Mean = 94.68$ (according to the principle of market neutrality). Equation 2.1.2 is applied to calculate historical

volatility on the basis of the 120-day history of close prices ($N = 120$). The obtained value is $HV = 30.29\%$. To derive variance from the HV value, we have to apply the correction factor that would take into account the time period from the present time point until the forecast date for which we calculate the criterion value. This factor equals the square root of the ratio of the number of days until the forecast date to 365. Because in this example the criterion is calculated for the expiration day (which is the forecast date), the time left from the present time point until the expiration equals 15 days. Thus

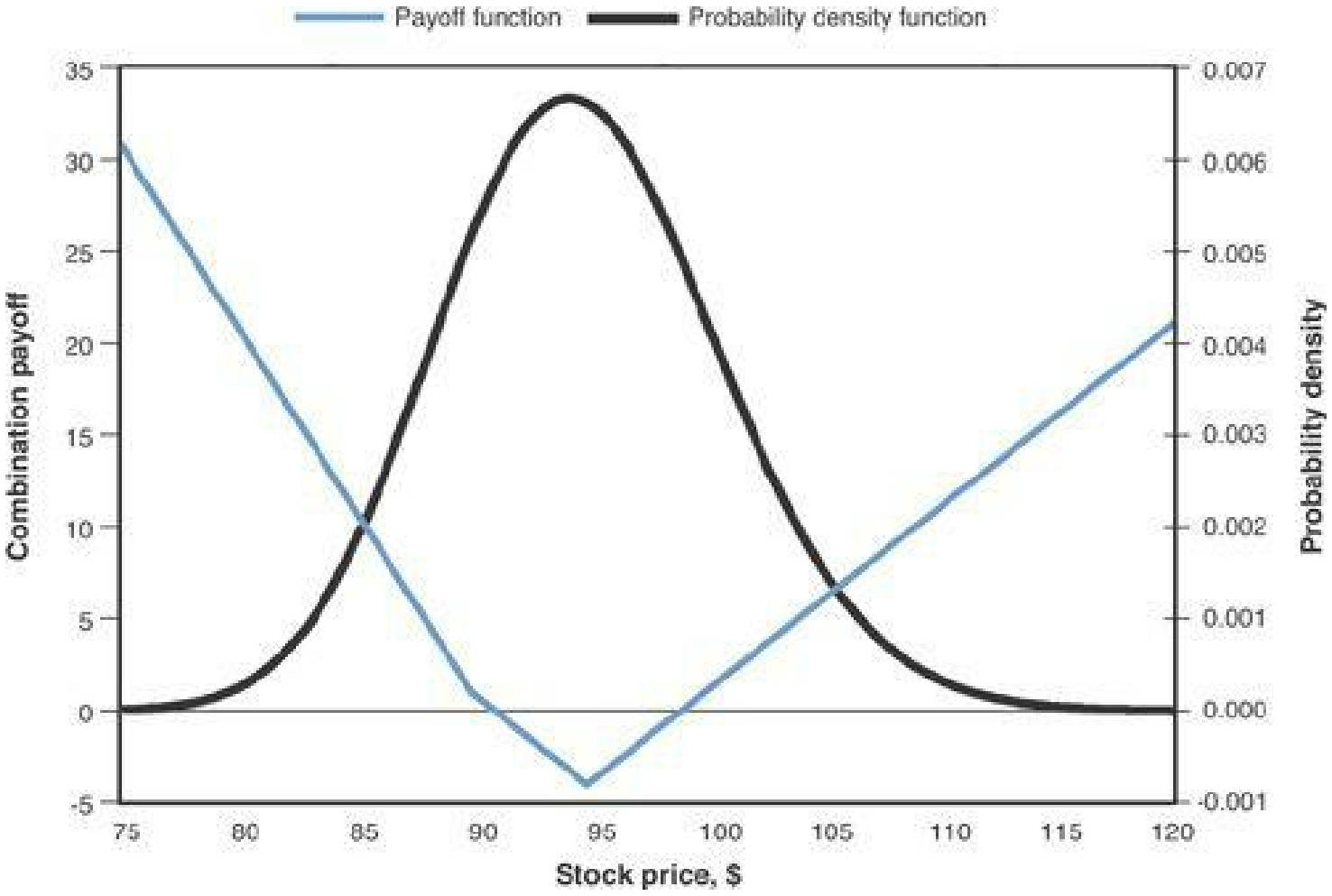
$$\sigma = \frac{30.29}{100}\sqrt{\frac{15}{365}} = 0.0614$$

.

Figure 2.1.2 presents the payoff function of the combination calculated for the expiration date (given that each option is bought at the ask price) and the probability density function of the lognormal distribution built with parameters equal to the values previously fixed. Integrating the product of the payoff function by the lognormal probability density function (equation 2.1.1), we obtain the expected profit of the combination at the time of options expiration: *MeanLogNorm* = 1.6052. What does this value mean in the context of combination evaluation? The criterion value is positive, which indicates potential combination profitability. However, we should not expect that

exactly this profit will be received from the investment in this combination. The obtained criterion value rather represents the most probable result. If we could run a series of statistical experiments with many possible outcomes, the average profit of all outcomes would work for the criterion value. However, in reality only one outcome will occur. It means that application of criteria based on probability distributions is reasonable only when they are systematically used over and over again in real trading. This implies that application of the probability theory for valuation of options and their combinations requires prudent interpretation of results.

Figure 2.1.2. Payoff function corresponding to the "proportional long strangle" combination (see the full description in the text) and the probability density function of the lognormal distribution.



2.1.3 Profit Probability on the Basis of Lognormal Distribution

Criterion Calculation Method

Depending on the future underlying asset price, the payoff function of any combination may be either negative (which corresponds to the combination loss) or positive (which corresponds to profit). Moreover, the magnitude of possible profits and losses varies substantially between different option strategies. Thus long strangles and straddles have unlimited potential profit if the underlying asset price moves significantly but have limited loss potential if the price doesn't change. On the contrary, short strangles and straddles are limited in their profit potential (they are profitable only if the underlying asset price does not change or changes just slightly) and have

unlimited loss potential if the price jumps considerably.

The criterion expressing profit probability estimates the probability that at the time of options expiration (or any other forecast time point) a payoff function of a given combination will have positive value. In other words, this criterion estimates the probability that the combination will be profitable. (That is, profit values will exceed zero.) Whether the payoff function is positive or negative is determined by the price of the underlying asset at the forecast time. In the example discussed in the previous section, the payoff function is positive for all AAPL prices lower than $91.12 and higher than $98.92.

The "profit probability on the basis of lognormal distribution" criterion is calculated by integrating the product of the combination payoff function by the probability density function over the price range (or ranges) for which the payoff function is positive. Following the preceding notation (equation 2.1.1) the criterion can be formalized as:

(2.1.3)

$$ProbLogNorm(B,S) = \int_0^{\infty} \theta(PF(B,S,x))LogN(x)dx \ ,$$

where $\theta(y)$ is the theta-function with argument $y = PF(B,S,x)$, which has the

following values: $\theta(y) = 1$, if $y > 0$, and $\theta(y) = 0$ in other cases.

To calculate this criterion value we use the same two parameters as in the case of expected profit: mathematical expectation of the underlying stock price at time t and the variance of normal distribution of the stock price logarithm.

The sense of expression 2.1.3 is easy to realize in the discrete case when a finite price series $\{x_t, t = t_1, t_2, ..., t_n\}$ is considered. (These prices constitute a set of possible future outcomes.) In such case we get the set of probabilities $\{p(x_i), i = 1, 2, ...n\}$ assigned to the corresponding price outcomes instead of the probability density function $LogN(x)$.

Price index i form two subsets: $I^+ = \left\{ i : PF\left(B, S, x_{t_i}\right) > 0 \right\}$, where the payoff function value is positive, and $I^- = \left\{ i : PF\left(B, S, x_{t_i}\right) \le 0 \right\}$, where the payoff function values are negative or equal to zero. Then our criterion turns into the sum of probabilities of elements forming the first subset:

(2.1.4)

$$ProbLogNorm(B,S) = \sum_{i \in I^+} p(x_{t_i}).$$

Criterion Calculation Example

Consider the complex combination consisting of three Call options on the AAPL stock. The combination is created on April 5, 2007 with the following options: one long Call 80 (ask $17,5), two short Calls 95 (bid $7,3), two long Calls 110 (ask $2,25). All options expire on July 21, 2007. As in the previous example, the stock price at the combination creation date was $94.68. According to the market neutrality principle (when we do not forecast price movement direction), the parameter of mathematical expectation of the underlying stock price is assumed to be equal to the current price (*Mean* = 94.68). Historical volatility was estimated on 120-day price history. (*N*

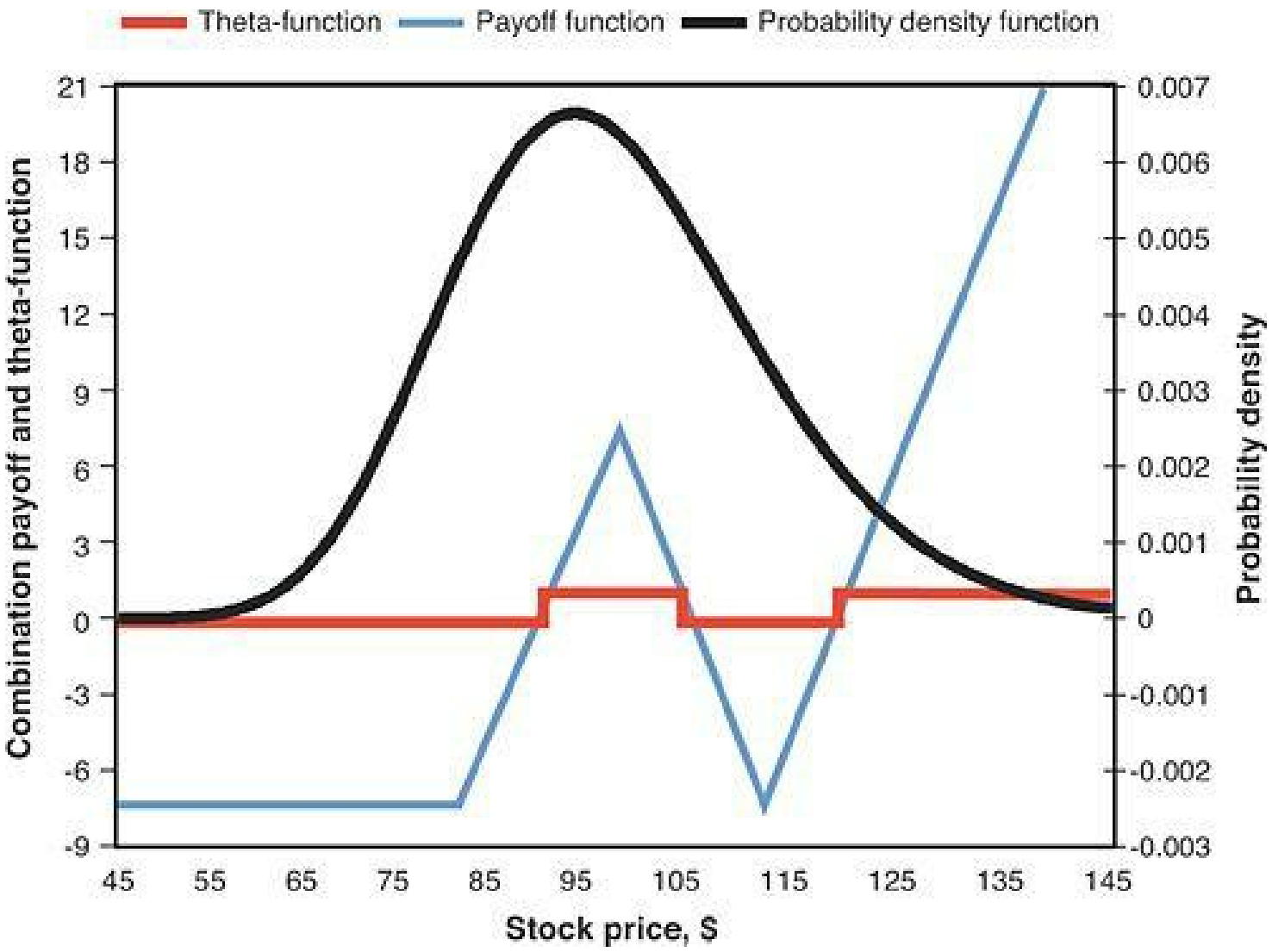
value is the same as in the previous example.) As in the previous example, *HV* amounts to 30.29%. There are 107 days left to expiration. Accordingly, multiplying the volatility by the correction factor, we get the variance value:

$$\sigma = \frac{30.288}{100}\sqrt{\frac{107}{365}} = 0.164$$

Figure 2.1.3 shows the payoff function *PF(x)* of the combination, the theta-function *θ(PF(x))* and the probability density function of the lognormal distribution. The theta-function equals zero over the price range where the payoff function is negative and equals

one for prices corresponding to the positive payoff function.

Figure 2.1.3. Payoff function of the combination consisting of three Call options (see the full description in the text), theta-function corresponding to this payoff function, and the probability density function of the lognormal distribution.



In this example there are two price ranges for which the payoff function is positive—from $87.43 to $102.40 and from $117.46 to infinity (Figure 2.1.3). Numerical integration according to formula 2.1.3 gives the criterion value of

0.45, which means that at the time of expiration this combination will be profitable with 45% probability. Of course, the same result will be obtained for the discrete case if equation 2.1.4 is applied. Selection of option combinations by maximizing this criterion minimizes the probability to choose unprofitable variants. Because this criterion does not estimate the magnitude of possible profit or loss values, it should be used not as an independent selection tool but jointly with other criteria expressing potential profit in an explicit form.

2.2 Criteria Based on

Empirical Distribution

2.2.1 Description of Empirical Distribution

Lognormal distribution describes the dynamics of market prices quite roughly. Settling its parameters *Mean* and σ we simplify information contained in historical price series due to averaging and application of other statistical procedures. On the other hand, it seems natural to assume that the future should resemble the past and to transfer the maximum information on price

movements from the known past to the forecast future. This is the aim of constructing the empirical distribution.

By analogy with the lognormal distribution, we assume that price change is a random variable. The additional assumption—specific for the empirical distribution—is that probability distribution of price increments can be built on the basis of their past values. Within the framework of this approach, we collect history of price movements and derive the probabilities of future movements from the frequency of their past occurrence (Izraylevich and Tsudikman, 2009a).

Formally the empirical distribution is

built in the following way. The current point of time is denoted by T_0 and the future time point for which we build the empirical distribution of the underlying asset price is denoted by T. The difference between these two time points $\tau = T - T_0$, $\tau > 0$ represents the forecast horizon. The length of the historical period used to build the distribution will be addressed as the historical horizon of the empirical distribution and will be denoted by L ($L > \tau$). The larger the horizon, the more extensive statistics are at our disposal. However, old historical data may give wrong information about patterns and trends prevailing in the present market. Thus the selection of the best L value is a nontrivial task that is as

important as finding optimal N in the case of lognormal distribution.

Having horizon values of τ and L fixed, we consider relative increments of prices $p(t) = C(t + \tau)/C(t)$ for all points of time t $(t = T_0 - L + 1, T_0 - L + 2, \ldots, T_0 - \tau)$. Thus we get $L - \tau$ observations of price increments. Multiplying the current price $C(T_0)$ by each of the relative increments we get a set of $L - \tau$ forecast price variants:

$$C_j^{empiric} = C(T_0)p_j, \text{ where } p_j = p(T_0 - L + j), j = 1, 2, \ldots, L - \tau$$

The obtained set of price values can be used to build a probability density function of the price distribution

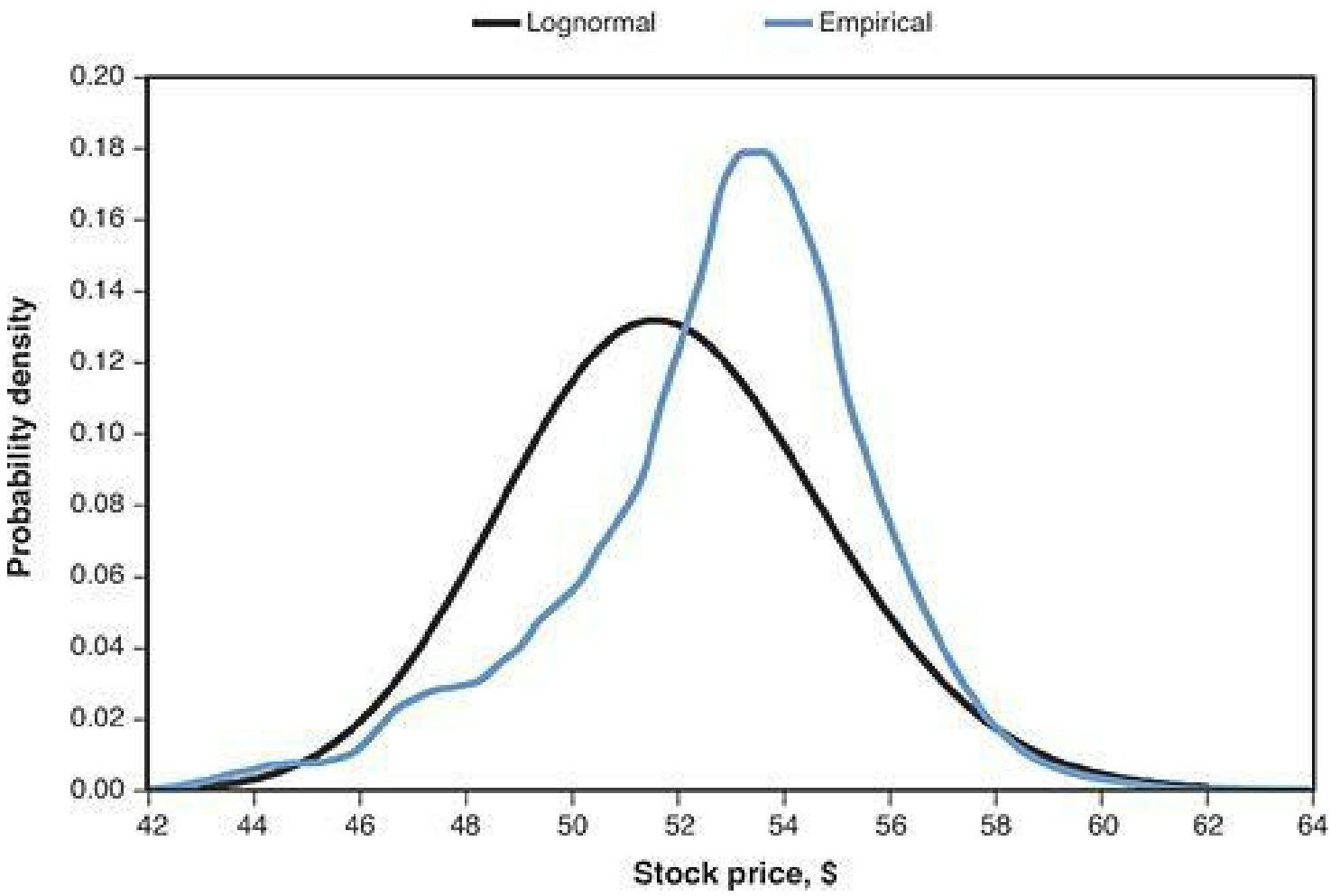
Empiric(x). This can be done on the basis of the histogram of $L - \tau$ forecast price values.

To illustrate the difference between lognormal and empirical distributions, we compare their density functions using INFY stock as an example. On April 5, 2007, the close price of this stock was $51.83. Its historical volatility calculated using equation 2.1.2 amounts to 28%. (The length of historical volatility calculation period N is 120 days.) Assuming that Mean parameter value is equal to the current stock price (according to the market neutrality principle), we obtain the probability density function of lognormal distribution (Figure 2.2.1). For

empirical distribution the forecast horizon τ determined by the expiration date (April 21, 2007) equals 11 trading days. The historical horizon L is 250 trading days. Figure 2.2.1 shows that the average and maximum values of the empirical distribution are shifted to the right (toward higher price values) relative to the lognormal function. Empirical distribution also has a higher and narrower middle part and fatter left tail.

Figure 2.2.1. Probability density functions of the lognormal and empirical

distributions of INFY stock.



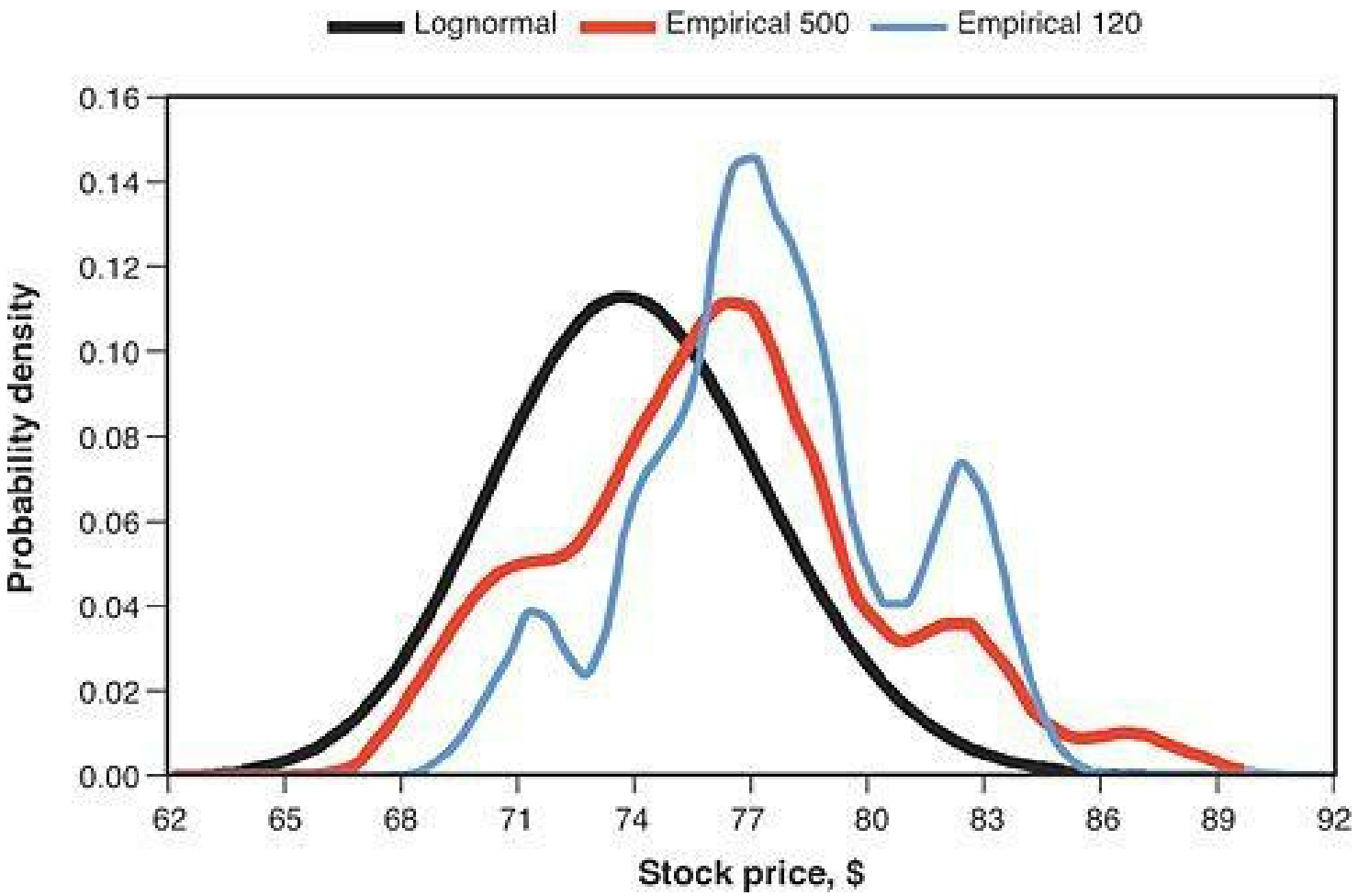
The divergence between lognormal and empirical distributions usually decreases when the historical horizon of empirical distribution (L) increases. Figure 2.2.2 shows the lognormal and

two empirical distributions built for NRG stock on April 5, 2007. The forecast horizon corresponds to the nearest expiration date of April 21, 2007. Lognormal distribution is based on historical volatility of 22.86% (N = 120) and the current stock price of $73.88. The probability density functions of empirical distribution are based on 120- and 500-day historical horizons. It is clear from Figure 2.2.2 that the shorter the historical horizon, the more peculiar the shape of the empirical distribution and the stronger its difference from lognormal distribution. The shift of empirical distributions to the right relative to the lognormal distribution reflects the stock price

uptrend. Unlike lognormal distribution, empirical distribution reflects past trends and strong price movements. Figure 2.2.2 illustrates that the selection of values for the "historical horizon" parameter L strongly influences the shape of empirical distribution. Consequently, L exerts a great influence on calculation of the criteria based on this distribution.

Figure 2.2.2. Probability density functions of the lognormal and two empirical distributions of the NRG stock. Empirical

distributions are built with 120- and 500-day historical horizons.



2.2.2 Expected Profit on the Basis of Empirical Distribution

Criterion Calculation Method

The main principle applied to the calculation of this criterion is similar to that of the criteria based on lognormal distribution. The "expected profit on the basis of empirical distribution" represents the integral of the combination payoff function taken over the probability density function of empirical distribution:

(2.2.1)

$$MeanEmpiric(B,S) = \int_0^{\infty} PF(B,S,x)Empiric(x)dx \ ,$$

where $PF(B,S,x)$ is a payoff function of combination S with underlying asset B;

$Empiric(x)dx$ is the probability density function of empirical distribution.

Equation 2.2.1 represents the formalized definition of this criterion. However, for the practical purpose of option valuation, it can be estimated easier by direct application of the set of forecast prices $\{C_j^{empiric} = C(T_0)p_j\}$ (see the previous section). The mathematical expectation of the underlying price at the forecast time is calculated as follows:

$$C^{empiric}(T) = \frac{1}{L-\tau}\sum_{j=1}^{L-\tau} C_j^{empiric},$$

and the expected profit on the basis of empirical distribution is

(2.2.2)

$$MeanEmpiric(B,S) \approx \frac{1}{L-\tau}\sum_{j=1}^{L-\tau} PF(B,S,C_j^{empiric}).$$

This expression provides a simple computational procedure to find the criterion value.

According to the classification introduced in Chapter 1, this criterion is obviously universal and forecasting.

Besides, it possesses an additional important feature. Calculating the criterion according to formula 2.2.2 means that its value is the average of $L - t$ values of the payoff function that is calculated using forecast underlying asset prices. This implies that in addition to the average we can calculate another characteristic expressing variability of payoff function values (either standard deviation or standard error). The ratio of the average to the variability allows the creation of a new criterion containing important additional information. This derivative criterion, and the original one, is universal, but it is not forecasting because it is an abstract indicator that does not refer to

any real characteristic.

Criterion Parameters

In contrast to the lognormal distribution model that has two parameters, empirical distribution possesses only one important parameter: the history horizon. The second parameter—the forecast horizon—is determined objectively by selecting the future date for which the criterion is calculated. It is widely known that selection of the parameter's values is strongly influenced by subjective factors. Hence the more parameters included in the criterion model, the more it is affected

by such unreliable factors. Besides, the large number of optimized parameters increases the risk of overoptimization (overadjustment to past data also referred to as curve fitting). Overoptimization means that trading systems showing impressive performance in back-testing experiments become unprofitable in real trading. Therefore, the fact that the empirical distribution has only one important parameter represents its indisputable advantage over the lognormal one.

On the other hand, the fact that parameter *L* is the only one requires even more prudence in selecting its value. While in a multiparametric model some inaccuracy in optimization of one of the

parameters can be offset by successful optimization of the others, in the case of one-parameter model any inaccuracy may become fatal.

The horizon of history used to build the empirical distribution influences the criterion value considerably. Moreover, mathematical expectation of profit may turn out to be either positive or negative depending on applied L values. This statement can be illustrated with the following example. A short strangle consisting of options on the YHOO stock is evaluated by "expected profit on the basis of empirical distribution" criterion. In section 2.2.4 we describe in detail the structure of this combination and the algorithm of criterion calculation

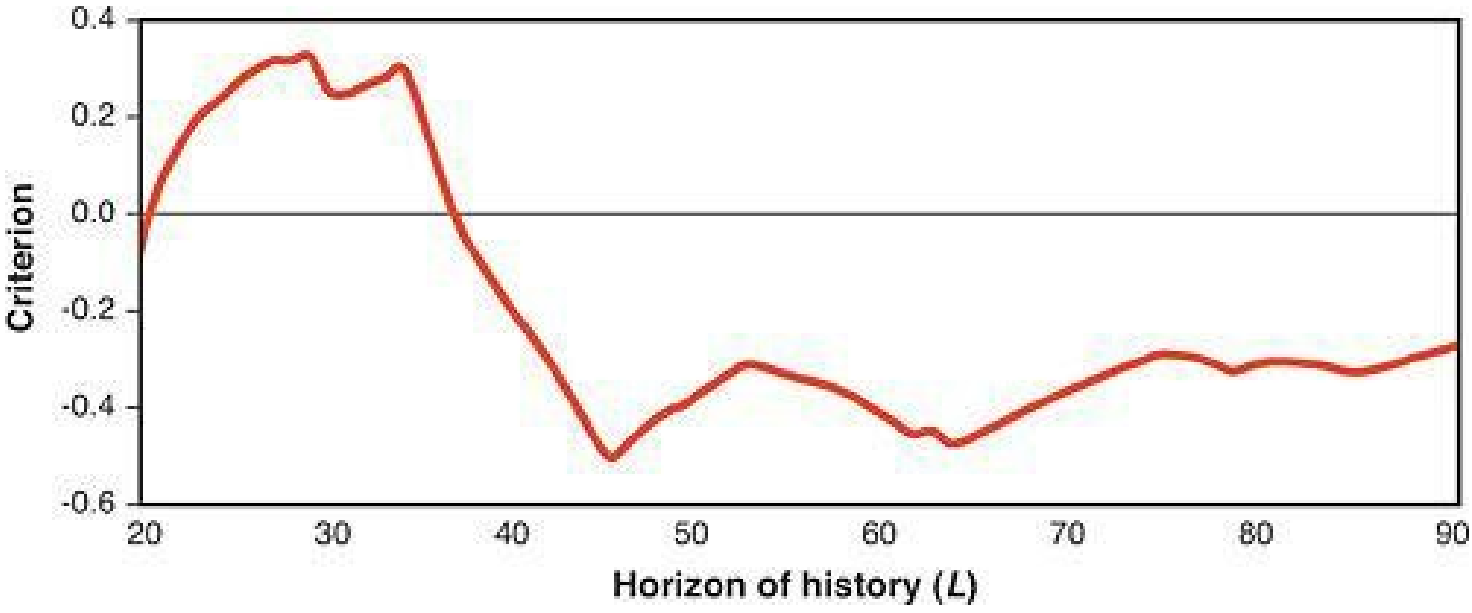
(for $L = 90$). Here to illustrate the extent of the influence exerted by L on the criterion output, we calculate $MeanEmpiric(B,S)$ values for all L from 20 to 90.

The relationship between criterion and L is shown in Figure 2.2.3. Application of long history horizons (above 40 days) results in negative criterion values. Reducing the horizon to around 30 days leads the criterion to the positive area. However, further decrease of L value brings about the criterion fall once again. Figure 2.2.3 shows that the criterion may be either negative or positive depending on the parameter value used in the calculation. It means that for some L values the combination

can be deemed potentially profitable and vice versa for other L values. This example vividly illustrates the importance of scrupulous analysis of all factors influencing the selection of the history horizon used for criteria that are based on empirical distribution.

Figure 2.2.3. Relationship between the "expected profit on the basis of empirical distribution" criterion and the "horizon of history" (L) parameter for the short strangle combination

created for the YHOO stock.



To complete the picture we should make a note about the second parameter of the empirical distribution: the forecast horizon τ. The name of this parameter indicates that its value is automatically defined at the point of fixing the future date for which the criterion is calculated. Despite this, nothing (except logic) prevents us from assigning any

whole number from 1 to $L - \tau$ to this parameter. Is there a sense in using the "horizon of forecast" value other than the real forecast horizon? Yes, there is! It is commonly known that the price changes more over longer time periods, that is, there is a straight relationship between absolute price changes and the length of time interval. It means that by increasing the horizon of forecast we can artificially raise the potential of possible future price movements and vice versa. (Remember that the real forecast horizon remains the same; only the parameter value changes.)

Is there any practical reason to modify the potential of price movements? Let us suppose that our criterion is applied to a

certain combination built according to the short strangle strategy. This strategy implies large losses when the price change is substantial. Therefore, if we prefer to be more conservative in calculating the criterion, we can increase the parameter value and thus make the amplitude of potential price movement a bit higher. And vice versa, if we prefer to take a more aggressive position, we can set the parameter value lower, which will lead to underestimation of the possible price change. Thus, control over the "horizon of forecast" parameter represents an efficient tool of adjusting the criterion to the individual risk profile of an investor.

2.2.3 Profit Probability on the Basis of Empirical Distribution

As in the case of lognormal distribution, this criterion estimates the probability that at the specified future time point a profit value of a given combination will have positive value. For empirical distribution this criterion is formalized as follows:

(2.2.3)

$$ProbEmpiric(B,S) = \int_0^{\infty} \theta(PF(B,S,x))Empiric(x)dx \ ,$$

where 1(y) is the theta-function with

argument $y = PF(B,S,x)$ which takes the following values: 1(y) = 1, if y > 0, and 1(y) = 0 in other cases.

In our simplified calculating scheme (when a finite series of discrete prices is considered), this criterion can be calculated as follows:

(2.2.4)

$$ProbEmpiric(B,S) \approx \frac{1}{L-\tau} \sum_{j=1}^{L-\tau} \theta(PF(B,S,C_j^{empiric})) .$$

2.2.4 Simplified Calculation Algorithm

Several different techniques can be used

to build empirical distribution and to compute criteria corresponding to it. Here we propose the method based on the simplified calculation procedure that operates on a finite series of discrete underlying prices (equations 2.2.2 and 2.2.4). The algorithm will be illustrated with calculation of two criteria: expected profit and profit probability on the basis of empirical distribution. The short strangle strategy applied to the YHOO stock will be used as an example. The date of position opening is May 30, 2007; values of both criteria will be calculated for the options expiration date (June 15, 2007). The combination consists of one short Call option with strike 30 sold at bid price

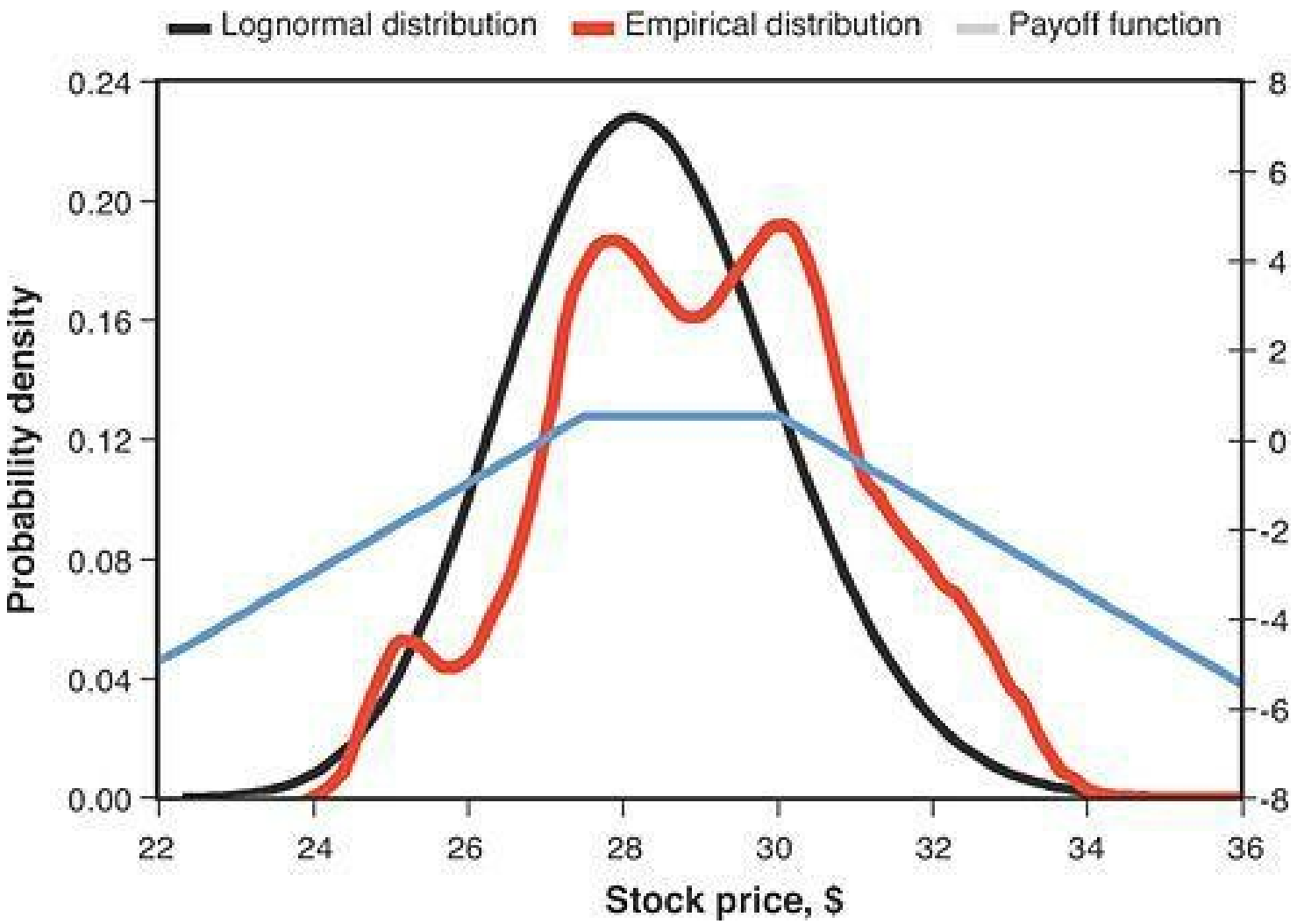
$0.3 and one short Put option with strike 27.5 sold at bid $0.25. The close price of the stock on the date of combination creation was $28.38.

The horizon of history for the empirical distribution will be $L = 90$ days. The horizon of forecast will be $\tau = 16$, which corresponds to the number of days until expiration. Figure 2.2.4 depicts the payoff function of our combination and the probability density function of empirical distribution. The lognormal probability density function is also shown on this figure to allow comparison between two distributions. Lognormal distribution was built with 120-day historical volatility (28.73%). Empirical distribution is shifted toward

higher price values relative to the lognormal distribution that reflects the prevalence of uptrends in historical data used to build the empirical distribution. It is also notable that the empirical distribution has two local peaks and none of them reaches the maximum of the lognormal distribution.

Figure 2.2.4. Probability density functions of the empirical and lognormal distributions built for the YHOO stock and the payoff function of the short

strangle strategy.



To calculate criteria values we list 90 close prices in historical order (the first column of Table 2.2.1). The second column contains close price ratios p_j calculated according to the horizon of

forecast (16 days). For example, the first p_j value is the ratio of the 17th day close price to the 1st day close price: 26.85 / 26.34 = 1.02. The third column of Table 2.2.1 contains estimates of the future stock price (forecast for 16 days in future from the corresponding close price). Values of C_j^{emp} are obtained by multiplying p_j by the current stock price (for example, the first C_j^{emp} value is calculated as follows: 28.38 × 1.02 = 28.93). The fourth column contains values of the combination payoff function PF_j estimated for the expiration time. The fifth column contains the theta-function of PF_j which is 1 for positive PF_j and 0 for other values.

Table 2.2.1. Data Required for Calculation of Expected Profit and Profit Probability Criteria on the Basis of Empirical Distribution

Close	p_j	C_j^{emp}	PF_j	θ	Close	p_j	C_j^{emp}	PF_j	θ	Close	p_j	C_j^{emp}	PF_j	θ
26.34	—	—	—	—	28.04	1.10	31.16	–0.61	0	32.09	1.03	29.14	0.55	1
26.49	—	—	—	—	27.87	1.09	30.88	–0.33	0	28.31	0.89	25.18	–1.77	0
26.75	—	—	—	—	28.04	1.04	29.64	0.55	1	27.51	0.87	24.66	–2.29	0
26.60	—	—	—	—	28.31	1.02	28.96	0.55	1	27.46	0.87	24.81	–2.14	0
26.87	—	—	—	—	28.35	1.02	28.82	0.55	1	27.88	0.89	25.25	–1.70	0
26.90	—	—	—	—	28.77	1.04	29.60	0.55	1	28.02	0.90	25.41	–1.54	0
26.30	—	—	—	—	28.56	1.00	28.24	0.55	1	28.06	0.90	25.46	–1.49	0
26.41	—	—	—	—	29.35	1.01	28.53	0.55	1	28.49	0.90	25.49	–1.46	0
25.59	—	—	—	—	29.89	1.02	28.96	0.55	1	28.34	0.90	25.44	–1.51	0
25.48	—	—	—	—	30.08	1.04	29.39	0.55	1	28.04	0.88	24.90	–2.05	0
25.55	—	—	—	—	29.74	1.06	30.01	0.54	1	27.61	0.87	24.77	–2.18	0
25.45	—	—	—	—	29.17	1.06	29.95	0.55	1	28.12	0.89	25.18	–1.77	0
25.75	—	—	—	—	29.56	1.08	30.59	–0.04	0	30.98	0.99	28.21	0.55	1
25.36	—	—	—	—	30.66	1.14	32.27	–1.72	0	30.38	0.97	27.63	0.55	1

25.54	—	—	—	—	31.25	1.08	30.65	−0.10	0	30.41	0.97	27.48	0.53	1
25.61	—	—	—	—	31.91	1.13	32.10	−1.55	0	30.22	0.96	27.13	0.18	1
26.85	1.02	28.93	0.55	1	31.66	1.13	32.04	−1.49	0	29.70	0.93	26.27	−0.68	0
27.74	1.05	29.72	0.55	1	31.41	1.13	31.98	−1.43	0	30.05	1.06	30.12	0.43	1
27.92	1.04	29.62	0.55	1	31.34	1.12	31.72	−1.17	0	29.31	1.07	30.24	0.31	1
27.58	1.04	29.43	0.55	1	31.29	1.11	31.37	−0.82	0	28.81	1.05	29.78	0.55	1
28.70	1.07	30.31	0.24	1	31.28	1.10	31.31	−0.76	0	29.21	1.05	29.73	0.55	1
29.20	1.09	30.81	−0.26	0	31.72	1.10	31.29	−0.74	0	28.57	1.02	28.94	0.55	1
29.29	1.11	31.61	−1.06	0	31.62	1.11	31.42	−0.87	0	29.75	1.06	30.09	0.46	1
29.05	1.10	31.22	−0.67	0	31.96	1.09	30.90	−0.35	0	29.35	1.03	29.24	0.55	1
28.12	1.10	31.19	−0.64	0	31.64	1.06	30.04	0.51	1	28.92	1.02	28.96	0.55	1
27.64	1.08	30.79	−0.24	0	31.69	1.05	29.90	0.55	1	28.61	1.02	28.96	0.55	1
27.42	1.07	30.46	0.09	1	31.17	1.05	29.74	0.55	1	28.41	1.03	29.20	0.55	1
26.96	1.06	30.06	0.49	1	31.21	1.07	30.36	0.19	1	28.58	1.02	28.84	0.55	1
28.94	1.12	31.90	−1.35	0	31.41	1.06	30.16	0.39	1	28.40	0.92	26.02	−0.93	0
28.21	1.11	31.57	−1.02	0	31.61	1.03	29.26	0.55	1	28.38	0.93	26.51	−0.44	0

Expected profit *MeanEmpiric*(*B*,*S*) and profit probability *ProbEmpiric*(*B*,*S*) on the basis of empirical distribution are calculated according to equations 2.2.2 and 2.2.4 respectively. Actually values of these criteria are obtained by averaging the fourth and the fifth columns of Table 2.2.1 respectively:

MeanEmpiric(*B*,*S*) = –0.27 and *ProbEmpiric*(*B*,*S*) = 0.53. These values suggest that the combination evaluated in this example is hardly perspective according to the criteria based on empirical distribution. However, we should not forget that the same criterion *MeanEmpiric*(*B*,*S*) has valuated the same combination as potentially profitable when its parameter *L* was valued within the range of 30 to 40 days (section 2.2.2, Figure 2.2.3). This fact points once again at the crucial importance of the thorough criteria parameterization and at the impact of this procedure on valuation of option combinations.

In the previously mentioned example,

criteria values are calculated for the expiration date. However, this should not always be the case. If an investor does not intend to hold a position until expiration, he can calculate criteria values for any intermediate date (depending on the prospective position holding period). This requires two modifications of the algorithm previously described. Firstly, we have to adjust the horizon of forecast according to the period of position holding (number of days from the position opening until the intermediate date). In our example, if we are interested in calculating criteria values not for the expiration date (June 15) but for June 10, the horizon of forecast will

be 11 days rather than 16 (the calculation of close prices ratio p_j will change accordingly). Secondly, the payoff function values should be calculated not for the expiration date but for the intermediate one. Here we face another obstacle: Although the calculation of payoff values for the expiration date is unambiguously defined, their estimation for any intermediate date is possible only by the application of option pricing models. Because all models—including the most sophisticated ones—have drawbacks arising from their basic assumptions, values of criteria calculated at intermediate dates may inherit various inaccuracies.

2.2.5 Modifications of Empirical Distribution

At first glance empirical distribution should produce better criteria than any other distribution because it is based on the price fluctuations that have actually occurred recently. (We have mentioned that empirical distribution can even reflect some past price trends.) However, the pitfall is right there. Market statistics is usually not stationary. The future does not always repeat the past. Strong trends and narrow trade ranges alternate unpredictably, and volatile periods come to the place of

tranquility irregularly without any distinguishable periodicity. That is why empirical distribution as a plain combination of past observations is not always the best way to create a probabilistic forecast. We developed several techniques to modify empirical distribution in the manner that makes it possible to moderate its drawbacks while maintaining its merits. Two of them are described here.

Weight Function as a Substitute for the Horizon of History Parameter

One of the modification methods is based on the introduction of weight

coefficients for past price movements. Higher weights are attributed to recent observations, whereas old data get relatively low weights. Such weighting permits to differentiate the influence of observations depending on their time of occurrence. Introduction of the weight coefficient modifies the equations 2.2.2 and 2.2.4 as follows:

(2.2.5)

$$MeanEmpiric(B,S) \approx \frac{1}{L-\tau} \sum_{j=1}^{L-\tau} \alpha_j PF(B,S,C_j^e) ,$$

(2.2.6)

$$ProbEmpiric(B,S) \approx \frac{1}{L-\tau}\sum_{j=1}^{L-\tau}\alpha_j\theta(PF(B,S,C_j^e)),$$

where $\alpha_j > 0, \sum_j \alpha_j = 1$ is a series of increasing positive weight coefficients.

Applying formulae 2.2.5 and 2.2.6 we can calculate criteria values so that recent price changes exert stronger influence on the final result than more distant ones. Implementation of weighting procedure leads to the gradual "forgetting" of old data. The function *a(j)*, which sets the weights $\{a_j\}$ can be of any form; it can be linear (with weight coefficients decreasing in time evenly), exponential, and so on. Selection of a

specific formula for this function can substitute the optimization of such important empirical distribution parameter as the horizon of history. It can be achieved in the following way: The horizon of history is fixed at its maximum (for example, more than several years or full historical database available). In this situation optimization of L is replaced by the selection of the way of "forgetting past data" that is done by choosing the appropriate equation for $a(j)$ function.

At first glance it seems that applying formulae 2.2.5 and 2.2.6 just replaces one complicated problem (L optimization) by another (selecting the formula for $a(j)$ function). Actually it is

not true. Because there are no objective criteria to optimize L, we are obliged to use the past data fitting that can turn to be just overoptimization. On the other hand, deliberate selection of $a(j)$ function can objectively reflect the investor's perception of market patterns and cycles.

Symmetrization of Empirical Distribution

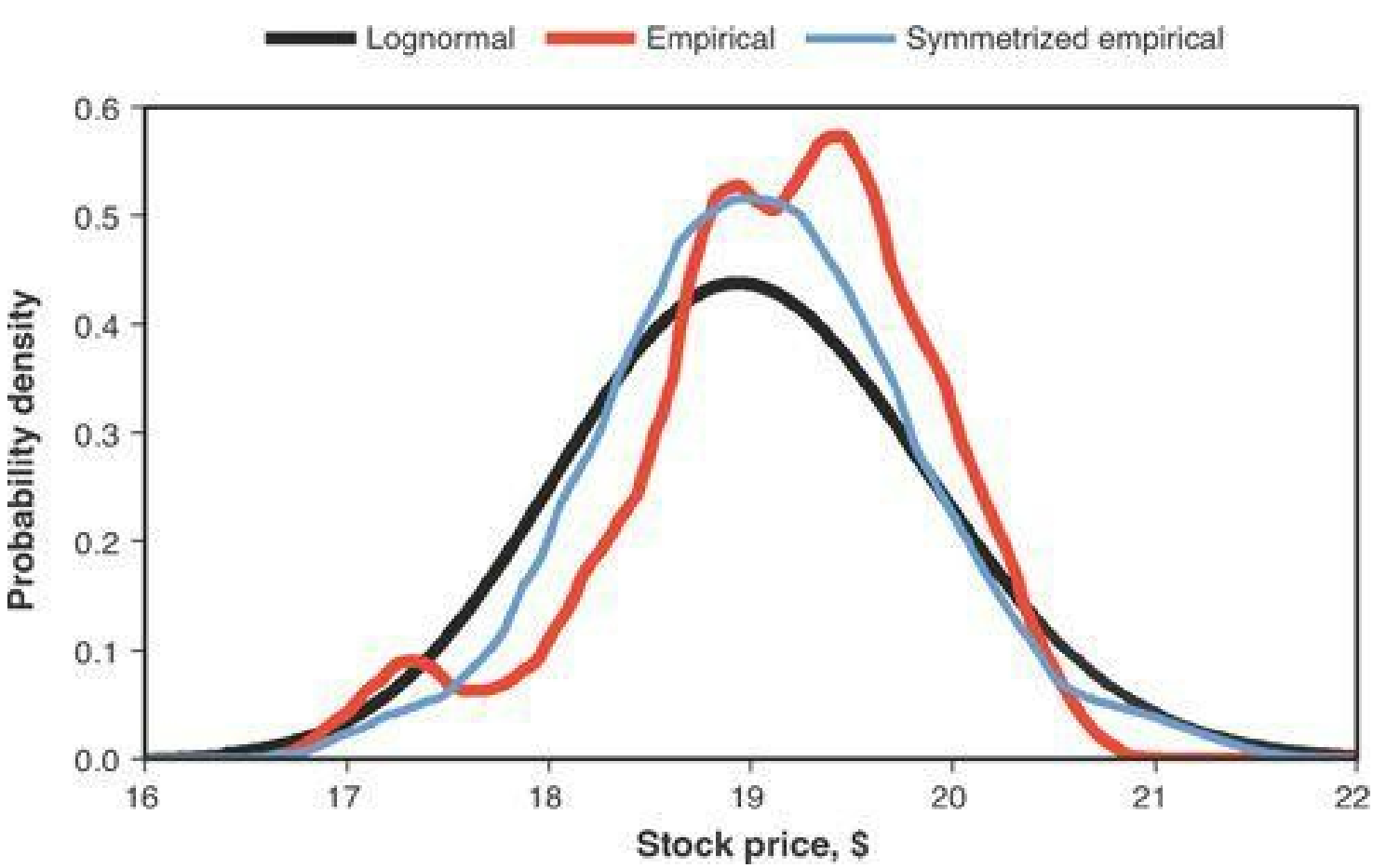
The second method of empirical distribution modification is based on its symmetrization. This procedure is performed through doubling the observations sample by adding to each $p(t) = C(t + \tau)/C(t)$ its inverse value p^{-}

$^{1}(t) = C(t)/C(t + \tau)$. This operation enriches the initial information used to calculate criteria by incorporating not only recorded past movements into the final distribution but their inverse values as well. Such modification allows for realizing two plausible assumptions. Firstly, it rejects the hypothesis of continuing market trends (empirical distributions reflect trends that prevailed in the past). All trends sooner or later turn around or enter the correction phase. There are no reliable instruments to estimate the probability of a trend continuation or turn. Secondly, future inverse price movements are likely to have characteristics similar to past movements.

Figure 2.2.5 demonstrates the effect of empirical distribution symmetrization. It depicts three distributions for the AMAT stock as of June 1, 2007, when its price was $19.34. All distributions are calculated as probabilistic forecasts for the nearest expiration date (June 16, 2007). Empirical distributions are based on a 120-day history. Historical volatility used for construction of lognormal distribution (HV=23.28%) was estimated on the same history length. The procedure of symmetrization does not only smooth the empirical distribution probability density function, but also eliminates its asymmetry while maintaining individual features and differences with lognormal distribution

(Figure 2.2.5).

Figure 2.2.5. Probability density functions of lognormal, empirical, and symmetrized empirical distributions built for AMAT stock.



Application of the symmetrization procedure produces two new criteria similar to those obtained using equations 2.2.1 and 2.2.2. The criterion expressing profit expected on the basis of symmetrized empirical distribution is formalized as follows:

(2.2.7)

$$MeanSymEmpiric(B,S) = \int_0^{\infty} PF(B,S,x)SymEmpiric(x)dx \ ,$$

and the criterion expressing profit probability is calculated as:

(2.2.8)

$$ProbSymEmpiric(B,S) = \int_0^{\infty} \theta(PF(B,S,x))SymEmpiric(x)dx \ ,$$

where *SymEmpiric*(*x*) is the probability density function of the symmetrized empirical distribution.

Formulae for simplified criteria calculation are analogous to expressions

2.2.2 and 2.2.4. Applying them and using the previously described algorithm we can estimate the criteria based on the symmetrized empirical distribution. To illustrate the procedure of simplified criteria calculation, we use the same option combination as in the example described in section 2.2.4. First, we add the inverse value to each p_j value shown in Table 2.2.1. For example, $p_j^{-1} = 1/1.02 = 0.98$ is added to $p_j = 1.02$. Then we compute all forecast prices and payoff function values related to them. In the previous example for $p_j^{-1} = 0.98$, we obtained the forecast price $C_j^{emp} = 0.98 \times 28.38 = 27.81$ and the payoff function related to it $PF_j = 0.55$. Averaging 148

values of the payoff function (74 values from Table 2.2.1 plus 74 new values based on inversed forecasts) and 148 values of the theta-function, we get the following criteria values: *MeanSymEmpiric*(*B*,*S*) = –0.37 and *ProbSymEmpiric*(*B*,*S*) = 0.45. In this particular case the symmetrized distribution gives worse values for both criteria compared with simple empirical distribution (section 2.2.4). In other circumstances—for other strategies, underlying assets or even with the same strategy and underlying assets, but at a different time point—it can be vice versa. Extensive statistical studies (not presented here) suggest that two criteria do not duplicate but rather supplement

each other.

2.3 Criteria Based on the Ratio of Expected Profit to Loss

2.3.1 Basic Concept and Criteria Calculation Method

In previous sections we described two main types of universal forecasting criteria based on probability distributions. The criteria forecasting future profit express it in terms of

mathematical expectations on the basis of a *certain* distribution. Similarly, the criteria forecasting profit probability express the probability of positive returns estimated on the basis of a *certain* distribution. Next we introduce new universal, but nonforecasting, criteria that combine the advantages of these indicators and possess additional useful features.

The argument values of any payoff function (that is, the set of possible future underlying asset prices) break up into two nonoverlapping subsets: profit and loss areas. The first area represents price values for which the payoff function is positive, and the second area includes prices for which the payoff

function becomes negative. If at the expiration date (or any other time for which we are calculating criteria values) the underlying price has a value falling within the first subset, then the combination will be profitable; otherwise it will not earn any profit. The subset of price values where the payoff function is positive is denoted by $I^+ = \{x : PF(B,S,x) > 0\}$. Correspondingly the subset of price values where the payoff function is negative is denoted by $I^- = \{x : PF(B,S,x) < 0\}$. Two integrals of the payoff function over profit $P(B,S)$ and loss $L(B,S)$ areas can be calculated separately for each subset:

(2.3.1)

$$P(B,S) = \int_{x \in I^+} PF(B,S,x) f(B,x) dx \ ,$$

(2.3.2)

$$L(B,S) = \int_{x \in I^-} PF(B,S,x) f(B,x) dx \ ,$$

where $PF(B,S,x)$ is the payoff function related to the combination S and the underlying asset B; x is the underlying asset price; $f(B,x)$ is a probability density function estimated for a certain future date. The difference between these two integrals $P(B,S) - L(B,S)$ is

nothing else but our "expected profit on the basis of $f(B,x)$ distribution" criterion. Its value can be the same under different $P(B,S)$ and $L(B,S)$ values (for example, if both integrals are increased or decreased by the same value).

Consider two option combinations S_1 and S_2 with the same expected profit values $P(B_1,S_1) - L(B_1,S_1) = P(B_2,S_2) - L(B_2,S_2)$, but $P(B_1,S_1) > P(B_2,S_2)$ and $L(B_1,S_1) > L(B_2,S_2)$. Suppose that we estimate two trading systems: The first one opens positions for the combinations having characteristics similar to that of S_1, whereas the second system opens positions for the combinations similar to

S_2. The equality of expected profit values means that in case of multiple trades both trading systems will generate the same profit. However, the first system is more volatile because it is characterized by higher potential profits and losses values. Thus, both trading strategies approach the same target but in different manners. Big losses of the first system can cause irretrievable capital losses, and when the funds allocated to trading are limited, it can result in bankruptcy before the system can win back. (The details of capital allocation deserve a separate discussion that is given elsewhere.) Similarly, target profitability of the first system can remain unachieved if the system is

limited by the maximum drawdown parameter that stops trading if a certain threshold value is reached. It means that given the same expected profit combination, S_1 is more risky than combination S_2. By maximizing the following function, we can maximize potential profit while minimizing possible losses (that is, risk):

(2.3.3)

$$PL_factor(B,S) = \frac{P(B,S) - L(B,S)}{P(B,S) + L(B,S)}$$

PL_factor(*B*,*S*) is a "ratio of expected profit to loss" criterion with values

ranging from –1 to 1. Minimum value –1 means that the combination is unprofitable for all possible underlying asset prices; maximum value +1 means that the combination is profitable for all underlying prices. (Both extremes are unrealistic.)

According to our classification (Chapter 1), *PL_factor*(*B*,*S*) is a universal nonforecasting criterion. It can be applied to all option strategies with any payoff functions. Actually, it represents a whole set of criteria because every probabilities distribution $f(B,x)$ generates its own criterion. Thus it is possible to use several *PL_factor*(*B*,*S*) criteria based on different distributions (lognormal, empirical, symmetrized

empirical, or any other) simultaneously in the multicriteria selection of option combinations.

The evident advantages of this criterion are (1) its abstract nature (it is not expressed in any particular units, such as percentage or currency, as is the case with criteria estimating expected profits and probabilities) and (2) the uniform scaling of its values which lie in the strictly defined interval. (The latter feature allows easy processing and manipulation of criterion values.) Besides, applying the ratio of expected profit to loss allows comparison of option combinations by their inner structure regardless of the investment amount and margin requirements. Due to

the specific construction of this criterion (formula 2.3.3), the form of profit expression and its normalization become unimportant. Calculation of criteria that estimate mathematical expectations in terms of relative yields require normalization by margin requirements and the time of position holding. However, any normalization executed via dividing profits and losses by investments and time would be offset because it has to be performed both in the numerator and the denominator (see expression 2.3.3). Thus the *PL_factor*(*B*,*S*) criterion is neutral to such operations.

Because the fluctuations of the underlying asset price during the life of

the combination constantly change margin requirements, the exact investment amount is hard to identify. Normalization by time generates some additional problems arising from uncertainty about the investment period. That is why the absence of such normalizations makes this criterion more reliable and universal for systematic evaluation of option combinations.

2.3.2 Criteria Calculation Example

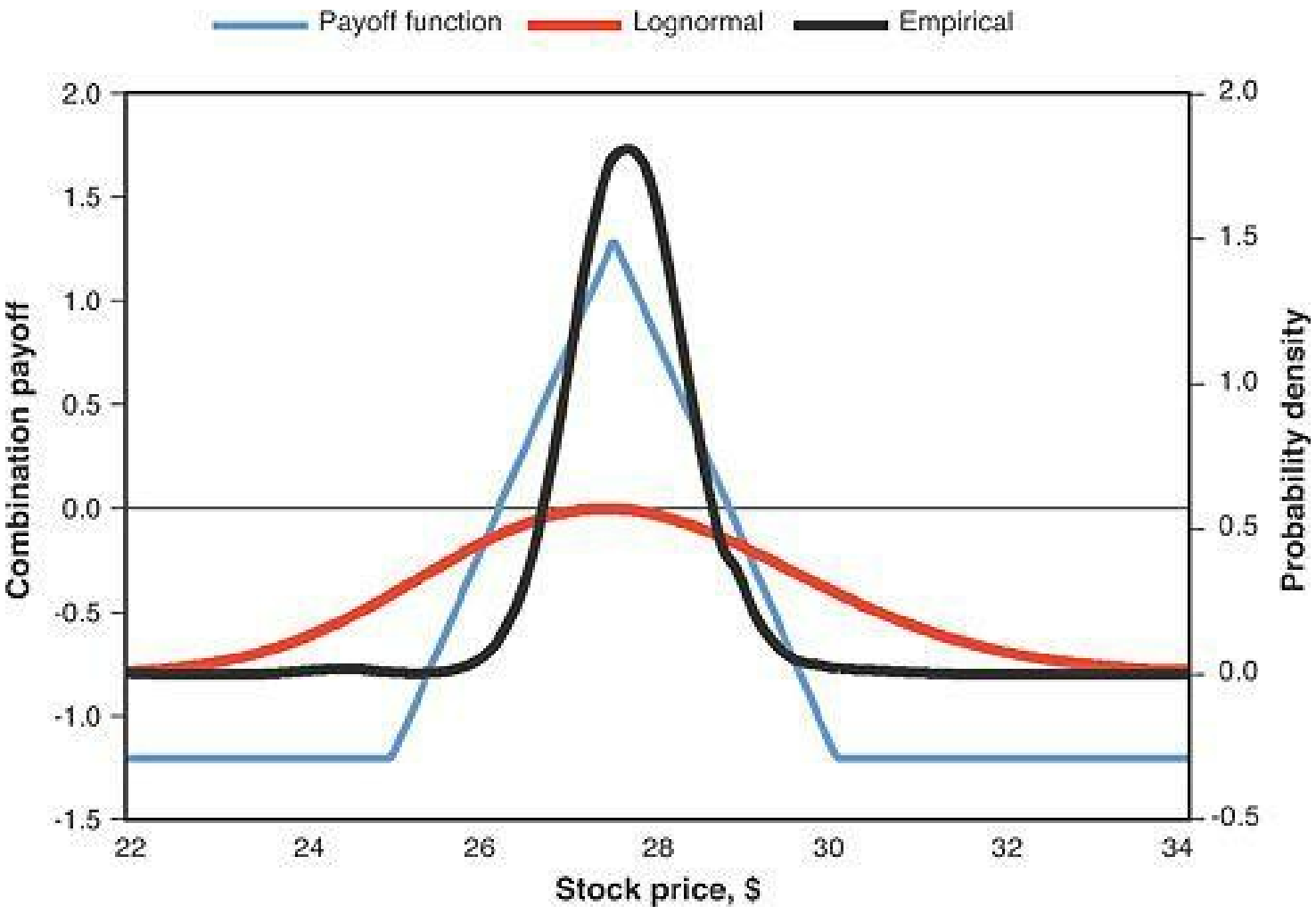
Consider the “butterfly” combination for the YHOO stock. Its close price on June 25, 2007, was $27.64. The combination consists of the following options

expiring on July 21, 2007: one long Call 25 (ask price $2.95); two short Calls 27.5 (bid price $1.05); one long Call 30 (ask price $0.35).

Figure 2.3.1 shows the payoff function of this combination (built for the expiration date) and probability density functions of lognormal (28.6% volatility) and empirical (250-day horizon of history) distribution. Note that the shapes of two distributions differ substantially. The empirical distribution has a much higher peak (meaning that it forecasts a frequent occurrence of small price movements) and its tails are thin (implying that strong price changes are considered as relatively rare events). To the contrary, the lognormal distribution is

characterized by a rather flat function predicting a more common happening of moderate and strong price movements and estimating small price fluctuations as infrequent.

Figure 2.3.1. Probability density functions of the empirical and lognormal distributions built for the YHOO stock and the payoff function of the butterfly strategy.



Simple numerical point-by-point integration (100 points were used in this calculation) gives the following values:

for lognormal distribution:

P(YHOO, *Butterfly*) = 0.3113;

L(YHOO, *Butterfly*) = 0.4402;

PL_factor(YHOO, *Butterfly*) = – 0.1715;

for empirical distribution:

P(YHOO, *Butterfly*) = 0.7623;

L(YHOO, *Butterfly*) = 0.0374;

PL_factor(YHOO, *Butterfly*) = 0.9064.

Note that the ratio of expected profit to loss calculated on the basis of lognormal distribution is negative, which means that this combination is estimated by this criterion as potentially unprofitable. At the same time the criterion calculated on the basis of empirical distribution identifies the same combination as profitable. Such “pluralism” enriches the

information obtained while evaluating option combinations by criteria. To make a final investment decision, we just have to decide which distribution is better in forecasting future price movements.

2.4 Criteria Based on Expert Distribution

2.4.1 Basic Concept and Criteria Calculation Method

Up to here we have been developing criteria based on probabilistic scenarios of future underlying asset price

fluctuations. Past data was the main source of information. Applying this approach we built different probability density functions forecasting price distributions at different future time points. Lognormal and empirical distributions described in previous sections represent two typical examples of this method. However, relying solely on historical data is fraught with risks of omitting details that might turn out to be essential for future events forecasting. Furthermore, setting the parameters of lognormal distribution—its mean and variance—implies that we assume market neutrality and specific volatility features. (These assumptions may turn out to be incorrect.) The form of

empirical distribution also depends on peculiarities inherent to its generation algorithm. An important feature is common for all probability distributions based on historical data: Although the algorithms used for construction and application of such distributions can be easily formalized, the reliability of their forecasts frequently raises many reasonable doubts.

Criteria discussed in this section are based on an absolutely different approach. As it follows from the heading, creating such criteria implies engaging specialists to evaluate qualities of option combinations on the basis of fundamental analysis of their underlying assets. Professional opinions of experts

must meet two main requirements. Firstly, they should not be based on past price data but only on fundamental characteristics of an underlying asset. (Otherwise we get a kind of empirical or similar distribution rather than the expert one.) Secondly, the expert opinion must be expressed in strictly formalized numerical form. This means that an expert must not only indicate the direction of anticipated price movement but also specify more and less probable future price ranges. The best way to formalize the expert forecast is to build the probability density functions—like we did for other criteria—with the only difference that the probabilities of different outcomes are estimated in this

case on the basis of expert opinions and not of past events.

Formalizing the expert forecast as a probability density function is an extremely complicated task. However, it can be successfully solved if a specialist is provided with a proper instrument—we call it the *interactive probability scenario builder*. Scenario builder is a software program consisting of several essential modules.

- The first and the main one represents a mathematical structure intended to calculate different probability distributions with parameter values fixed by the expert.

- The second module combines the set of separate distributions created in the first module into a unified probability density function.
- The third module calculates criteria on the basis of the combined density function produced in the second module.

Application of the *interactive probability scenario builder* software reduces the expert's task to consecutive execution of the following steps:

1. From the variety of basic standard distributions, select the most appropriate ones to create the

combined probability density function.

2. Determine optimal values for parameters of distributions selected at the first step.
3. Determine criteria that can reflect the forecast of future events most adequately and calculate their values on the basis of the unified density function.

The main and necessary elements of the first module are uniform, lognormal, empirical, symmetrized empirical, exponential distributions, and at least one distribution with fat tails. Although other distributions may also be included in this basic set, practice shows that

these previously mentioned main elements represent the sufficient kit required to structure a forecast of any complexity. Two features of any distribution are of particular interest for the expert: its position relative to the price axis and the spread. Position is a coordinate of the most typical distribution point, that is, the point where the density function reaches its maximum (in statistics it is called *mode*). In the case of nonexpert criteria, the position is often taken as equal to the current underlying asset price. Here it is not necessary because this is the parameter that most fully reflects the specialist's view on the future price evolution. Analogously, the spread is not

necessarily derived from historical volatility. The expert must deeply understand the nature of these distribution parameters—position and spread—to manipulate them freely to select their optimal values. The change of these parameters in the probability scenario builder software should be easy to make by a simple dragging operation or by inputting respective values.

2.4.2 Set of Standard Distributions

Every element of the first module of the *interactive probability scenario builder* should be a simple standard distribution

with the minimum number of parameters. Its interpretation must be obvious for an expert and realize some simple idea of a future scenario expressed in the probabilistic form. Here we dwell on some of the basic elements of the first module.

Uniform Distribution

This distribution has the following density function:

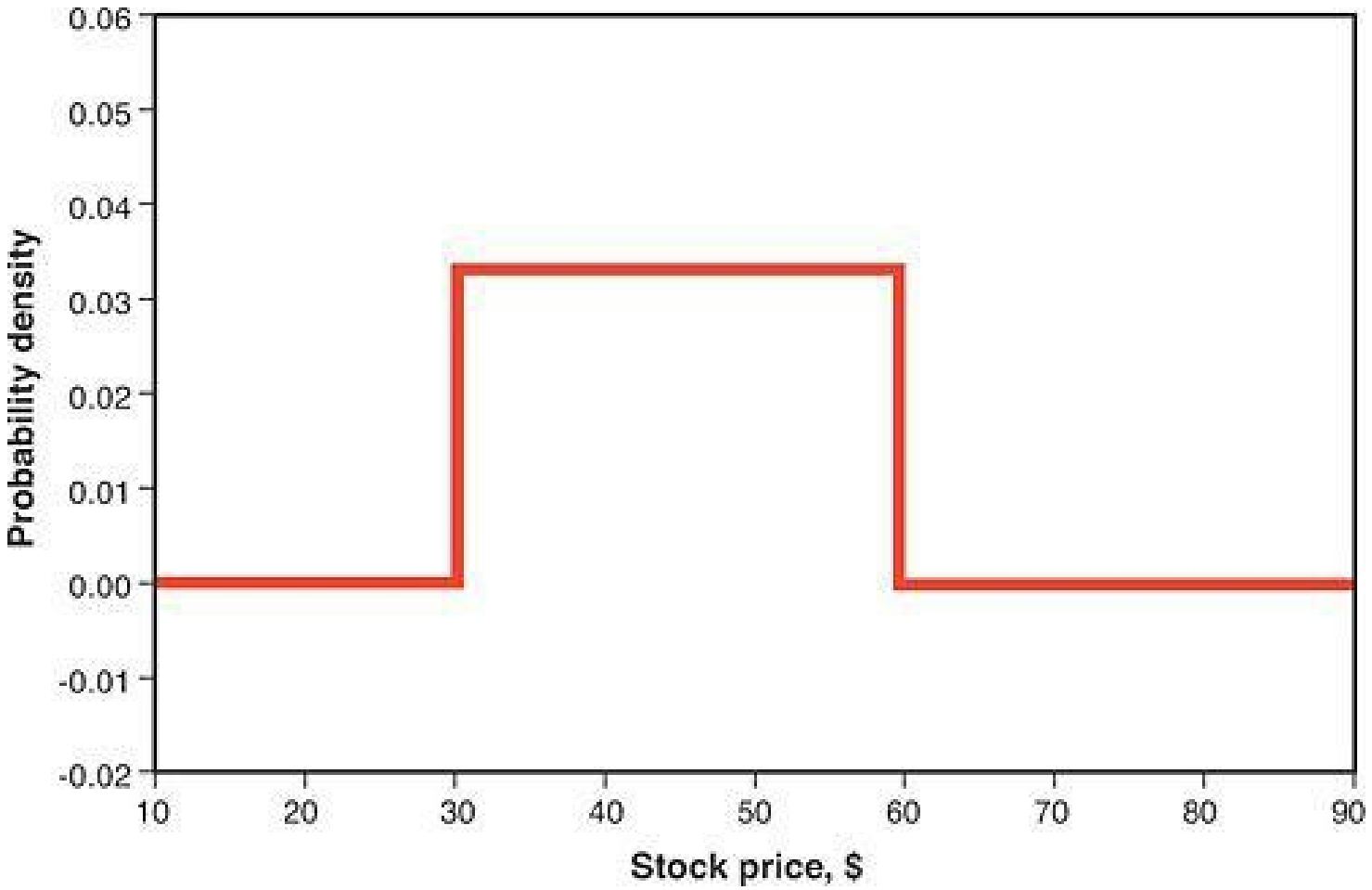
$$f(x)=\begin{cases}1/(C_{\max}-C_{\min}), x\in\left[C_{\min}, C_{\max}\right],\\ 0, x\notin\left[C_{\min}, C_{\max}\right]\end{cases}$$

Here C_{min} and C_{max} are the minimum and maximum possible values of the underlying asset price. Applying uniform distribution the expert claims that at some future time T the price will be at any point of the $[C_{min}, C_{max}]$ range with equal probability and will not go beyond it. Mathematical expectation of the future price value is $x = \frac{C_{min} + C_{max}}{2}$. The parameters that must be fixed by the expert are C_{min} and C_{max}. The middle of the $[C_{min}, C_{max}]$ segment represents the mathematical expectation. The spread of the uniform distribution is automatically determined by the length of the segment $C_{max} - C_{min}$.·

Figure 2.4.1 shows the density function of uniform distribution for $C_{min} = 30$ and $C_{max} = 60$. This distribution forecasts that the price will take any value between \$30 and \$60 with equal probability and under no circumstances will go beyond this range. Such expert estimates can follow from fundamental analysis of threshold oversold and overbought levels of a stock. It is important to emphasize that we do not discuss the final forecast, but rather one of the basic elements required to compile it.

Figure 2.4.1. Probability density function of the

uniform distribution built for $C_{min} = 30$ and $C_{max} = 60$.



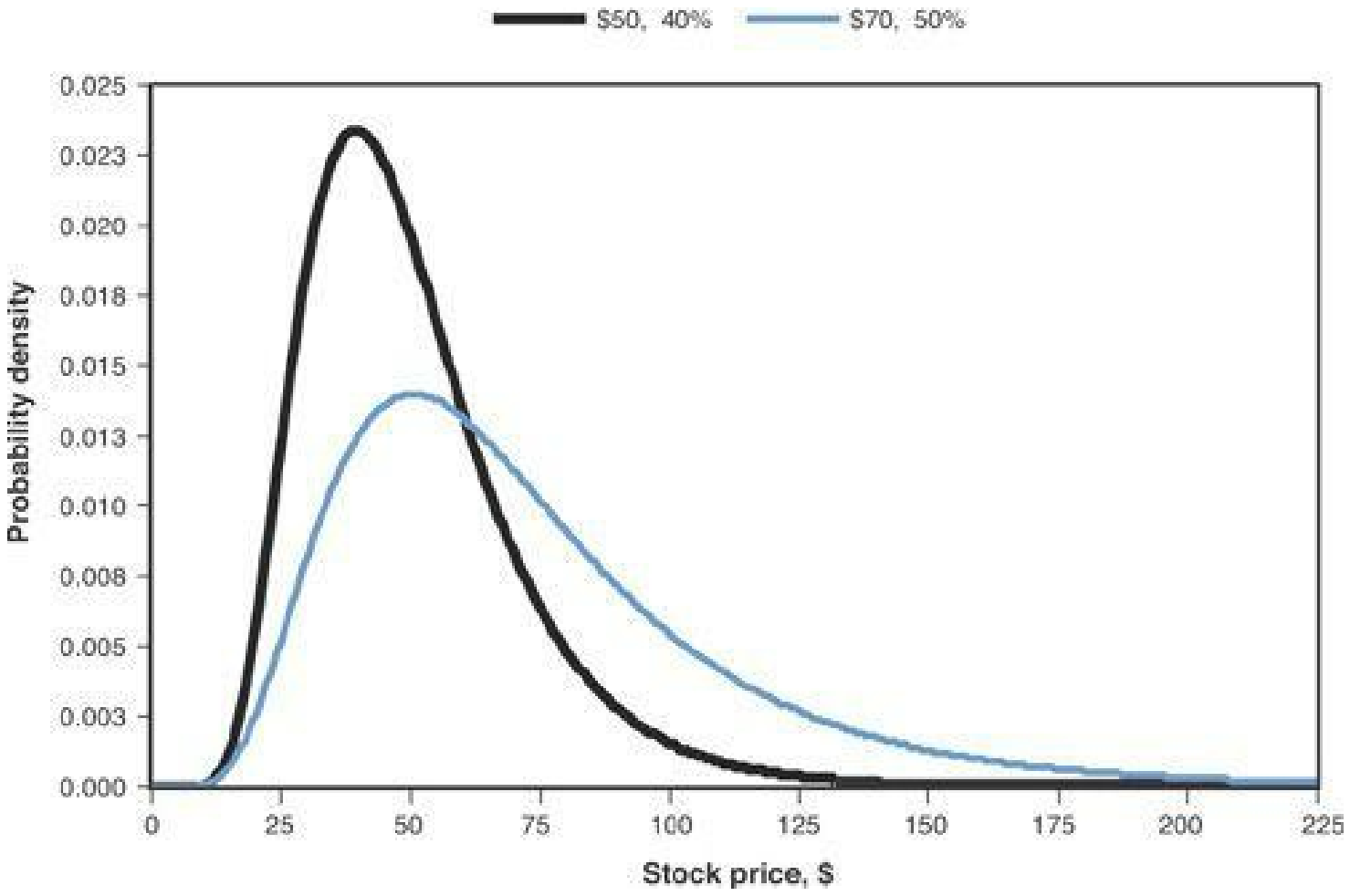
Lognormal Distribution

This distribution is the most popular one in the option pricing theory. The mathematical background of its density function was described in section 2.1.1. The most convenient way to handle the lognormal distribution is to manipulate with the mathematical expectation of the underlying asset price (which is the average price weighted by probability) and with its variance (expressed through volatility). The values of these parameters, which are available from numerous sources of market information, are continually analyzed and interpreted by experts.

Figure 2.4.2 shows two probability density functions of lognormal distribution for a weighted-average

price of $50 and a 40% volatility, and for a weighted-average price of $70 and a 50% volatility. Because the lognormal distribution is asymmetric, its peak (formally speaking—mode) corresponding to the most probable price value does not coincide with the mathematical expectation. In Figure 2.4.2 the most probable price (mode) of the function with a weighted-average price of $70 is around $50, and for the function with a weighted-average price of $50, the peak is at $40. Thereby, the expert gets an additional parameter—the most probable price—to handle the lognormal distribution.

Figure 2.4.2. Probability density functions of lognormal distribution built for different parameters of mathematical expectation and volatility.



How could this additional third parameter be used? To answer this question we have to discuss the general scheme of handling the lognormal distribution by experts.

An expert who uses lognormal distribution as one of the basic elements forming a unified probability density function assumes that fluctuations of the underlying asset price represent a geometrical Brownian motion with a certain mathematical expectation value at moment T and variance. However, this assumption follows solely from the mathematical form of lognormal distribution. In reality, the expert, as a fundamental analyst, considers price

movement not as a random Brownian process but as a movement toward a certain target. He expects that this target will sooner or later be achieved (or, at least, approached) by the pseudo-chaotically fluctuating price. To proceed with the Brownian motion analogy, we can say that the analyst should think of the price as of a charged Brownian particle that moves not in the neutral environment but in the electrostatic field. In this situation the expert has to estimate the intensity of the electrostatic field (which determines the speed of the price —"particle" on the way to its target) and to locate the position of the electrode with the opposite charge (which determines the direction of the price

—“particle”). The theoretically straight path of this Brownian particle is modified by the unpredictable impact of chaotically moving surrounding molecules which—coming back from this physical allegory to the reality of the financial world—can be compared to the influence of economic and corporate news entering the market unpredictably and irregularly. This philosophy should be used by a fundamental analyst when applying the lognormal distribution to market analysis and forecasting.

Based on his vision of future market events, an expert has to determine values for lognormal distribution parameters. Namely, he must predict the average future price and its variance. The former

is not complicated, if the analyst has a deep understanding of the fundamental properties of the considered underlying asset. On the other hand, determining the variance may become an almost unrealizable task if the analytical tools available to the expert are strictly limited to the instruments of fundamental analysis. Remember that the expert should base his forecasts only on the fundamental characteristics of an underlying asset whereas an estimate of the variance is hard to derive from the information based on economical indicators. There are several ways to solve this issue. Variance can be forecast on the basis of past volatility. However, in this case we get hybrid

criteria that combine both approaches—expert forecast (determining the average future price) and approach based on past data analysis (determining variance). Alternatively, we can obtain the variance value from option market prices by deriving their implied volatility. However, in this case instead of forming our own view on underlying volatility, we are forced to rely on the opinion of the broad market. Below we introduce the third method: using the most probable price (mode) as an additional parameter replacing variance.

To use lognormal distribution, the expert has to construct the probability density function similar to that presented in Figure 2.4.2. Such functions are defined

by fixing two parameters: mean and variance. If the expert cannot define the variance, he can fix the values of mean and mode (the most probable price) and then select the variance so that the final function corresponds to these two parameters. In this way we obtain the "implied variance" value allowing construction of the lognormal distribution probability density function. How should the expert assign different values to these close parameters, the mean, and the mode? We propose the following sequence of steps.

1. The expert defines the first parameter: the mode. This is easy because the mode, which is the

most probable price, coincides with the future price forecast by the expert's fundamental analysis.

2. Then the expert assigns probabilities to other price outcomes. To do this, he chooses some reasonable (not too wide) price range, and each price from within this range (with a certain step) is given a probability of its realization for the time of forecast. The sum of all assigned probabilities must be equal to one.
3. These probabilities are multiplied by the corresponding prices, the results are summed up, and the sum is divided by the number of items.

This operation provides us with a value of the second parameter: the mean (which is the mathematical expectation of future price value). The variance value, corresponding to the values of these two parameters, is then obtained iteratively. Further density function creation is obvious.

Exponential Distribution

This distribution plays an important role in the basic set of standard distributions (the first module of the *interactive probability scenario builder*). It is very useful for forecasting the outcomes of corporate upcoming events. Consider the

situation when the FDA (Food and Drug Administration) decision on some drug developed by a biotechnological company is expected on a certain date. According to the expert's opinion, if the FDA approves the commercialization of this drug, the stock price will rise above a certain threshold level. The higher the extent of price excess over the threshold level, the lower its probability. In case of the opposite FDA decision—prohibition of drug commercialization—the stock price will plunge and break another threshold level. As a result, the price will be lower than the predetermined level. And again the lower the price is relative to the threshold level, the lower the

probability of such outcome. The most convenient way to express the forecast of the upcoming event is to combine two exponential distributions (one for each variant of the FDA decision).

The probability density function of exponential distribution is defined as follows:

$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}, x \geq 0,$$

where μ is the mathematical expectation of the exponential distribution (its variance equals to μ^2).

Consider the situation when the expert

fixes the following values: $x_u = 60$ as the upper threshold level (which corresponds to the positive outcome of the forecast corporate event, such as drug approval in our example) and $x_d = 45$ as the lower threshold level (corresponding to the negative outcome of the forecast corporate event, such as drug refusal in our example). The distribution describing the positive outcome (price moving up) has the following form:

$$f_u(x) = \frac{1}{\sigma_u} e^{-(x-x_u)/\sigma_u}, x \geq x_u,$$

and the distribution describing the negative outcome (price moving down)

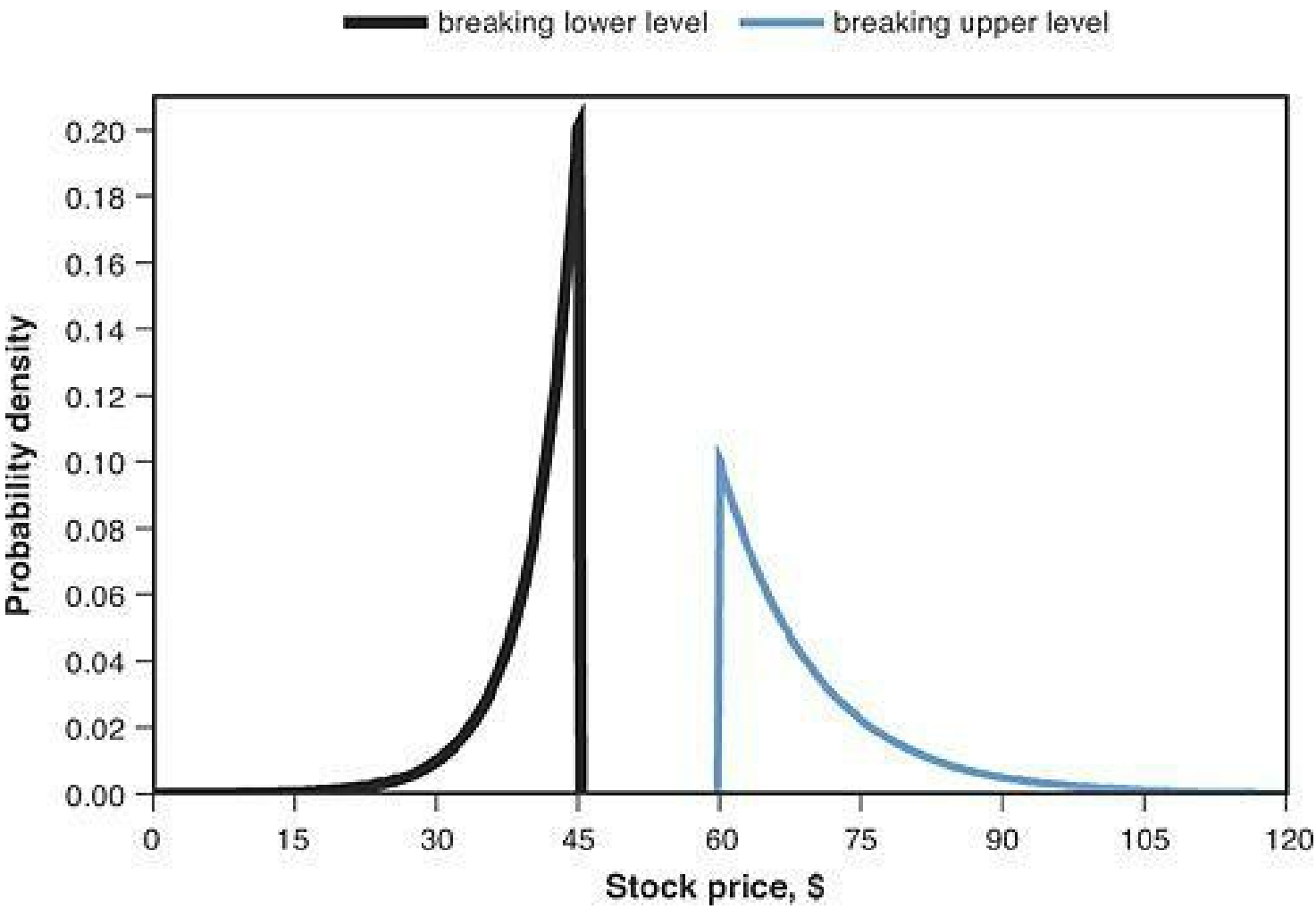
has the following form:

$$f_d(x) = \frac{1}{\sigma_d} e^{-(x-x_d)/\sigma_d}, x \le x_d .$$

The variance parameters σ_u, σ_d should also be selected by the expert individually for each case. These parameters define the rate of decrease of the probability corresponding to the prices upper (positive outcome) or lower (negative outcome) than fixed threshold levels. Suppose that the expert sets $\sigma_u = 10$ for the exponential distribution describing the up movement of the price and $\sigma_d = 5$ for the down movement. Then the probability density functions for breaking the \$60 threshold

level by the price moving up and the $45 threshold level by the price moving down will have the shapes shown on Figure 2.4.3.

Figure 2.4.3. Probability density functions of exponential distribution built for two cases: price breaking upper threshold level ($x_u = 60$, $\sigma_u = 10$) and price breaking lower threshold level ($x_d = 45$, $\sigma_d = 5$).



The three distribution types described here (uniform, lognormal, and exponential) form quite a representative basic set. To enhance the expert's flexibility in the process of creating the final combined probability density function, this set should be enlarged with

more complicated distributions. It would be useful to add two main forms of empirical distribution and several distributions simulating "fat tails."

2.4.3 Combining Separate Standard Distributions into a Unified Probability Density Function

By choosing a certain distribution, the expert fixes a model intended to describe a certain variant of the possible future market developments. For a high-quality forecast the expert should take into consideration many potentially possible alternatives. This concept may be illustrated by our example of a

biotechnological company anticipating an approval of its new drug. By now we have discussed only two scenarios: its approval and disapproval. Actually, there are at least two other possible outcomes. The first possibility is that the decision of the FDA will be postponed for an uncertain period of time. The second possibility corresponds to taking an interim uncertain decision, neither approval nor refusal (for example, requirement to perform additional clinical tests of the drug). Considering the situation in this way implies that price breakings of the upper and lower thresholds (modeled by means of exponential distributions) represent just two separate components of the final

unified forecast that has to be made. To construct it, we have to select distributions adequately describing two additional possible outcomes. For the case of the postponed decision (when there are no events in the nearest future), we can use lognormal distribution with parameters defined by application of standard fundamental analysis methods. The second case (interim decision) is much harder to simulate. Market reactions to the decisions of such kind are unpredictable and range from price soaring to a sharp fall (depending on the details of information). The lack of any reaction or just a minor reaction is also possible. Such outcomes can be described by uniform distribution with a

sufficiently large price range.

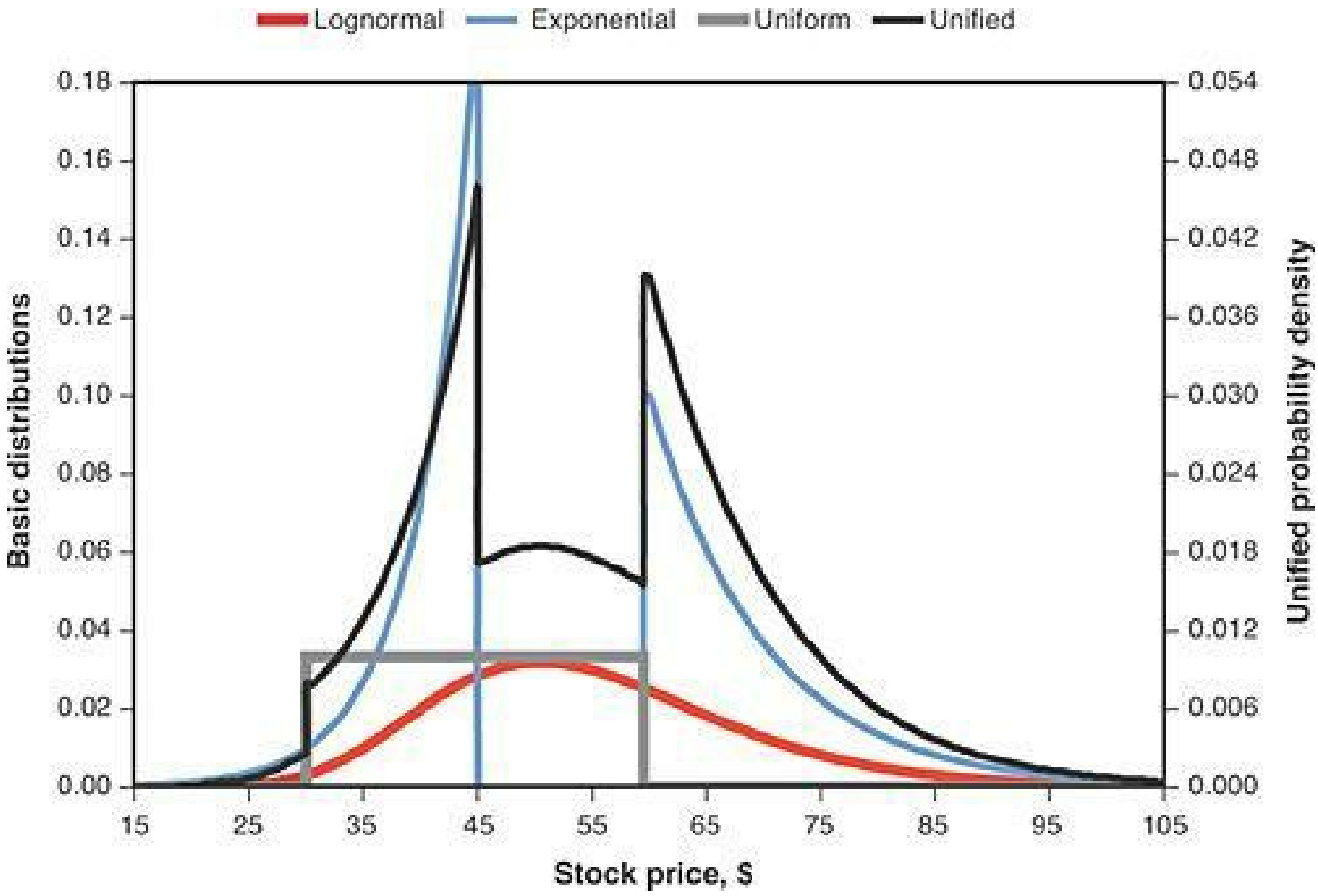
We have selected distributions for all the four possible outcomes. The next step is to define the probability of each of these outcomes. The expert should make an estimate of these probabilities in the form of the weight coefficient assigned to each distribution; the greater this weight, the more probable the outcome. For example, if the expert thinks that breaking the lower threshold level is twice as probable as breaking the upper threshold level, he can assign the weight coefficient 2 to the former and the coefficient 1 to the latter. Finally, taking into account all distributions, their parameters and weights, the expert creates the final probability density

function as a weighted sum of separate distributions.

The colored lines on Figure 2.4.4 show the four distributions from the example described: a lognormal distribution with the average price value of x = \$55 and volatility of 24% (weight of 3); an exponential distribution corresponding to breaking the lower threshold level of $x_d = 45$ with $\sigma_d = 5$ (weight of 1); an exponential distribution corresponding to breaking the upper threshold level of $x_u = 60$ with $\sigma_u = 10$ (weight of 2); and a uniform distribution reflecting the idea of equal probability for the price to take on any value within the \$30-\$60 range (weight of 1). The unified distribution

shown in Figure 2.4.4 has a complex shape resulting from the combination of different basic distributions.

Figure 2.4.4. Unified probability density function created by combining separate basic distributions.

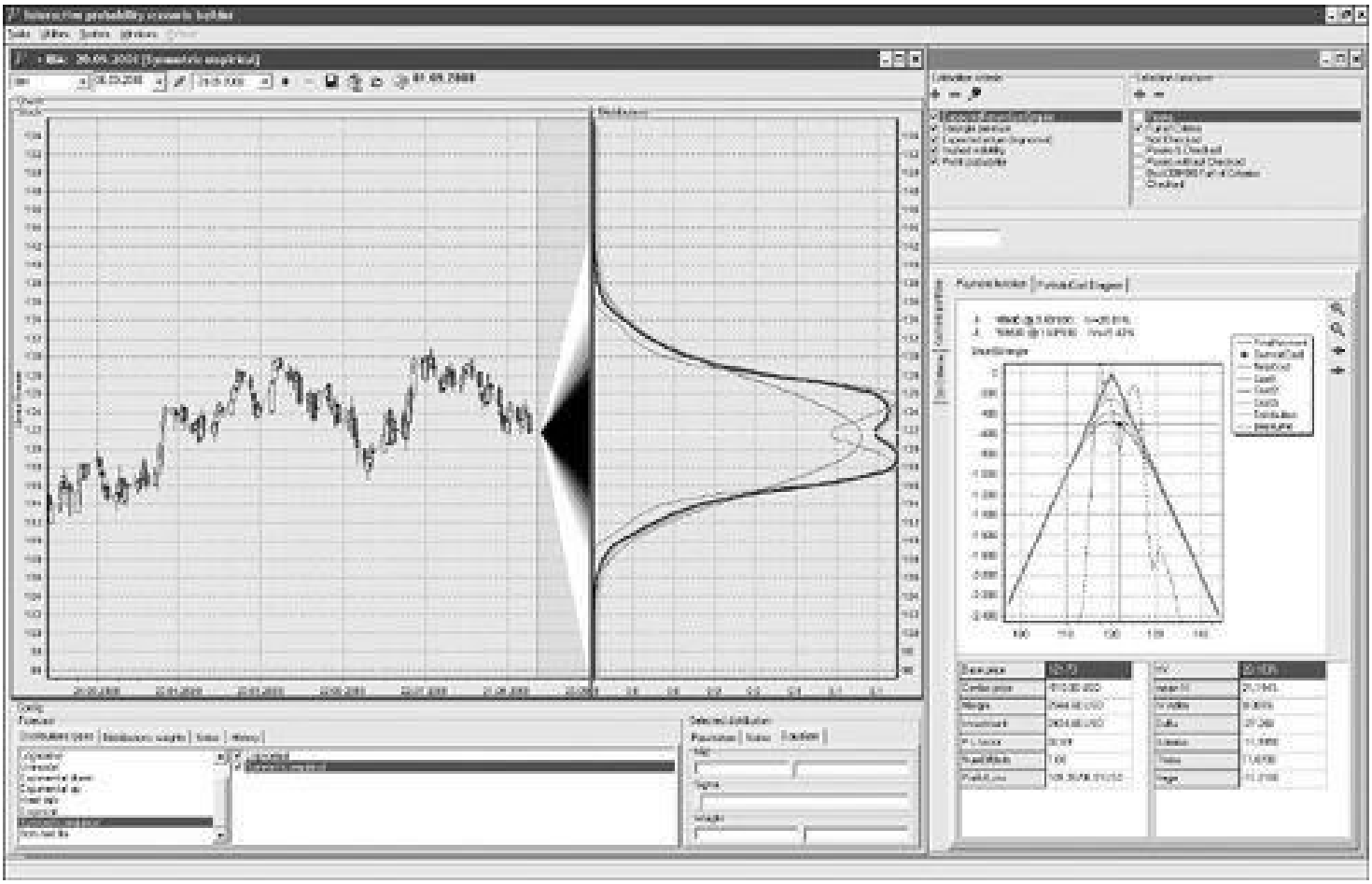


Consider another example of creating a unified probability density function on the basis of several basic distributions. Suppose that an expert was requested to create a forecast for IBM stock for the September 2008, expiration date. (The date of the forecast is August 29, 2008.) After the fundamental research was

accomplished, he decided to use the lognormal and the empirical distributions and selected the following values for their parameters. The expected price of the lognormal distribution was set as equal to the current price, and the variance was derived from 120-day historical volatility. For the empirical distribution the expert selected an 80-day horizon of history. Both distributions were combined with equal weights. The thin lines in Figure 2.4.5 depict probability density functions of the lognormal and empirical distributions, whereas the bold line shows the unified distribution. Note that the final distribution has a concave plateau in its central part. It

means that the forecast based on both the lognormal and empirical distributions assigns rather high and similar probabilities to all prices within a relatively broad range of $117-$126.

Figure 2.4.5. Unified probability density function (bold line), created by combining the empirical and the lognormal distributions of IBM stock price.



2.4.4 Criteria Calculation on the Basis of the Unified Probability Density Function

As described in the previous section, an expert obtains the unified density function $f(B,x)$ for price x of underlying

asset B by combining several basic distributions (each of which represents a separate scenario forecast for a future date T), selecting their parameters and assigning each of them a weight corresponding to the probability of this particular scenario. The unified function can be used to calculate different criteria in the form of the following integral:

(2.4.1)

$$I(B,S) = \int_0^\infty P(B,S,x,T) f(B,x) dx \ .$$

If the function $P(B,S,x,T)$ is a payoff function of combination S, the integral

$I(B,S)$ will estimate the expected profit on the basis of unified distribution $f(B,x)$. If S is a trivial combination consisting of the only option, this integral will evaluate the fair value of the option. Naturally, the accuracy of this estimate fully depends on the choice of basic distributions, their parameters and weights.

If the value of the function $P(B,S,x,T)$ represents the profit of combination S normalized by margin requirements and time of position holding, then the integral $I(B,S)$ will estimate the expected yield on the basis of unified distribution. Other criteria (like profit probabilities and ratios of expected profit to loss) can be calculated in a similar way (sections

2.1 to 2.3).

2.4.5 Construction and Valuation of Complex Strategies Based on the Unified Probability Density Function

In the context of the systematic approach, we define the option strategy as a specific form of the payoff function corresponding to a specific structure of an option combination. The majority of simple option strategies have unimodal and, in most cases, symmetric payoff functions. Unimodality implies that the payoff function has a single peak or bottom. Combinations corresponding to

simple option strategies can be adequately valued by one of the basic probability density functions because all of them are unimodal. For example, payoff functions of such simple strategies as long or short strangle/straddle, long or short calendar spread have one and only one minimum or maximum. Consequently, option combinations belonging to these strategies can be effectively estimated by application of one of the basic density functions.

Unified density functions constructed by combining several basic distributions are usually polymodal with several local maximums. For such functions it is preferable to create complex option

strategies with payoff functions that also have several local peaks. The main point here is to superimpose peaks and bottoms of the payoff function with those of the unified density function. The closer the extreme values, the higher the criteria values. The increase in criteria values implies that profit probability of the complex option strategy also increases. One of the main advantages of options in comparison to other financial instruments is the possibility to construct payoff functions of almost any desired shape. We exploit this quality to create complex option strategies with payoff functions matching the unified expert distributions.

Step by step we consider the following

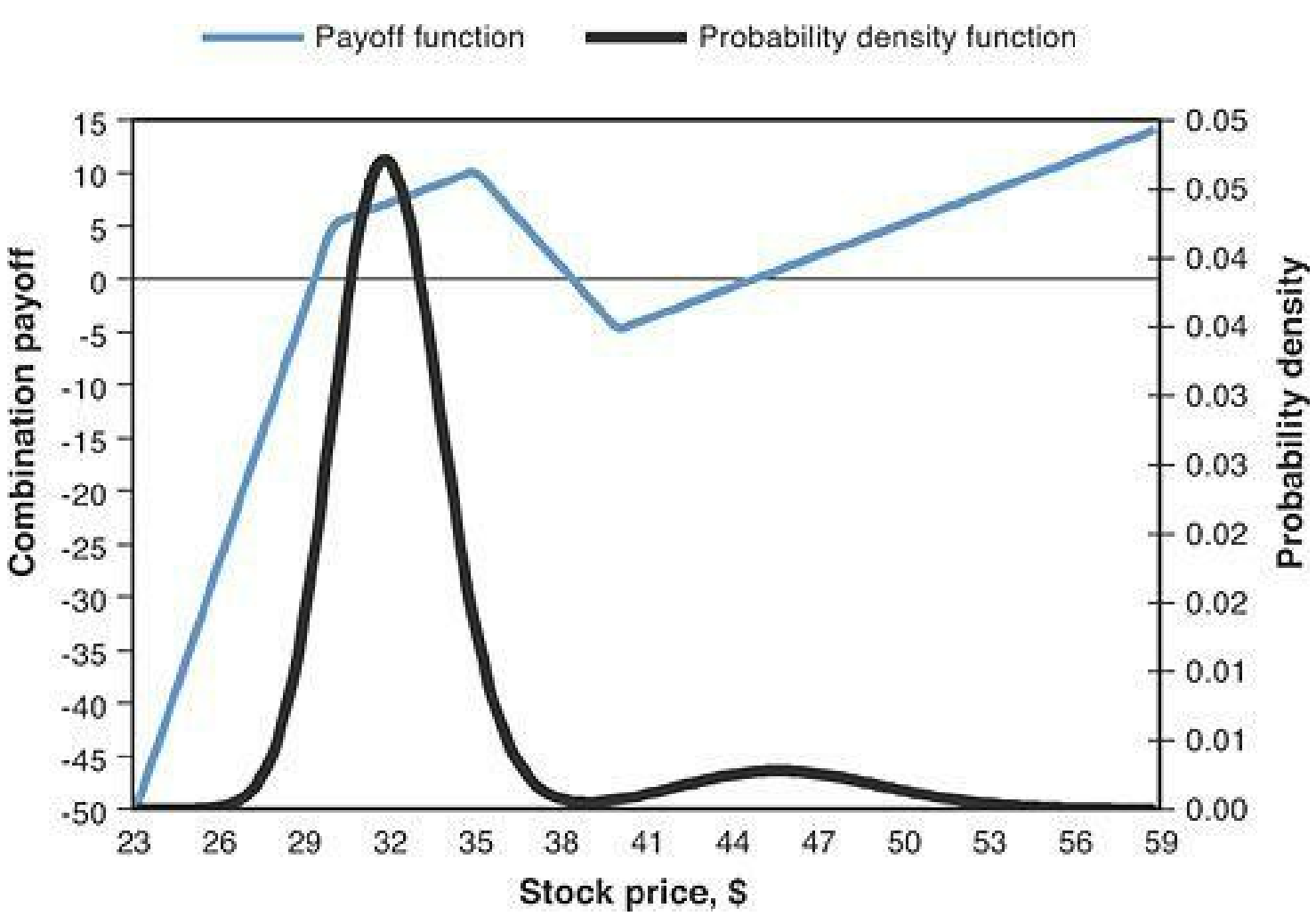
example of (1) making an expert forecast, (2) building a unified distribution on its basis, and (3) creating a corresponding complex option strategy.

The expert reckons that the VCLK company may be acquired by Microsoft corporation until the next expiration date, June 16, 2007. On May 23, 2007 this stock was worth $32.63. According to experts' estimation, the possible acquisition price is around $46. After evaluating the possible acquisition price range, the expert formulates the first scenario in the form of a lognormal distribution with $46 mean and variance corresponding to the width of the price range. The second scenario implies that

the acquisition will not take place until expiration. In this case, the expert forecasts a slight decrease in price and describes it by a lognormal distribution with $32 mean and variance derived from historical volatility. Based on the probabilities of each scenario, the expert assigned the weight of 0.1 to the first scenario and the weight of 0.9 to the second one.

After combining two lognormal distributions into the unified probability density function, the expert gets the polymodal function with two local maximums to the right and to the left from the current stock price (Figure 2.4.6).

Figure 2.4.6. Expert probability density function obtained by combining two lognormal distributions and the payoff function of the complex option strategy.



Now the expert has to create such an option combination in which positive areas of its payoff function would correspond to the unified density function peaks and negative areas would be close to the low values of the unified probability density function. Quite a few combinations satisfying this condition

can be created.

What combination of this variety has the best performance potential? The answer is obvious: Use the criteria to find out. By applying formula 2.4.1 we can calculate several criteria based on integration of the payoff function over the unified probability density function. As an example we calculate three criteria—expected profit value, profit probability, and the ratio of profit to loss—all on the basis of the density function presented in Figure 2.4.6.

We do not discuss all possible combinations that were rejected due to obviously inferior criteria values. The structure of the three best combinations

and their corresponding criteria values are shown in Table 2.4.1. The second combination is the best one according to all the criteria. (Its payoff function for the expiration date is shown in Figure 2.4.6.) Thus, we created a complex option strategy matching the unified expert probability density function. The real profitability of this strategy will fully depend on the quality of the expert forecast.

Table 2.4.1. The Structure and the Criteria Values of Three Option Combinations Created for the Unified

Expert Probability Density Function Shown in Figure 2.4.6.

Options	Combination 1	Combination 2	Combination 3
Put 30	–4	–7	–4
Call 35	–4	–3	3
Put 35	0	–1	3
Call 40	6	4	2
Criteria Values			
Expected profit	3.34	5.63	0.65
Profit probability	0.88	0.90	0.38
Ratio of profit to loss	0.76	0.85	0.09

2.5 Specific (Nonuniversal) Criteria

All criteria previously discussed are

based on applying different probability distributions of the future underlying asset price. Calculation of their values is based on integrating the payoff function or its derivatives over the probability density function. Such criteria can be used to evaluate and compare any option combinations corresponding to different strategies regardless of their payoff function shapes. According to our classification (Chapter 1), such criteria are referred to as universal ones.

However, the structure of many option strategies allows creation of additional criteria suitable only for them or for a limited number of other closely related strategies. Such criteria are not of the universal nature, because they cannot be

used to compare combinations belonging to different strategies and possessing payoff functions with different shapes. Next we consider some of them.

2.5.1 Break-Even Range

The mathematical structure of this criterion is simple. Its basic idea is also rather ordinary and, being nonformalized, is frequently used in evaluating option combinations. This criterion is based on the notion of break-even points, which are defined as underlying asset price values where the payoff function is equal to zero. Option combinations may have one break-even

point (bull or bear spread), two points (long and short strangle, straddle, calendar spreads), or a multitude of such points (different complex strategies). The underlying asset itself has the only break-even point that coincides with the price of this asset at the time of position opening.

The idea of this criterion is based on the statement that trading attractiveness of an option combination increases when the underlying asset price range, in which the payoff function is positive, broadens. Because break-even points are the boundaries of this range, it is natural to use them for criteria calculation. The position of the current price in relation to break-even points is also important. If

the price is situated between two break-even points at the time of position opening (which is true in most cases), it is not only the range that is important, but the distance between the current price and the nearest break-even point as well. Note that the absolute range value is not so informative. For a stock worth $10, a range of $2 is rather big (because it is 20% of the price); the same range for a stock worth $50 is much narrower (only 4% of the price). Accordingly, criteria values based on the break-even range should be normalized either by the current price of the underlying asset or by another characteristic.

Criterion Calculation Method

The criterion estimating how wide the break-even range is in relation to the current underlying asset price is calculated as

(2.5.1)

$$ProfitRange(B,S) = \frac{\rho(B,S,X)}{X}$$

where $\rho(B,S,X)$ is the distance between the break-even points of combination S, B is the underlying asset, and X is the current price of the underlying asset.

This version of the criterion does not take into account the position of the current underlying asset price relative to

the break-even points. We can fix it by defining $\rho(B,S,X)$ as a distance between the current price and the nearest break-even point.

For the short straddle strategy, high value of this criterion (calculated according to equation 2.5.1) corresponds to the higher potential profitability of a combination. When applying the same criterion to the long strangle strategy, $\rho(B,S,X)$ should be defined as the distance between the current price and the most remote break-even point. In this case, low criterion values indicate higher profitability of the evaluated combination.

Different underlying assets may have

different variability values. Therefore, it seems reasonable to apply an additional normalization factor. Then the break-even range criterion is formalized as

(2.5.2)

$$ProfitRange(B,S) = \frac{\rho(B,S,X)}{X \cdot V(B)},$$

where $V(B)$ is the underlying asset volatility and $\rho(B,S,X)$ is either the distance between break-even points or the distance between the current price and the nearest break-even point. The product of the current price by volatility (the denominator of equation 2.5.2) characterizes absolute price deviations

from the current value. The choice of volatility estimation procedure, and of the optimization algorithm of its main parameter—the horizon of history—is a separate complicated task. (We discussed it in section 2.1.2.) In some cases it is reasonable to use implied volatility as *V(B)*; in other cases one of the numerous variants of historical volatility should be applied.

Criterion Calculation Example

Consider two short straddles created on April 05, 2007, using options on AMZN and ADBE stocks. The AMZN close price was $41.68; bid prices for Call

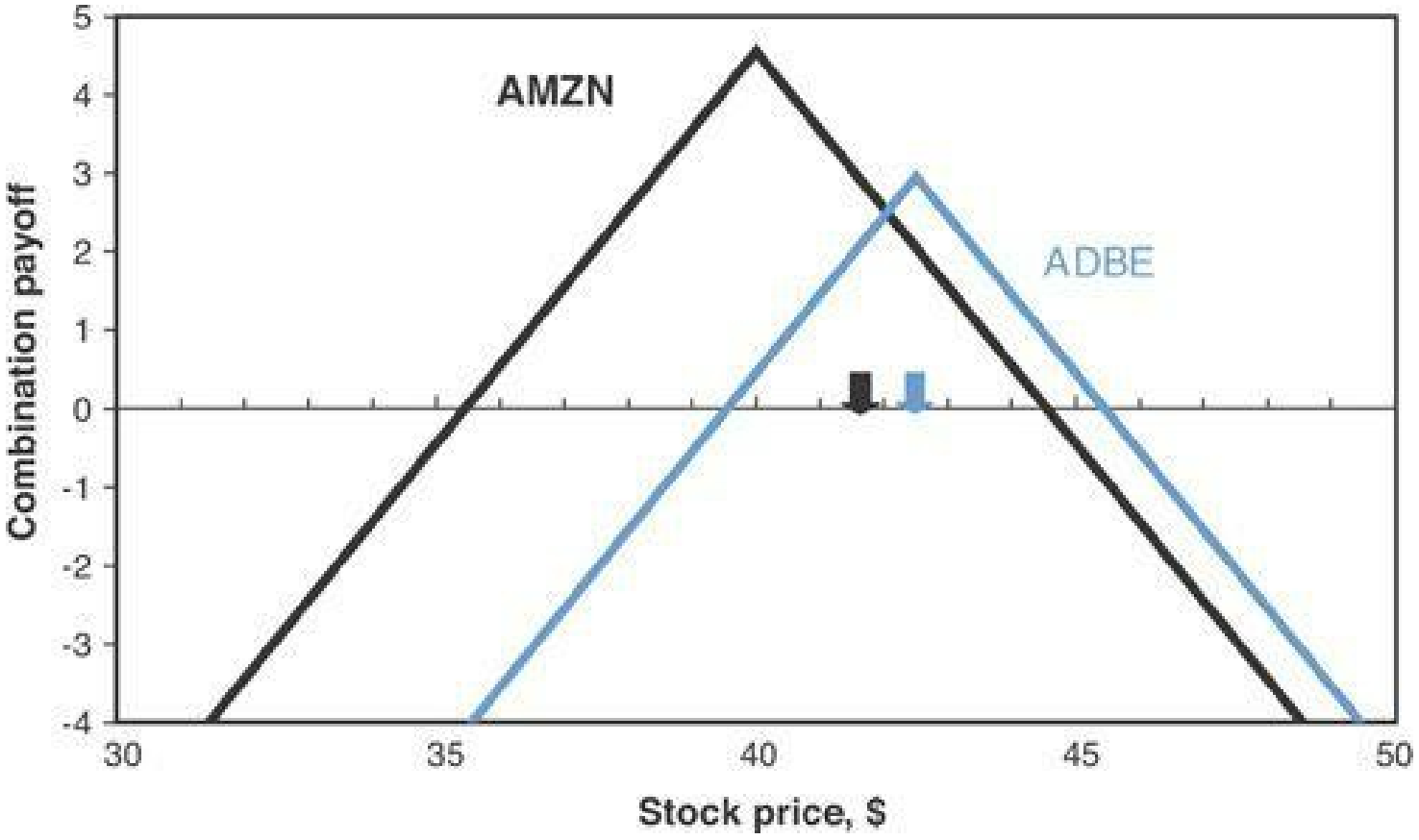
and Put options with strike 40 were $3.2 and $1.35 correspondingly. The close price of ADBE was $42.61; bid prices of its options with strike 42.5 were $1.65 and $1.3 for Call and Put correspondingly. The expiration date of all the options is May 19, 2007. Suppose that each combination consists of one short Call and one short Put option. To evaluate the potential attractiveness of these two combinations, we estimate and compare both variants of the criterion calculated according to formula 2.5.1.

Figure 2.5.1 shows positions of both short straddles relative to their underlying prices. At first glance, the AMZN combination seems to be better than the ADBE combination because its

premium is higher. The break-even range of the AMZN combination—$9.10 (44.55 – 35.45)—is also wider than that of the ADBE combination—$5.90 (45.45 – 39.55). The criterion value calculated according to that version of formula 5.2.1, where $\rho(B,S,X)$ is the distance between break-even points, also indicates the advantage of the AMZN straddle ($ProfitRange(B,S) = 0.22$) over the ADBE straddle ($ProfitRange(B,S) = 0.14$).

Figure 2.5.1. Payoff functions of two short straddles created using

options on AMZN and ADBE stocks. Arrows indicate current stock prices at the time of combination creating.



To obtain a different insight into the

comparison of these two combinations, we evaluate the distance from the current price to the nearest break-even point. For the AMZN combination it is 44.55 – 41.68 = $2.87, and for the ADBE combination it is 42.61 – 39.55 = $3.06. The criterion calculated according to the second version of formula 5.2.1 (where $\rho(B,S,X)$ is the distance between the current price and the nearest break-even point) has the following values: AMZN *ProfitRange(B,S)* = 0.69, *ADBE ProfitRange(B,S)* = 0.72. It means that, according to the second version of the same criterion, the ADBE combination is preferable (which is contrary to the conclusion based on the first version of the criterion).

What should we do if the same criterion calculated in two different ways gives opposite results? Which of two criterion variants should we rely on when they contradict each other? The following approach can be applied. If we are market-neutral with regard to the underlying asset (that is, we do not forecast the price direction), then the numerator of formula 2.5.1 should express the distance between the current price and the nearest break-even point. Alternatively we may suppose that the probability of certain price direction is higher (which means that we are not market-neutral). In this case we can build the combination in a way that either maximizes or minimizes the

distance between the current price and the nearest break-even point (depending on the strategy and the predicted price direction). To evaluate such combinations the numerator of formula 2.5.1 should express the distance between two break-even points.

2.5.2 *IV/HV* Ratio

One of the ideas of option trading is that opening short positions implies receiving a premium that turns into profit as the time value of options decays. However, profits generated by time decay are accompanied by losses induced by price changes. Strategies like

short straddle, short strangle, and those with a similar payoff function profile are good examples vividly illustrating this idea. (Such strategies are usually classified as selling volatility.) For strategies with an inverse payoff function profile (classified as buying volatility), the opposite statement is correct: The profit from price changes is offset by the time decay of the option premium paid for the combination.

Choosing between buying or selling volatility, you should determine which factor—time decay or price change—will prevail. (This is in addition to the forecast of the volatility itself.) To develop a criterion based on the comparison of these two factors, we

have to express them numerically.

The possibility of deriving profit from time decay depends, among other things, on the expensiveness of options that may be expressed through the implied volatility (*IV*). The magnitude of price movements can be characterized by historical volatility of the underlying asset (*HV*).

Considerable excess of implied volatility *IV* over historical volatility *HV* suggests the superiority of selling volatility strategies where short option positions are preferable to long ones. The opposite statement also holds true: Excess of historical volatility over implied volatility suggests the

superiority of buying volatility strategies where long option positions dominate over short ones. In the first case options are considered to be overvalued, in the second case—undervalued. These considerations may be formalized to form a criterion expressing the *IV/HV* ratio:

(2.5.3)

$$IV / HV(S,B,X) = \frac{IV(S,B)}{HV(B)}$$

The ratio of two volatilities looks rather ordinary. However, this simplicity hides a multitude of potential variants that may be used to compute this ratio. These

variants differ not only in their computation algorithms, but also in selection of parameters used to calculate volatility indicators.

Criterion Parameters

The numerator of equation 2.5.3 contains implied volatility of combination *S* corresponding to underlying asset *B*. Implied volatility is a market estimate of the future variability of the underlying asset price, which will be realized from the current time until the expiration date. In other words, implied volatility is a value characterizing option expensiveness expressed through the

Black-Scholes formula (or any other model).

The classical definition of implied volatility relates to a single option. *IV* is derived from the Black-Scholes equation by substituting the current market price of the option into the model's formula. Because option combinations usually contain several options, it is preferable to use an index of implied volatility combining the implied volatilities of several separate options. The simplest solution is to average the implied volatilities of all options included in the combination. More complicated averages can also be used. Companies specializing in market data delivery offer their versions of implied volatility

index for different underlying assets. Their indexes are based on averaging *IV* corresponding to different options; higher weights are assigned to series that are closer to the current date and to strikes, which are closer to the current price. Precise formulae are considered to be the intellectual property of data vendors and usually are not disclosed. However, such indexes are inappropriate for calculation of the *IV/HV* criterion because they relate to the underlying asset in general rather than to a combination of specific options. (Here we describe a situation where these indexes can still be applied.)

There are a lot of alternative techniques

for assessment of implied volatility. Investment companies usually develop their own methods and do not disclose them. Nevertheless, it is possible to give some general recommendations.

1. For the comparability of results, all option prices should be fixed simultaneously (that is, should correspond to the same moment in time).
2. One serious problem giving rise to considerable inaccuracies in estimation of implied volatility is the spread between bid and ask option prices. Which of them should be substituted into the model? Can we use their average?

These questions should be answered to obtain the reliable results. A smart approach to solving this problem is to gather statistical data about your own trading and determine the extent to which your execution prices are close to the best market prices existing in the market at the time of execution. Consider the case of options selling (when bid is the best price and ask is the worst price). For each trade we calculate the execution efficiency indicator:

$$ExecutionEfficiency = (Price - \text{Bid}) / (\text{Ask} - \text{Bid}),$$

where *Price* is your real execution price, Bid and Ask are bid and ask

prices at the time of trade execution (to be more exact, before the order was placed). Averaging this indicator for all trades executed over a certain time period gives *MeanExecutionEfficiency*. Using *MeanExecutionEfficiency* we can estimate the prices (*ModelPrice*) appropriate for derivation of implied volatility:

(2.5.4)

$$ModelPrice = \text{Bid} + MeanExecutionEfficiency \times (\text{Ask} - \text{Bid}),$$

where Bid and Ask are current bid and ask prices at the time of *IV* calculation.

3. When calculating the implied volatility index, we suggest using the previously described weight principle but taking into account only the options that are used in the combination under consideration.
4. Because option price data vendors often provide inaccurate, lagging, and even false information, it is necessary to develop and regularly apply the monitoring algorithms intended to control the accuracy of incoming data.

Now consider the denominator of equation 2.5.3. The details and peculiarities of historical volatility (*HV*) calculation were discussed in section

2.1.2. Here we merely repeat the main principles. Historical volatility reflects the past variability of the underlying asset price. In the simplest of cases, *HV* can be calculated by formula 2.1.2. The length of history *N* used to calculate *HV* is an extremely important parameter that strongly influences the results. Owing to statistical nonstationarity of market characteristics, a very low *N* can give nothing but local information on volatility, whereas a very high *N* can include events and data that are of little importance today. When calculating the *IV/HV* ratio, it is optimal to use time periods ranging from 30 to 240 days. In each particular case the precise *N* value should be determined by the specificity

of a given trading strategy.

Criterion Peculiarities

The *IV/HV* ratio criterion has one peculiarity that the majority of the criteria do not possess. It can be used not only to analyze option combinations but also to compare underlying assets in respect to their suitability for implementation of the whole group of option strategies (selling or buying volatility). High criterion value (when *IV* exceeds *HV*) indicates that selling volatility strategies are more appropriate for this underlying asset. On the contrary, low criterion value often

indicates that buying volatility is preferable. Such interpretation of criterion values relates to underlying assets themselves rather than to combinations of their options. If the *IV/HV* ratio is used in this way, the implied volatility should be estimated on the basis of all traded options (assigning higher weights to series that are closer to the current date and to strikes situated closer to the current price). It is even possible to use *IV* indexes supplied by independent information agencies. However, using "ready-made" indexes makes it impossible to calculate *IV* values more accurately using formula 2.5.4.

Applying this criterion in selecting a

group of strategies—buying or selling volatility—allows us to divide the initial set of underlying assets into two subsets. Each subset corresponds to one or another group. However, the next step requires applying additional criteria intended to select the most suitable underlying assets and specific strategies within each subset. Actually, the same criterion (*IV/HV* ratio) can be used again for this purpose, but in this case the numerator of equation 2.5.3 should contain another *IV*, calculated specifically for each combination rather than the universal index value.

Another peculiarity of *IV/HV* ratio is rather contra-intuitive: Extremely high and low values of this criterion usually

indicate the inappropriateness of a combination (or an underlying asset) rather than its superiority. The reason is that extreme values of implied and historical volatility are frequently caused by extraordinary fundamental events, which have just taken place or are anticipated in the nearest future.

When *IV* exceeds its average value, it usually creates opportunities to realize option strategies based on volatility selling. However, an abnormally high *IV* may arise from unique temporary circumstances, which are not suitable for implementation of standard option strategies. Extremely high *IV* usually means that the market anticipates news concerning some essential event such as

a court decision, M&A announcement, new products launch, and so on. The exact event date, its probability, and sometimes even the essence itself are often unknown to the public. But if such an event occurs, it frequently causes a considerable price move. Sometimes the essence of the event and its date are predefined and known to the wider public (such as publication of accounting information with fixed terms and periodicity). A classic example is a quarterly earnings release. It can also be publication of important industry or macroeconomic statistical data. Such events often induce sharp changes in underlying asset prices or *IV* values. In these cases it is useless to predict future

price variability on the basis of historical volatility because the price movement caused by the news can be several times greater than past price fluctuations. Such situations are extremely risky for standard selling volatility strategies, and they should either be avoided or combined with forecasts based on creation of expert probability distributions.

The opposite situation of abnormally low *IV* values usually takes place after the announcement of the acquisition. When this fact becomes known, the stock price sharply jumps up to the acquisition price level (thereby raising the historical volatility), and the option price plunges (thereby decreasing the implied

volatility). Afterward the stock price will not change in the near future. (It will fluctuate within a narrow range around the acquisition price.) In this situation some buying volatility strategies may look very attractive, having no prospects at all (because there will be no considerable price and volatility changes).

2.5.3 Relative Frequency Criterion

The idea of this criterion (and of the previous one and all the other criteria intended to evaluate option combinations) is based on the divergence between the premium

(expressed by *IV* or otherwise) and the variability of the underlying asset price (expressed by *HV* or otherwise). The main distinction of this criterion is that instead of analyzing the absolute divergence between these characteristics (expressed by their ratio or otherwise), it estimates the relative frequency of their occurrence.

Implied volatility $IV(S,B,t)$ of combination S built for underlying asset B at the moment in time t will be used as a characteristic of the option expensiveness. Methods of $IV(S,B,t)$ calculation were described previously. For this criterion we suggest using simple averaging of implied volatilities of all options composing the

combination. To compute IV of each separate option, we substitute its market prices adjusted according to formula 2.5.4 into the Black-Scholes model. Because implied volatility sharply increases as the expiration date approaches, the options with an expiration date that is less than 5 trading days away should be discarded from the averaging procedure.

The underlying asset price variability will be expressed as the absolute value of the logarithm of daily price changes. The following notations will be used: $C(t)$ is the underlying asset close price at the moment in time t; L is the length of history horizon; and T_0 is the current moment in time. We calculate price

changes according to the equation $p(B,t) = \left|\ln(C(t)/C(t-1))\right|, t \in \Omega$ at the time interval $\Omega = \{T_0 - L, T_0 - L + 1, ..., T_0\}$.

As a result, we obtain L pairs of $IV(S,B,t)$ and $p(B,t)$ values. The criterion is defined as the relative frequency of days on the given horizon of history, when implied volatility of the combination was higher than the real price change:

(2.5.5)

$$FrIV(B) = \frac{1}{L}\left|\{t \in \Omega : IV(B,t) > p(B,t)\}\right|$$

Here the symbol |{...}| denotes the

number of elements in the set. There are several variants of this criterion. Firstly, we can use 2-day, 3-day, and such price changes instead of 1-day changes. (The period length is one of the parameters to be optimized.) Secondly, we can define the threshold value for the difference between $IV(S,B,t)$ and $p(B,t)$ (another parameter). Thirdly, the frequency of this excess can be expressed in different ways. The simple way was used in formula 2.5.5; the advanced variant would be to build a complete frequency distribution of the differences between $IV(S,B,t)$ and $p(B,t)$. Finally, we can create several versions of this criterion changing the method of determining the length of the history horizon. (The

importance and the extent to which *L* influences the criterion value were discussed in the section devoted to empirical distribution.)

The disadvantage of the relative frequency criterion consists in its inappropriateness to evaluate combinations containing deep in-the-money and out-of-the-money options. (They are usually illiquid or have negligible volumes.) Due to the volatility smile phenomenon, such options may have extremely high (or low) *IV* values. In these cases, whatever the price change is, *IV* would be higher (lower) than the corresponding price move. Therefore, this criterion can only be used to compare combinations

consisting of at-the-money options.

The interpretation of this criterion is straightforward. High *FrIV(B)* values indicate that options are overvalued relative to the underlying asset price variability potential. Thus, when selecting combinations containing more short options than long ones (strategies of selling volatility), the variants with higher criterion values should be preferred. Low criterion values indicate that the underlying asset price variability is underestimated (not fully reflected in the current option prices). Therefore, when selecting combinations containing mainly long options (buying volatility strategies), the alternatives with lower criterion values should be preferred.

2.5.4 The Ratio of Normalized Time Value to the Coefficient of Absolute Price Changes Distribution

The structure of this criterion is similar to that of the *IV/HV* ratio criterion. It is the ratio with the option expensiveness indicator in the numerator and the underlying asset price variability indicator in the denominator. The difference is that both indicators are based on fundamentally different principles without using historical and implied volatilities. This allows avoiding inaccuracies and assumptions related to these indicators. Like the

IV/HV ratio this criterion can be used not only for comparing option combinations but also for selecting the underlying assets appropriate for strategies of buying or selling volatility. High criterion values indicate that selling volatility is preferable for the particular underlying asset and vice versa. The criterion can also be used to compare option combinations, but with one important restriction: Only combinations with identical structure are comparable. It means that the "ratio of normalized time value to the coefficient of absolute price changes distribution" is a specific (nonuniversal) criterion. (The reason for this lies in the numerator calculation method and will become

clear later.)

We begin with the numerator that shows how expensive the options are at the current moment in time. First, we express the time value of the combination as a fraction of the underlying asset price. To make underlying assets comparable by this criterion, two conditions have to be met. Firstly, all combinations must have an identical structure. To be more precise, they must be straddles with the strike that is closest to the current underlying asset price. (Strangles can also be used; we discuss it next.) Secondly, for all combinations the price of the underlying asset must be exactly equal to the corresponding strike price. (Then the

option premium will consist only of time value.) Although the first condition is easy to fulfill (it depends only on our own choice), the second one is an idealization that can never be realized in real life for a number of assets simultaneously. Nevertheless, we can approximate this idealization by calculating what would be the value of options if the underlying asset price has changed abruptly and coincided with the nearest strike. It can be done by application of the option's delta, which shows by how many points the option price will change if the underlying asset price changes by one point. To reduce real prices of the Put option (P_p) and the Call option (P_c) to hypothetical prices

(P_p' and P_c'), which would have been formed if the current underlying asset price (X) coincided with the nearest strike (S), the following transformations need to be executed:

(2.5.6)

$$P_p' = P_p - \Delta_p(S - X) \text{ and } P_c' = P_c + \Delta_c(S - X)$$

where Δ_c *and* Δ_p *are Call and Put option deltas*.

The next step is to normalize the time value of the combination by the underlying asset price (which is equal to S after transformation 2.5.6):

(2.5.7)

$$TimeValueStraddle = (P_p' + P_c') / S$$

To obtain the same indicator for strangles, you should take into account an important peculiarity. Consider the following unified strangle structure: The combination is formed of Put and Call options with the nearest to the current price (X) out-of-the-money strikes(S_p and S_c correspondingly). The option premium of the combination having such structure consists only of time value. The peculiarity mentioned is that distances between adjacent strikes expressed as

percentage of the current price $_X$ are different for different underlying assets. This implies that, all other terms being equal, the combination relating to the stock with lower $(S_c - S_p)/X$ value will have a higher time value. This is quite natural because strikes of both options of such stock are situated "closer to the money." To eliminate this effect and make all strangles comparable by their time value, we normalize time value by the strike step: $(P_p + P_c) / (S_c - S_p)$. This expression should be further normalized by the current price of the underlying asset to obtain the indicator for strangles similar to the one developed for straddles (equation 2.5.7):

(2.5.8)

$$TimeValueStrangle = (P_p + P_c) / [X (S_c - S_p)]$$

Note that normalizing time value by $(S_c - S_p)$ does not fully solve the problem of underlying assets comparability. As the current price moves away from the strike, time value decreases nonlinearly. Consequently, cheaper underlying assets (those with more infrequent strikes expressed as a percentage of the price) will have overpriced time value compared to more expensive underlying assets. Thus we suggest using formula 2.5.8 to compare only underlying assets

with the price difference of no more than 20% to 25%.

Expressions 2.5.7 and 2.5.8 allowing comparison between different underlying assets by time value of their options serve as the numerator of the criterion. It is important to note that indicators calculated by application of equation 2.5.7 are incomparable with those obtained with equation 2.5.8 because time value of the straddle is always higher than that of the strangle. Besides, these indicators express different values. The first one expresses time value as a percentage of the underlying asset price. The second one expresses time value (also as a percentage of the underlying asset price) accounting for each dollar

of the distance between strikes.

The idea of the indicator serving as the criterion's denominator consists in expressing the underlying asset price variability without resorting to historical volatility or variance. This can be solved in various ways; here we discuss only one of them. Putting aside price trends, we express its variability by application of the exponential function to analyze the absolute price changes.

We fix the horizon of history L, relative to the current point in time T_0. For each underlying asset b at the time interval $\Omega = \{T_0 - L, T_0 - L + 1, \ldots, T_0\}$ we obtain the set of L absolute values of daily price changes: $p(B,t) = \left|\ln(C(t) / C(t-1)\right|, t \in \Omega$

. This set is then ranked in ascending order of $\{\rho(B, t_i)$: $\rho(B, t_i) \leq \rho(B, t_{i+1})\}$ values and the cumulative empirical distribution function is constructed on the basis of this ordering:

$$F(B,x)=\begin{cases} 0, x \leq p(B,t_1) \\ \frac{i}{L}, p(B,t_i) < x \leq p(B,t_{i+1}) \\ 1, x > p(B,t_L) \end{cases},$$

Applying the least-squares method we approximate the function *F(B,x)* with the exponential distribution function $f(\lambda,x)=1-e^{-\frac{x}{\lambda}}$. For every underlying asset *B* we obtain its $\lambda(B)$ value. The probability density function of the

exponential distribution is $f(x)=\frac{1}{\lambda}e^{-\frac{x}{\lambda}}$. If two different stocks *B1* and *B2* have different values of the coefficient $\lambda(B)$ and $\lambda(B1) > \lambda(B2)$, then stock *B1* has fewer small price changes while big price fluctuations are more frequent. That is, the distribution of price changes of the stock *B1* has fatter "tails." Such a property indicates that the price of the first underlying asset is more variable than the price of the second one.

Evaluating the ratio of TimeValue calculated by application of equation 2.5.7 (or 2.5.8) to λ(B), we obtain the value of the criterion called "ratio of normalized time value to the coefficient of absolute price changes distribution."

Finally, we would like to emphasize that this criterion and the *IV/HV* ratio can be used to create additional hybrid criteria. For example, we can divide *IV* by λ(B) or, on the contrary, calculate ratio of *TimeValue* to *HV*.

chapter 3. Evaluation of Criteria Effectiveness

3.1 Introduction

Evaluation of criteria effectiveness requires addressing the following question: To what extent is the potential effect of their exploitation consistent with our expectations? In previous chapters we described the philosophy,

methodology, and main approaches used to create criteria. While developing and constructing criteria we pursue quite concrete and definite goals. Hence, upon completion of this comprehensive creative process, it would be natural to check if the result is satisfactory and whether the goals have been achieved.

Before judging on the suitability of a particular criterion, it has to be tested thoroughly. To carry out such a test, we have to establish a set of numerical characteristics that would reflect the performance of a criterion. Hereafter such characteristics will be referred to as "indicators of criteria effectiveness." (Using other terms, that is criteria quality, suitability, and such, we will

imply the same indicators.) To maximize the test reliability, it is desirable not to be restricted by one indicator but to use as many of them as possible (within reasonable limits, of course). Next we consider a set of effectiveness indicators. Despite the fact that each of them has strengths and weaknesses, their joint use allows you to evaluate the criteria effectiveness adequately.

The criteria effectiveness strongly depends on the type of the task it is intended to deal with. This should be taken into account while selecting indicators and developing evaluation procedures. Talking about types of task, we mean different areas of criteria application such as selecting option

combinations, strategies, and underlying assets (see Chapters 4 through 6). In Chapter 1, "General Presentation and Review of Criteria Properties," we distinguished between two basic groups of criteria: universal and specific ones. As follows from the name, the former can be applied to resolve almost any kind of problems whereas the latter are not as multifunctional. However, even universal criteria show heterogeneous suitability for dealing with different tasks.

The criterion often shows high effectiveness in certain situations but may become useless in other circumstances. Hence, in this chapter we concentrate on evaluation of criteria

quality when they are used to accomplish one general task: sorting option combinations by their potential profitability. We consider this task to be general because its realization is required in all areas of criteria application. The methods intended to evaluate the effectiveness of criteria when they are used to fulfill specific tasks are described in the second part of this book dedicated to the main areas of criteria application.

The criteria effectiveness varies depending on the type of option strategy. A certain criterion may successfully detect profitable variants within the set of combinations created under one strategy, but it may be ineffective in

selecting combinations created under another strategy. This feature is described in detail here, and it must also be taken into account when evaluating criteria effectiveness.

Four option strategies are used in this chapter: short strangle/straddle, long strangle/straddle, short calendar spread, and long calendar spread. (Here, strangles and straddles are considered as combinations relating to one strategy.) A *short calendar spread* is a combination consisting of a short Call (Put) with the nearest expiration date and a long Call (Put) with the same strike price but with the next expiration date. (Thus, this combination is short because its payoff function is similar to

short positions.) Accordingly, a *long calendar spread* is a combination consisting of a long Call (Put) with the nearest expiration date and a short Call (Put) with the same strike price but with the next expiration date. (Its payoff function is similar to long positions.) Note that these definitions differ slightly from the common ones.

These particular four strategies were chosen because they represent a wide range of payoff functions that are opposite to each other and neutral to an underlying asset market (see the appendix). In this chapter we develop a set of effectiveness indicators intended to evaluate criteria described in Chapter 2, "Review of the Main Criteria." The

quality of each criterion will be evaluated using many indicators within different option strategies.

3.2 Methods of Criteria Effectiveness Evaluation

3.2.1 Correlation Between a Criterion and Profit as the Main Effectiveness Indicator

You cannot gain full insight into the effectiveness of the criterion relying solely on a single indicator. Nevertheless, there is one fundamental

characteristic—the correlation between the criterion value and the realized profit —which for several reasons should be considered in the first place. Firstly, the correlation between the outcome and the forecast (most criteria do include a forecast in one form or another) is the most direct and intuitively understandable indicator of the criterion's suitability for determining trading opportunities. Secondly, this indicator is universal in the sense that it is suitable for evaluation of any criterion within all option strategies. Finally, it is easy to compute and does not require any complicated data processing except building a simple regression model.

The evaluation of criterion effectiveness

by means of the correlation is based on the relationship between criteria values and realized profit. Applying a regression model to this relationship allows an estimation of the fraction of the profit variance explained by the criterion variation and of the fraction of the profit variance caused by random factors. The square of the correlation coefficient (also referred to as "coefficient of determination") shows how well the regression equation (which reflects the linear relationship between the profit and the criterion) fits actual data. In other words, it reflects the dispersion of data around the regression line. The coefficient of determination ranges from zero (when the profit does

not depend on the criteria) to one (profit changes are fully determined by criterion changes).

One particular issue should be taken into account to perform a high-quality evaluation of criterion effectiveness using the correlation method. The value of the correlation coefficient expressing the {criterion × profit} relationship is usually small and variable in time even when the criterion is correctly parameterized and possesses good forecasting properties. This holds true even when the predictive power of the criterion is known in advance and was verified both empirically and in back-testing. This feature is not specific to this particular indicator, but is rather

common for most indicators that will be discussed next. However, this should not upset us or make us think that it is impossible to develop adequate methods for evaluation of criteria effectiveness. In section 3.2.2 we show how to overcome these difficulties by applying a simple transformation procedure.

The previously mentioned issue can be illustrated by the following examples. Five hundred U.S. stocks were chosen according to the liquidity of their options. From the beginning of 2001 until September 2007, for each stock and each date we create combinations within short and long strangle/straddle strategies. The combinations are constructed using four strikes that are the

closest to the current stock price (two strikes below the current price and two strikes above it). Thus, 10 combinations are created for each stock: 4 straddles and 6 strangles (provided that Call strikes in a strangle are always higher than Put strikes). Combinations are made of contracts with the nearest expiration date and with a Call-to-Put ratio of 1:1. These 10 combinations are evaluated using a certain criterion, and three best variants are selected. In this way 1,500 combinations are produced for 500 stocks for each date. After all these combinations are evaluated by the corresponding criterion; their profit (or loss) values are recorded as of the expiration date. (Values of both the

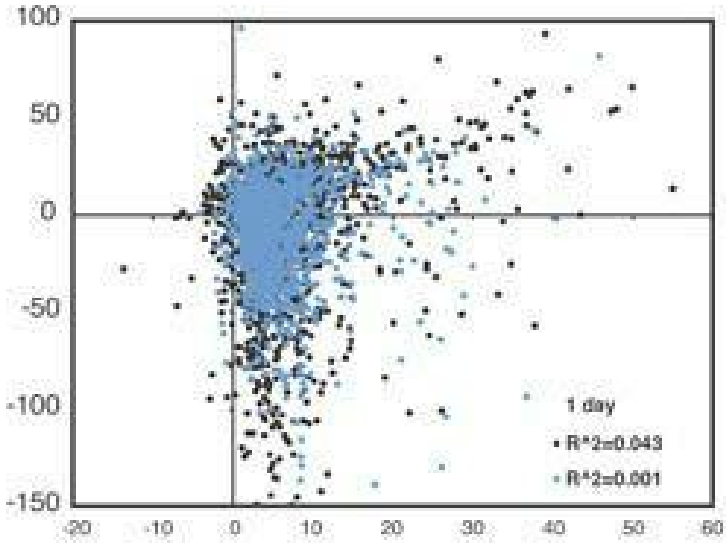
criterion and the profit are expressed in percentage of margin requirements.) Relationships between criterion and profit values are used to obtain correlation coefficients. The whole procedure is repeated for each criterion under examination.

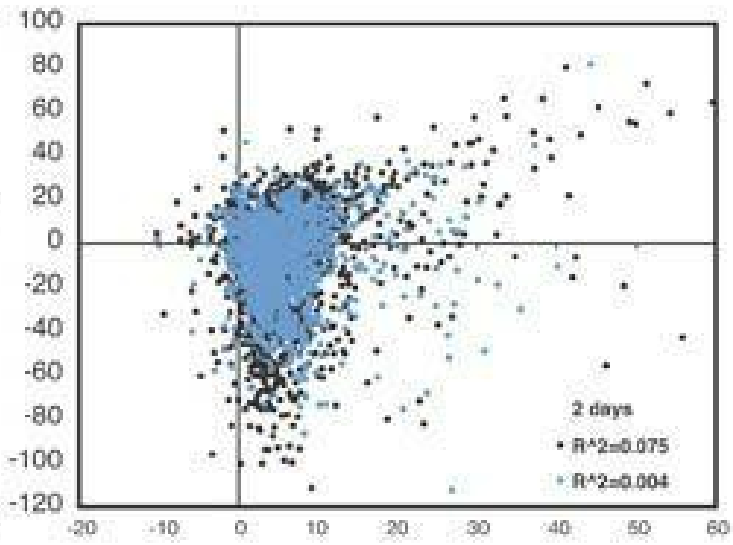
We begin with considering the correlation between criterion and profit values for several randomly selected dates. As an example we chose the "expected profit on the basis of lognormal distribution" criterion and the short strangle/straddle strategy. Two dates were randomly selected: August 27, 2007 and September 10, 2007. The regression model for {criterion × profit} relationship is shown in the top-left

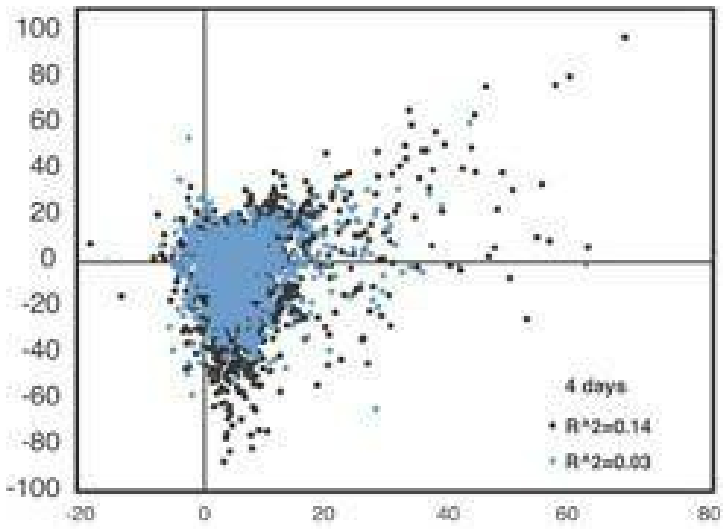
corner of Figure 3.2.1. Each point here represents a certain combination, and all points of the same color correspond to combinations built within one date.

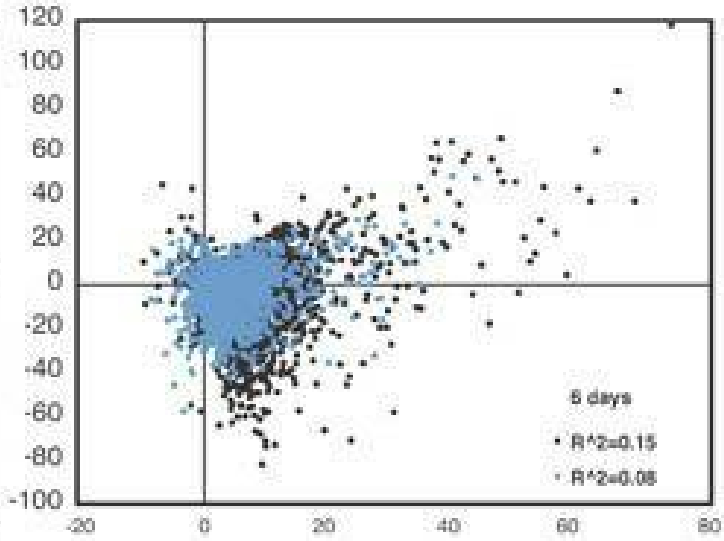
Figure 3.2.1. Regression model for the relationship between the "expected profit on the basis of lognormal distribution" criterion (horizontal axis) and the profit of option combinations (vertical axis). Option strategy: short

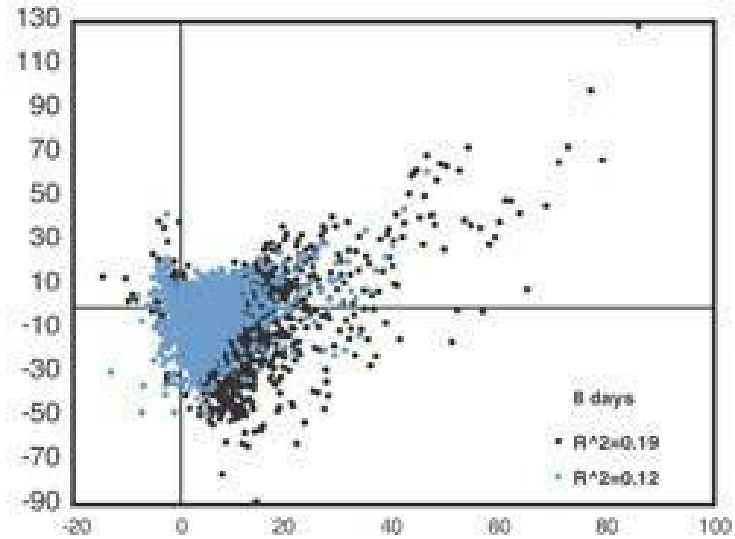
strangle/straddle. The black points correspond to August 27, 2007, the gray points to September 10, 2007. The legend shows the values of determination coefficients. Each chart corresponds to a certain amount of averaging days (from 1 to 10).

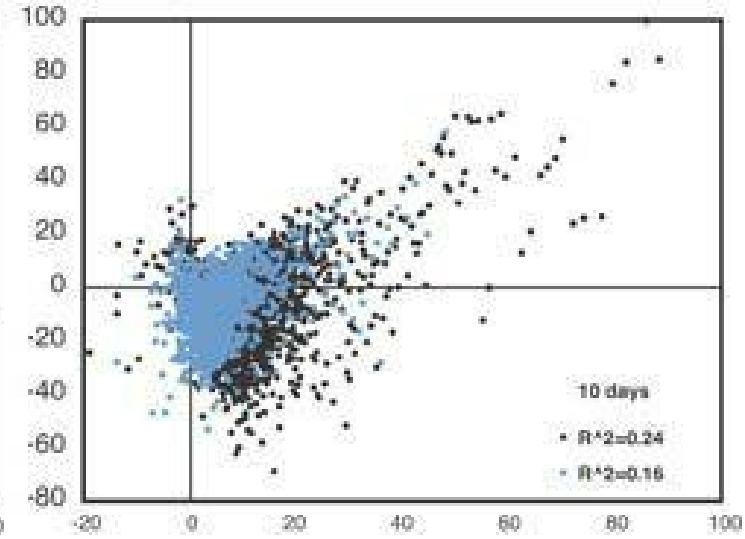

1 day
R^2=0.043
R^2=0.001


2 days
R^2=0.075
R^2=0.004


4 days
R^2=0.14
R^2=0.03


6 days
R^2=0.15
R^2=0.08


8 days
R^2=0.19
R^2=0.12

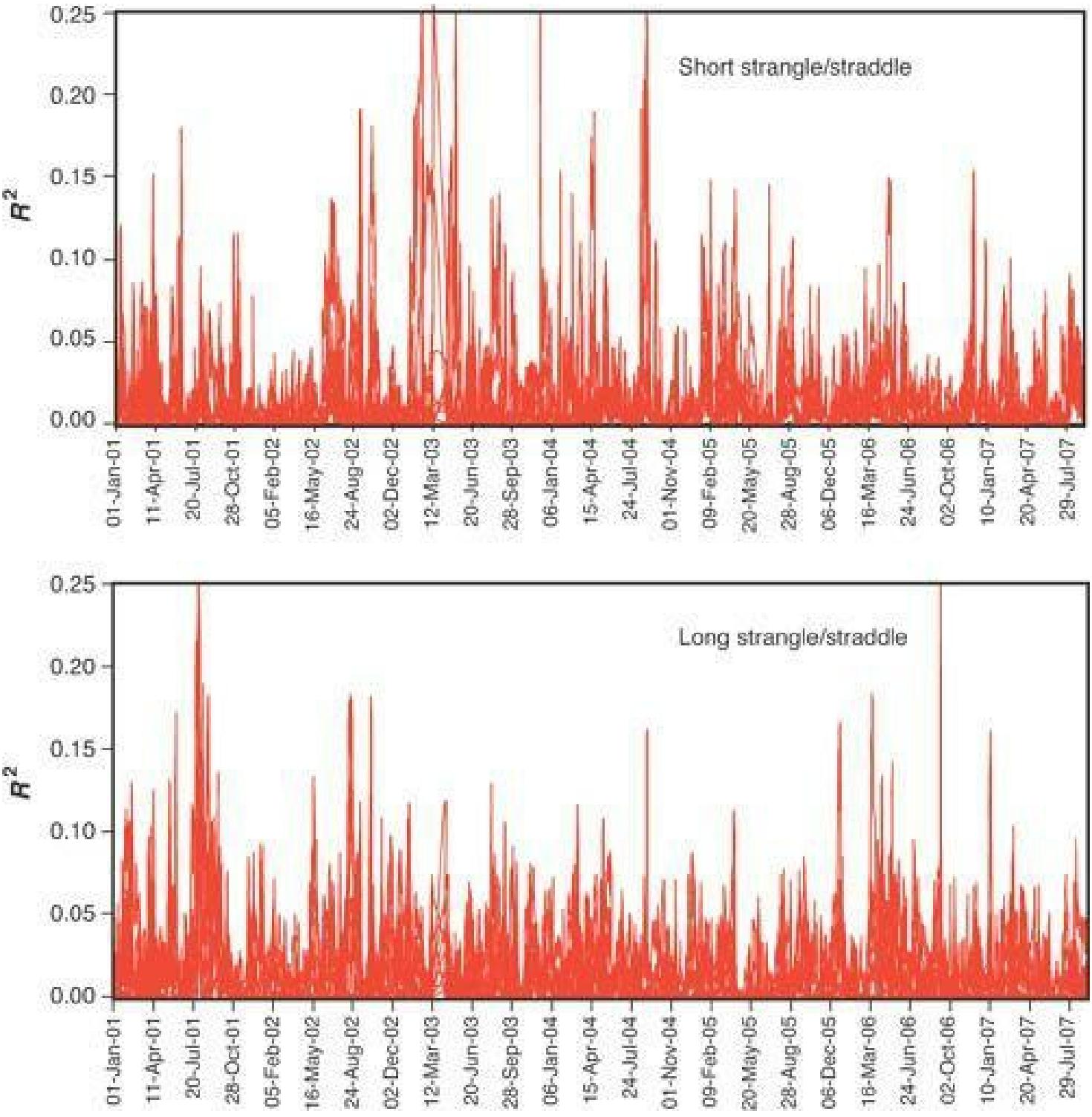

10 days
R^2=0.24
R^2=0.16

As we already mentioned, the correlations between the criterion value and the realized profit are very low. (Determination coefficients are negligibly small: $R^2 = 0.043$ and $R^2 = 0.001$, see the top-left chart of Figure 3.2.1.) However, only one strategy, one criterion, and just two dates (out of almost 7 years) were used in this analysis. Therefore, it is necessary to verify if the obtained results are representative. To extend this study we estimated the effectiveness of many criteria, for both long and short strategies and considered their dynamics during the 7-year period.

Regressions similar to those shown on the top-left chart of Figure 3.2.1 were

constructed for each {date × strategy × criterion} conjunction, and their determination coefficients were calculated. We found again that in the course of all 7 years the correlations between criteria values and profits of option combinations were extremely low (Figure 3.2.2). The coefficients of determination chaotically fluctuated within the range from almost 0 to 0.2 without any definite cycles. Such relationships were observed for all the 11 criteria regardless of the strategy. This extensive dynamic analysis confirms the validity of our prior conclusions based on two random samples.

Figure 3.2.2. Dynamics of determination coefficients corresponding to {criterion × profit} regressions for long and short option strategies. Each line corresponds to one of 11 criteria.



Does it mean that our criteria are ineffective and do not possess real forecasting abilities? Not necessarily! It

only shows that this particular method in its present form is unsuitable for the analysis of criteria effectiveness. In the next section we show that applying a simple transformation technique allows you to increase the significance and productivity of this evaluating procedure substantially.

3.2.2 Transformation of the Criteria Effectiveness Indicator

Let us have another look at the top-left chart of Figure 3.2.1. It is quite reasonable to suppose that the weak relationship between the criteria and the profit values is caused by the influence

of external factors that add noise to the system and reduce correlation. We suppose that at least some of these factors have a chaotic nature, and their impact on profit can change a vector in the course of time. If this assumption is true, these factors can be neutralized by combining several days in the analysis.

Combining does not mean mixing data that belongs to several days in one analysis (like merging black and gray points on the top-left chart of Figure 3.2.1). This could possibly increase the correlation coefficient but just due to the increase in the sample size. We propose the following procedure of data processing. Two successive dates are selected and combinations

corresponding to each of them are sorted by criterion values. Then we combine these two orderings by calculating the average of two criterion values for combinations occupying the two first positions. The average profit value for these two combinations is calculated in a similar way. Then we go on to the second positions in both orderings and calculate the averages for the criterion and the profit. The same procedure is applied for all positions up to the 1,500th.

This transformation provides us with a set of data consisting of averaged criterion and profit values. Their interrelationship is presented on the top-right chart of Figure 3.2.1. Compare it

with the similar relationship before averaging (the top-left chart of Figure 3.2.1). The averaging of data corresponding to August 27, 2007 and the adjacent date induced the increase of the determination coefficient from 0.043 to 0.075 (black points on the figure). In another case, when averaging included September 10, 2007 and the adjacent date, the increase was from 0.001 to 0.004. Although such augmentations seem to be negligible, it could be due to the fact that only two dates have been combined so far. To obtain a more pronounced effect, we should proceed further.

Averaging 4-days data (beginning from the same dates as in the previous

example) resulted in further increase of correlations. In one case R^2 doubled in comparison with 2-days averaging, in another case it increased almost tenfold (Figure 3.2.1). The more dates we include in the averaging, the higher the determination coefficients are. In the case of 10 days the strong correlation between criterion and profit values becomes obvious (Figure 3.2.1) (coefficients of determination are $R^2 = 0.24$ for August 27, 2007 and $R^2 = 0.16$ for September 10, 2007).

It is interesting to trace the evolution of interrelationships between the criterion and the profit as the averaging period increases. Going from the top-left corner

of Figure 3.2.1 to its bottom right corner, we can easily observe the gradual increase in correlation.

The data presented so far proves the effectiveness of the single criterion when applied to one strategy. Simple transformation was sufficient to acquire a notion of it. At the same time many questions still remain unconsidered. Examining the averaging period, we stopped at the tenth day. It is necessary to continue expanding this period to obtain the optimal value of this parameter. Furthermore, we have to find out how the averaging procedure influences the evaluation of other criteria. The applicability of this procedure to other option strategies

should also be ascertained. The influence of the transformation procedure on the dynamics of the correlation coefficients should be analyzed as well.

3.2.3 The Dynamics of Transformed Effectiveness Indicators

To investigate the influence exerted by the transformation procedure on the dynamics of {criterion × profit} relationships, we analyze three averaging periods: 30, 60, and 100 days. Determination coefficients will be calculated for each date through the entire 7-year period for both long and

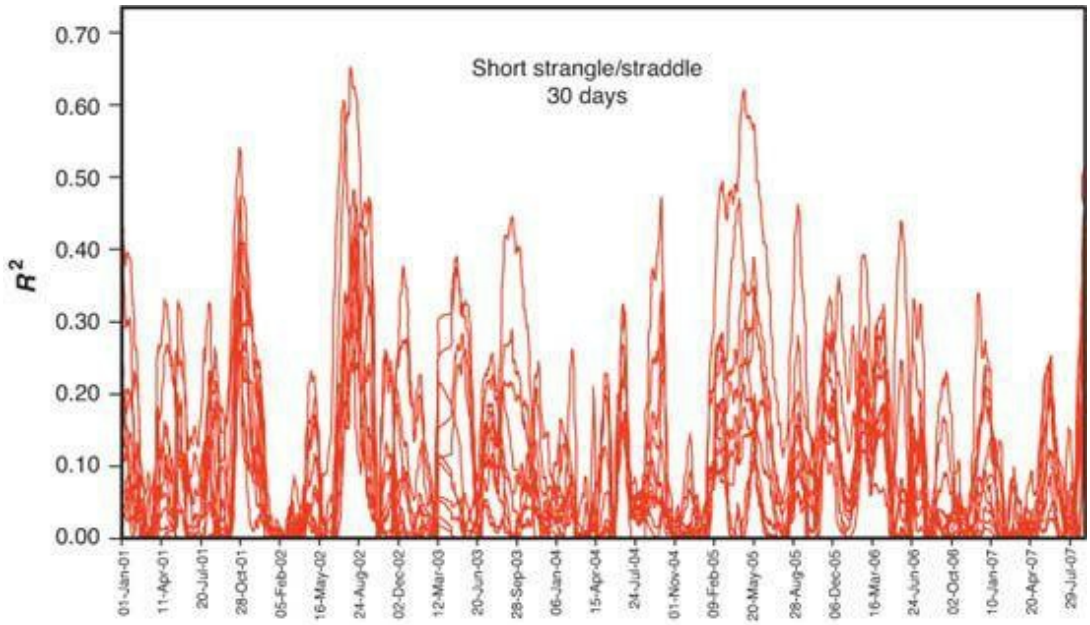
short strategies and for all the 11 criteria.

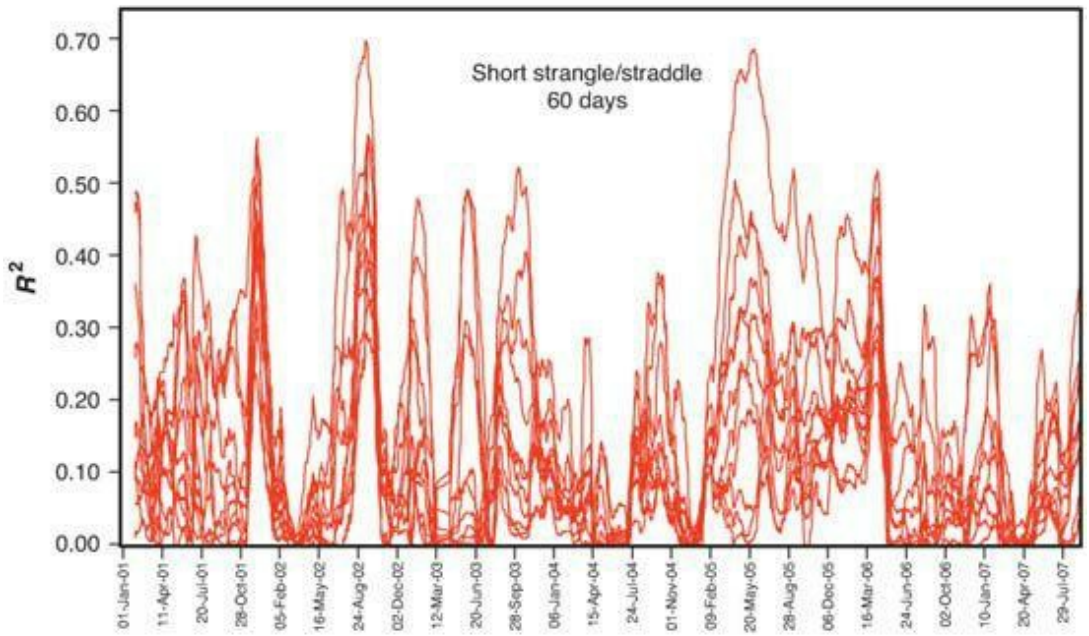
Figure 3.2.3 shows the dynamics of the effectiveness indicator for three averaging periods and two opposite-directed strategies. Comparison of diagrams with increasing averaging periods reveals that the transformation procedure allows detecting cycles in the dynamics of the criteria effectiveness. Although the dynamics is absolutely chaotic in the absence of transformation (Figure 3.2.2), 30-day averaging favors the appearance of some regularities in fluctuations of the effectiveness indicator (Figure 3.2.3). Increasing the period up to 60 days makes the regularities even more observable, and

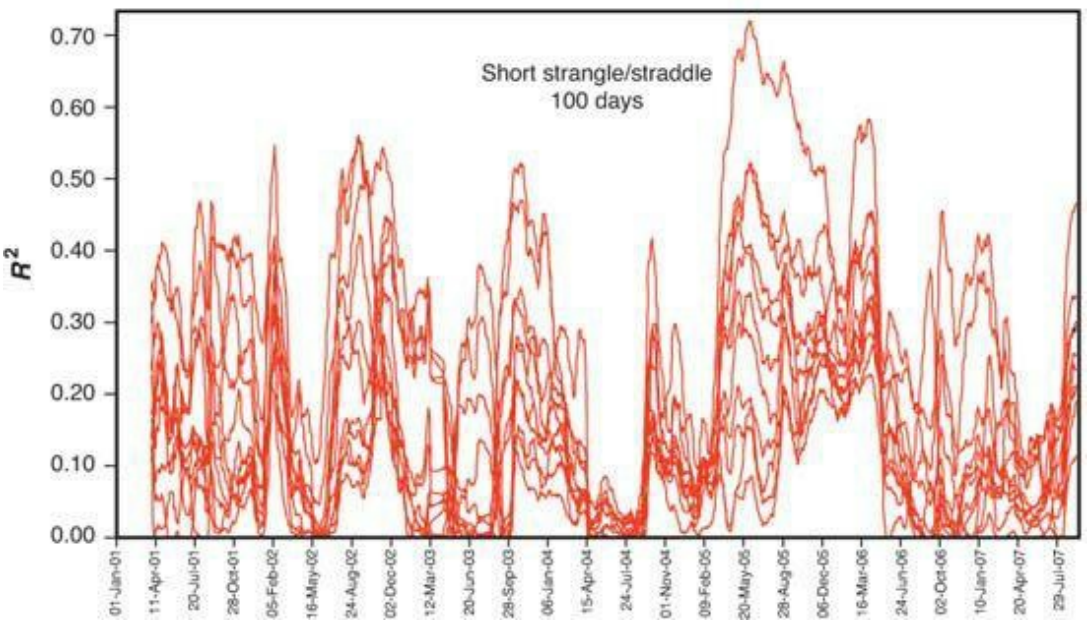
the 100-day period reveals obvious cycles in the dynamics of the effectiveness indicator. These regularities are more apparent for the short strangle/straddle strategy. Although the long strategy shows a similar dynamic, the cycles here are less obvious and harder to recognize (Figure 3.2.2).

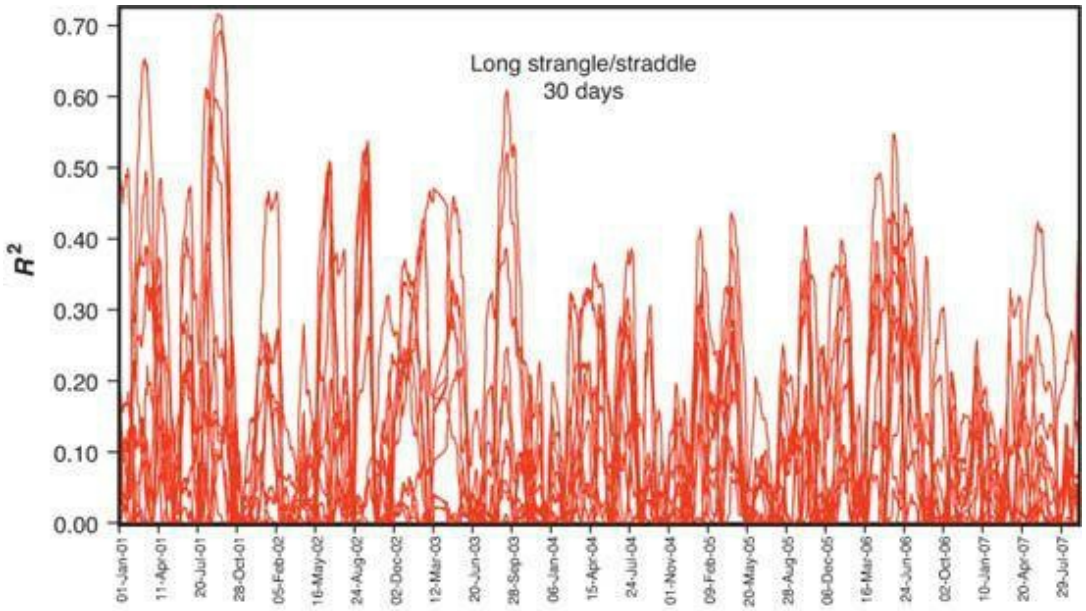
Figure 3.2.3. Dynamics of the determination coefficient corresponding to {criterion × profit} regressions for long and short strategies and for

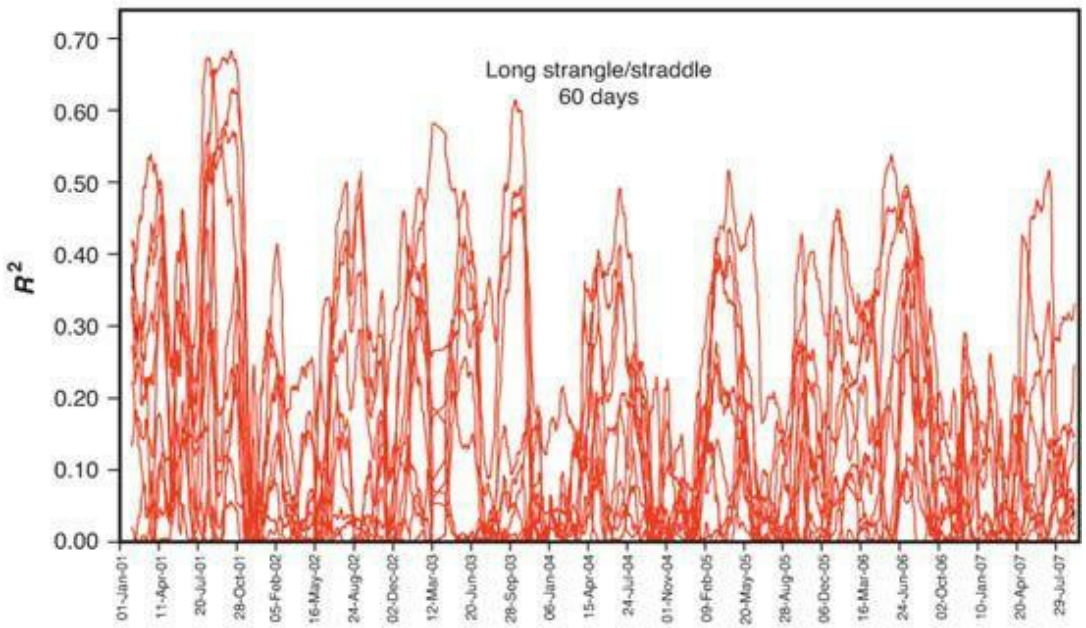
three averaging periods (30, 60, and 100 days). Each line corresponds to one of 11 criteria.

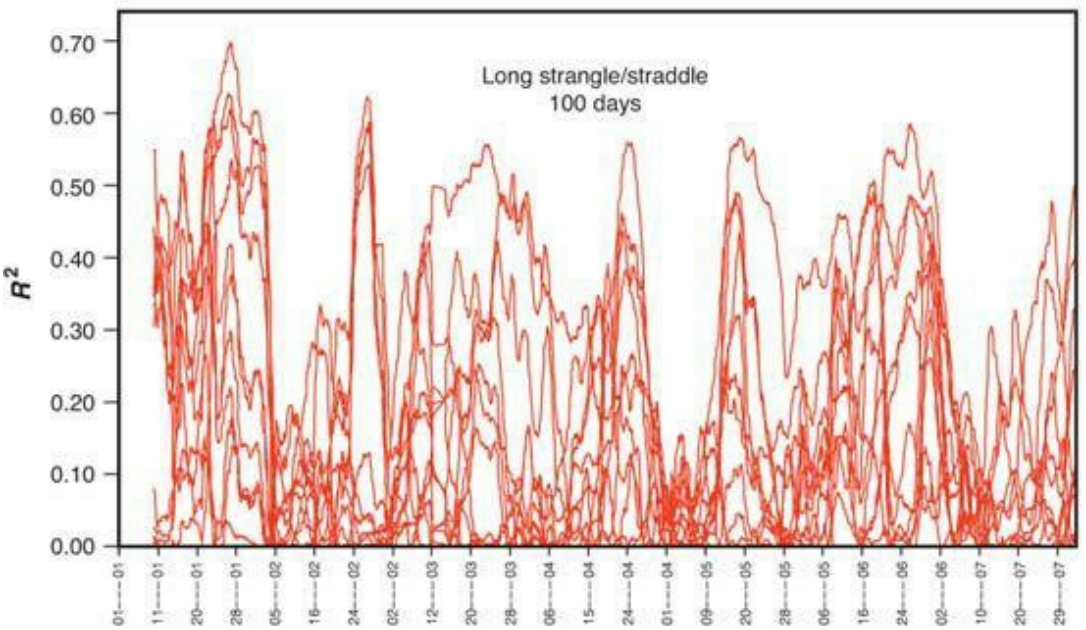

Short strangle/straddle
30 days
R^2
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
01-Jan-01
11-Apr-01
20-Jul-01
28-Oct-01
05-Feb-02
16-May-02
24-Aug-02
02-Dec-02
12-Mar-03
20-Jun-03
28-Sep-03
06-Jan-04
15-Apr-04
24-Jul-04
01-Nov-04
09-Feb-05
20-May-05
28-Aug-05
06-Dec-05
16-Mar-06
24-Jun-06
02-Oct-06
10-Jan-07
20-Apr-07
29-Jul-07


Short strangle/straddle
60 days
R^2
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
01-Jan-01
11-Apr-01
20-Jul-01
28-Oct-01
05-Feb-02
16-May-02
24-Aug-02
02-Dec-02
12-Mar-03
20-Jun-03
28-Sep-03
06-Jan-04
15-Apr-04
24-Jul-04
01-Nov-04
09-Feb-05
20-May-05
28-Aug-05
06-Dec-05
16-Mar-06
24-Jun-06
02-Oct-06
10-Jan-07
20-Apr-07
29-Jul-07


Short strangle/straddle
100 days
R^2
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
01-Jan-01
11-Apr-01
20-Jul-01
28-Oct-01
05-Feb-02
16-May-02
24-Aug-02
02-Dec-02
12-Mar-03
20-Jun-03
28-Sep-03
06-Jan-04
15-Apr-04
24-Jul-04
01-Nov-04
09-Feb-05
20-May-05
28-Aug-05
06-Dec-05
16-Mar-06
24-Jun-06
02-Oct-06
10-Jan-07
20-Apr-07
29-Jul-07


Long strangle/straddle
30 days
R^2
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
01-Jan-01
11-Apr-01
20-Jul-01
28-Oct-01
05-Feb-02
16-May-02
24-Aug-02
02-Dec-02
12-Mar-03
20-Jun-03
28-Sep-03
06-Jan-04
15-Apr-04
24-Jul-04
01-Nov-04
09-Feb-05
20-May-05
28-Aug-05
06-Dec-05
16-Mar-06
24-Jun-06
02-Oct-06
10-Jan-07
20-Apr-07
29-Jul-07


Long strangle/straddle
60 days
R^2
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
01-Jan-01
11-Apr-01
20-Jul-01
28-Oct-01
05-Feb-02
16-May-02
24-Aug-02
02-Dec-02
12-Mar-03
20-Jun-03
28-Sep-03
06-Jan-04
15-Apr-04
24-Jul-04
01-Nov-04
09-Feb-05
20-May-05
28-Aug-05
06-Dec-05
16-Mar-06
24-Jun-06
02-Oct-06
10-Jan-07
20-Apr-07
29-Jul-07


Long strangle/straddle
100 days
R^2
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00

The data presented in Figure 3.2.3 also throws light upon the individual features of certain criteria. In general, most of the criteria show similar cycling within the short strategy. Nevertheless, they do not always coincide in terms of phase, amplitude, and duration of cycles. In the case of the long strategy, such discrepancies are even more apparent. Such property of the criteria to follow individual patterns in their behavior is quite a useful feature in view of their practical use. When several criteria are used simultaneously to evaluate option combinations (see Chapters 7 and 8), low effectiveness of some criteria is balanced with higher effectiveness of the

others, which do not coincide with them in time phase. This increases the combined effectiveness of the whole set of criteria.

The cycling depicted on Figure 3.2.3 means that periods of high criteria effectiveness are succeeded by periods of their lower effectiveness and vice versa. If that is the case, how should we derive a unique numerical indicator that reflects the general effectiveness of a certain criterion regardless of its current position in the time cycling? This is necessary in order to (1) compare the effectiveness of different criteria as applied to a given option strategy; (2) compare the effectiveness of a certain criterion as applied to different

strategies; and (3) solve other problems that require selecting the most suitable criteria. This issue is discussed in the next section.

3.2.4 Selection of the Averaging Period

In view of the dynamic patterns displayed by the determination coefficient and the influence of the averaging period on this indicator, we proceed to the issue that is essential for evaluation of criteria effectiveness. To conduct a thorough analysis of different criteria we need to determine the optimal averaging periods for all of

them. These periods can be either individual for each criteria (and even change depending on the option strategy used) or be quite universal. Given the goals of the current investigation, we argue in favor of universality. (The arguments will be given next.) At the same time an individual approach could be preferred for practical purposes.

In the previous section we saw that despite the averaging procedure, the determination coefficient is quite variable in time and sometimes drops to rather low values. To obtain the generalized characteristic of criteria effectiveness, we need to express the determination coefficient in the form that describes the criterion regardless of the

phase of its dynamic cycle. Now we propose the procedure that generalizes the effectiveness indicator.

Consider Figure 3.2.3 (short strategy, 30 days) again. Averaging all determination coefficients related, for example, to the EPLN criterion, gives $R^2 = 0.16$. This indicator describes the generalized effectiveness of the EPLN criterion given the 30-day averaging period. Application of the 60-day period gives $R^2 = 0.21$ and application of the 100-day period gives $R^2 = 0.25$. Reproducing such calculations for other periods (the range from 2 to 900 days is sufficient) we obtain the relationship between the generalized coefficient of determination

and the averaging period. Having such type of analysis performed for all criteria (for long and short strategies separately), we obtain a set of relationships that serve as the basis for solving the problem of selecting the optimal averaging period.

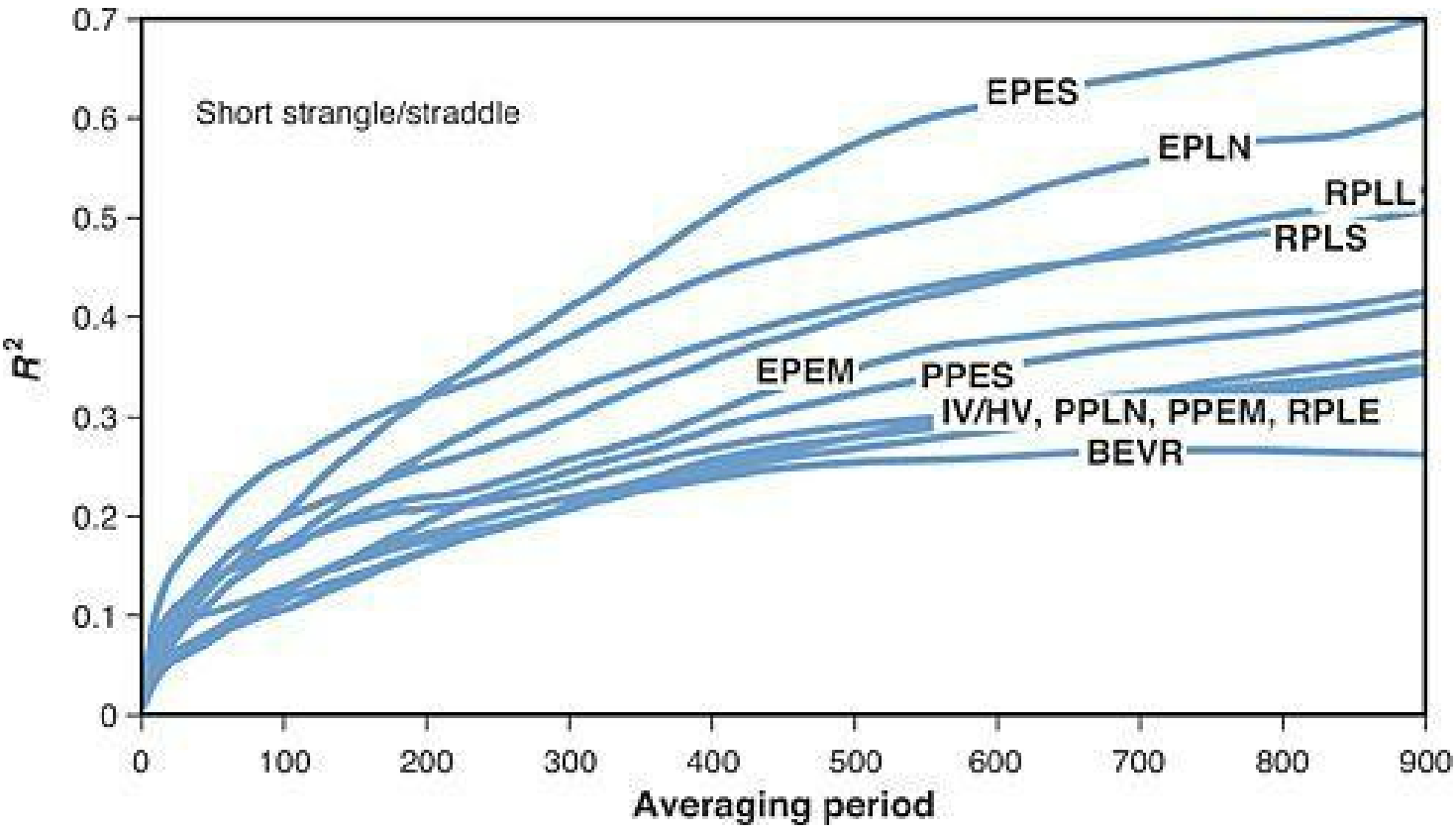
The results of the short strategy analysis are displayed in Figure 3.2.4. For all the criteria the increase of the averaging period causes an increase of the generalized determination coefficient (Figure 3.2.4). The relationships are nonlinear. As the averaging period increases, the correlations first grow rapidly. Further increase of the averaging period induces further growth of the effectiveness indicator but at a

slower rate. This pattern can be viewed as an approximate description matching the behavior of most criteria. However, despite the similarity, some criteria demonstrate their own specificity. For example EPLN and EPES have the highest determination coefficients almost at all averaging periods. The former dominates the latter when the averaging period is small (less than 200 days) and vice versa when the period is large (more than 200 days). Other criteria show weaker effectiveness although some of them are better than EPEM when the averaging period is short.

Figure 3.2.4. Relationship

between the determination coefficient and the averaging period for the short strategy. Criteria abbreviations: EPLN, EPEM, EPES—expected profit on the basis of lognormal, empirical, and symmetrized empirical distributions; PPLN, PPEM, PPES—profit probability on the basis of the same distributions; RPLL, RPLE,

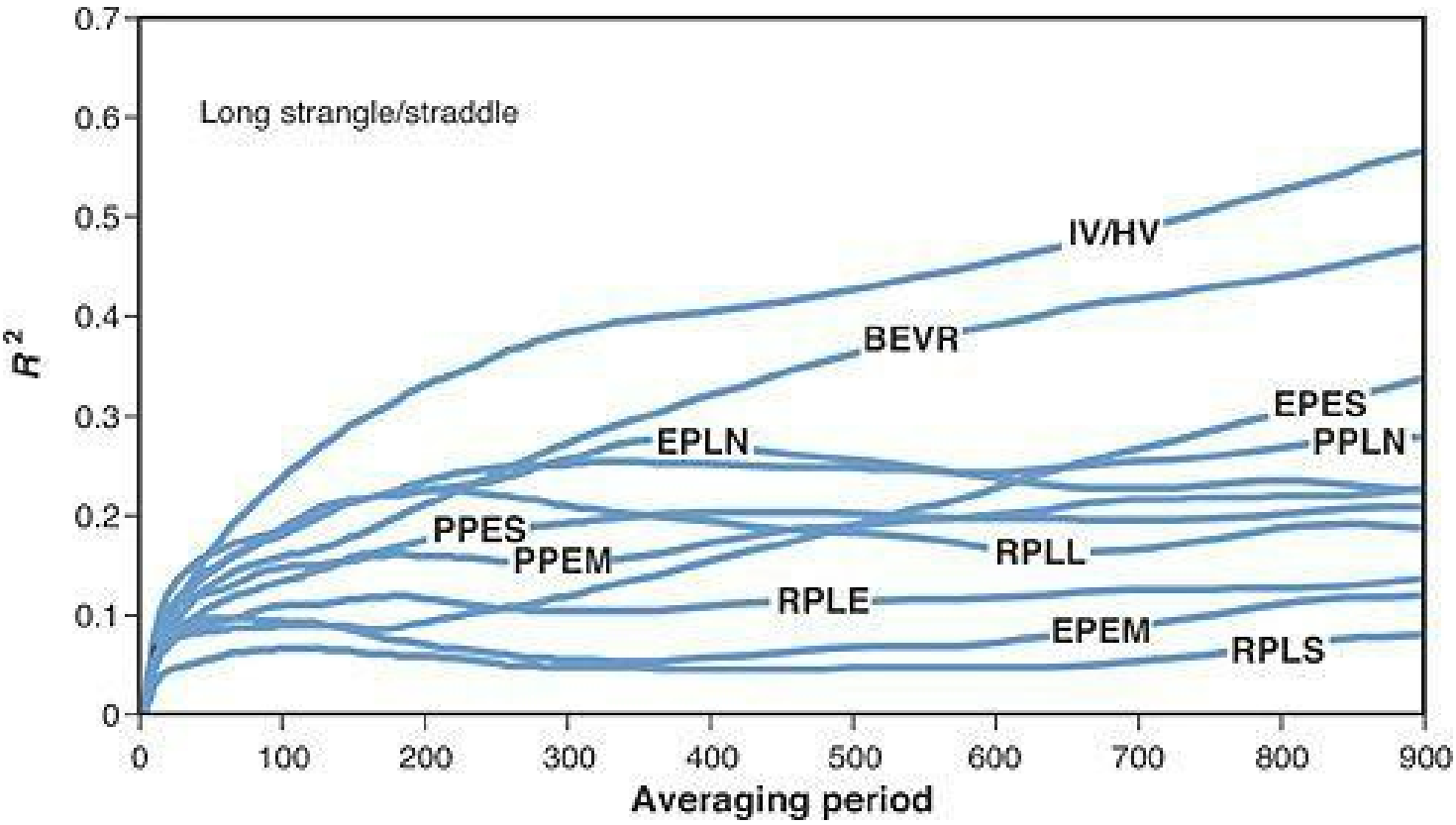
RPLS—the ratio of expected profit to loss based on these distributions; IV/HV—the ratio of implied to historical volatility; BEVR—break-even range.



In the case of the long strategy, the relationships between the generalized determination coefficient and the averaging period are similar to those observed in the case of the short strategy; the correlation rises as the period increases (Figure 3.2.5). However, the nonlinearity here is much stronger. Starting from the 50th day, the increase in the averaging period results in the steep fall of the correlation growth rate. When the number of days exceeds 100, the correlation growth completely stops. Such trend is typical for most but not all criteria. For example, EPEM and EPLS reach their maximums earlier than other criteria. BEVR and *IV/HV* do not have any evident points of saturation. To

summarize, for the long strategy the generalized indicators of criteria effectiveness are lower, and the saturations are reached earlier than in the case of the short strategy (compare Figures 3.2.4 and 3.2.5).

Figure 3.2.5. Relationship between the determination coefficient and the averaging period for the long strategy. Criteria abbreviations are the same as in Figure 3.2.4.



What averaging period should be used to evaluate criteria effectiveness? Should this period be universal for all criteria or chosen on an individual basis? These questions are essential not only to the indicator considered up to now ({criterion × profit} correlation) but also regarding other criterion effectiveness indicators that have not

been discussed yet. Considering these questions, we should not forget that averaging is just an auxiliary procedure intended to increase the significance of criterion effectiveness evaluation. The procedure of evaluation is used not merely to judge about the quality of a certain criterion but also to compare it with other criteria. Therefore, it is preferable to use the same averaging period for all criteria used in the same research to ensure their comparability. Hence, the most acceptable solution would be to determine the universal (that is, suitable for all the criteria) value for the averaging period.

How should we define the value for the universal averaging period? The

simplest decision to use the maximum period (of 900 or more days) seems to be erroneous. Although in most cases the correlation reaches its maximum under the longest period, such approach does not comply with one of the fundamental principles of trading systems development—to refrain from using excessively long historical intervals to avoid over-optimization and other adverse effects of data fitting. Therefore, we suggest using a relatively small threshold period corresponding to the point at which correlation growth reaches saturation. In other words, the optimal period value is the moment when the growth rate of the determination coefficient starts

dropping.

Adopting this approach and analyzing the data in Figures 3.2.4 and 3.2.5, we face difficulties with fixing an averaging period that could be considered as common for all the criteria. The figures clearly show that the optimal period value, as it was previously defined, is individual for each criterion and can vary depending on the option strategy. Nevertheless, we need to fix a compromise solution that will be at least to some extent suitable for different situations that we discuss next.

Bearing in mind all these arguments and considerations, we decided that the most acceptable value for the averaging

period is 100 days. This value will be used further to evaluate the criteria effectiveness. However, it is not supposed to be regarded as an invariable, permanent, and the most optimal solution for criteria quality evaluation. This is just a convenient compromise allowing us to demonstrate different aspects of criteria effectiveness evaluation. (This averaging period is equally suitable for several option strategies, many criteria, and numerous effectiveness indicators.) On the contrary, when selecting the averaging period for practical use, such global universality is not required. It would be enough to adhere to it only within the local system of compared criteria and

strategies. Hence, when creating a real trading system, we can use any number of averaging periods depending on the criteria and strategies used by the system.

3.3 Peculiarities of Criteria Effectiveness Evaluation

3.3.1 Number of Combinations Used in the Analysis

Assessing the criterion effectiveness through the correlation between criterion and profit values requires the calculation

of the determination coefficient on the basis of a certain sample. We decided that this sample was going to include 1,500 combinations: three for each of the 500 stocks. However, if we used only one combination instead of the three best ones, then the sample would consist of just 500 elements. Likewise, using five combinations for each stock would expand the sample to include up to 2,500 elements. Moreover, the number of stocks itself was determined arbitrarily. We could restrict the number of underlying assets to 100 or, on the contrary, we could use several thousand of them.

In fact, the sample size represents an additional parameter influencing the

evaluation of the criterion effectiveness (another parameter, the averaging period, was previously discussed). We call this parameter "the number of combinations used in the analysis." Here we examine the sensitivity of the effectiveness indicator to changes in the number of combinations and determine the optimal value for this parameter.

Careful examination of Figure 3.2.1 reveals that the distribution of points on the coordinate space deviates significantly from the normal cigar-shaped pattern. (You can clearly see it on the charts with high averaging periods: 8 and 10 days.) It is rather pear-shaped, linearly stretched in the area of high criterion values and with a

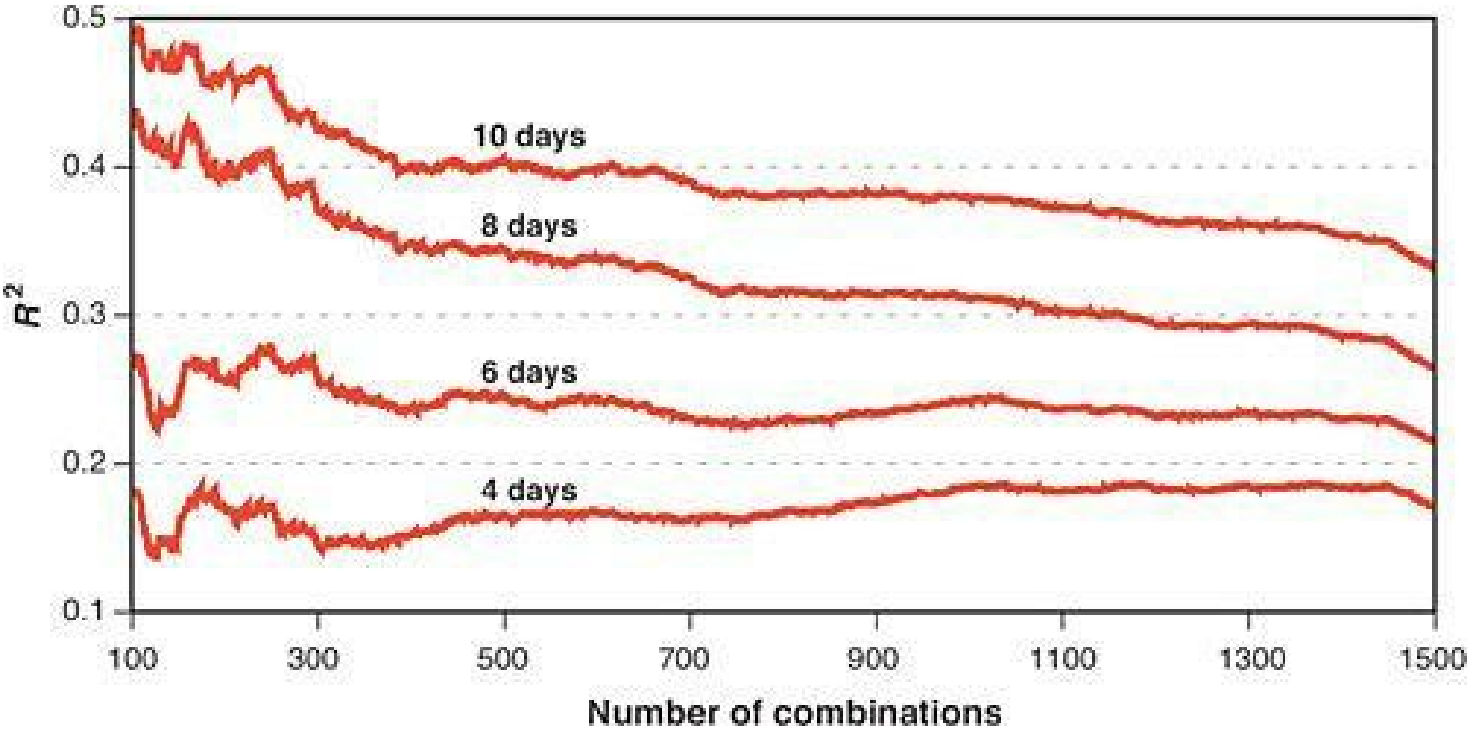
considerable dispersion of points, which seems to be random, in the area of low criterion values. Visual analysis suggests that if the sample consists of exclusively combinations with high criterion values, the coefficient of determination would be much higher than if the sample includes elements with both high and low criterion values. In other words, adding combinations with low criterion values into the sample weakens the correlation.

Now we present the market data providing formal grounds for these visual observations. Using the data from Figure 3.2.1 (August 27, 2007) we begin with the 10-day averaging period. All combinations are sorted according to

criterion values and the determination coefficient is calculated for the first 100 combinations. Then we add the 101st combination and compute the effectiveness indicator value again. We continue adding combinations one after another right up to the last one (1,500th), every time obtaining the coefficient of determination. The result is depicted by the top line in Figure 3.3.1.

Figure 3.3.1. Relationship between the coefficient of determination and the number of combinations used in the regression

analysis. Criterion: expected profit on the basis of lognormal distribution; strategy: short strangle/straddle; date: August 27, 2007. Four averaging periods are shown.



As was expected, the maximum value of the determination coefficient corresponds to the minimal sample of 100 combinations. The gradual increase of the sample size to 400 combinations entailed a sharp drop in the criterion effectiveness indicator. Afterward, adding new elements up to the 700th element did not change the correlation value. Further increase in the number of

combinations led to the decrease of the determination coefficient down to its minimum reached upon inclusion of the 1,500th combination (Figure 3.3.1, the top line). The same trend was detected for the 8-day averaging period (Figure 3.3.1, the second line from the top). At the same time, lower averaging periods did not show any definite dynamics (Figure 3.3.1, the two bottom lines). This once again confirms the usefulness of the averaging procedure.

The analysis presented in Figure 3.3.1 indicates the negative influence of the sample size on the criterion effectiveness indicator. Beside this, we can draw another important conclusion: The criterion used in this investigation

more accurately (that is, producing fewer errors) identifies combinations as "good" rather than as "bad" ones. This means that although unprofitable combinations would not be mistakenly included in the portfolio, many potentially profitable combinations could be undervalued and wrongly excluded from the potential investment universe.

Nevertheless, we refrain from making any generalizations regarding the shape and the degree of the relationship between the effectiveness indicator and the sample size. After all, we presented data corresponding to only one criterion, one strategy, the only randomly selected date, and one effectiveness indicator.

Hence, we recommend treating these results merely as an indication that the sample size actually has influence upon the value of the criterion effectiveness indicator.

We do not overcharge this chapter with illustrations of all possible relationships between the effectiveness indicator and the number of combinations used in the analysis. However, it is worth mentioning that in other cases these relationships could become positive rather than negative. In fact, positive relationships are quite natural because, all other things being equal, adding data to the sample normally causes an increase of the correlation coefficient. Yet this phenomenon could be further

amplified by more efficient identification of "bad" combinations accompanied by less effective recognition of "good" ones.

Deciding on the optimal number of combinations is a difficult task, and the solution depends on a considerable number of interrelated factors such as criterion, strategy, effectiveness indicator, and so on. Just as in the case of parameterization of the averaging period, it is preferable to select optimal values for different situations individually. Nevertheless, for the purpose of reviewing various effectiveness indicators (see section 3.4) we need to set one universal value for the number of combinations used in the

analysis. It is essential for the comparability of the results obtained in different studies.

The optimal parameter value is chosen based on our experience and a large number of preliminary investigations. In the following research we continue to use 500 stocks as underlying assets, but the number of combinations selected for each of them will be reduced from three to one. Thus, all samples will consist of 500 combinations. This compromise value seems to be quite optimal if it has to be universal.

3.3.2 Expressing Profit

Evaluation of criterion effectiveness is based on two variables: criterion and profit. Their values must be estimated for all the combinations used in the analysis. In the examples discussed we expressed the values of the EPLN criterion and the values of profit in U.S. dollars normalized by the margin requirements. Thus, both the criterion and the profit were expressed in the same units; they were similarly scaled and, in fact, represented variables with the same meaning.

When evaluating the effectiveness of criteria forecasting other values (rather than expected profit), we are obliged to choose between two alternatives. One

possibility is to express realized profits in normalized U.S. dollars regardless of what the criterion forecasts. Another alternative is to express profit in the same way as it is forecasted by the criterion.

Unfortunately, it is not always possible to express profit exactly in the same way as forecasted by the criterion. Still, even in these cases it can at least be expressed in the same units as criterion values. For example, when evaluating the effectiveness of the criterion forecasting profit probability, it is impossible to express the realized profit of a single combination as a probability. However, if we consider a group of combinations (grouping is based on the

positions occupied by combinations in the orderings), we can calculate the proportion of profitable combinations (ratio of profitable combinations to the total number of combinations). This value is easily comparable with profit probability (that is, with the criterion value averaged for the group) and is expressed in the same units.

In most cases the second alternative—to express profit as it is forecasted by the criterion—is preferable. The following examples will support this statement.

We continue using the determination coefficient of the {criterion × profit} relationship as the indicator of criterion effectiveness. It will be applied to

analyze two criteria: expected profit and profit probability based on the lognormal distribution (EPLN and PPLN, respectively)—within the short strangle/straddle strategy. The parameters are fixed as follows: the 100-day averaging period, 500 combinations used in the analysis. Every group of combinations is formed according to its position in the orderings. (The first group corresponds to the first positions in the orderings, the second group to the second positions, and so on.) Because we use a 100-day averaging period, there will be 100 elements in each group. The total number of groups is 500 (according to the number of combinations used in the

analysis). The realized profit values (calculated separately for every group) are expressed in two different ways: (i) in U.S. dollars normalized by margin (which corresponds to the EPLN criterion) and (ii) as proportion of profitable combinations (which corresponds to the PPLN criterion).

Here we analyze four {criterion × profit} relationships:

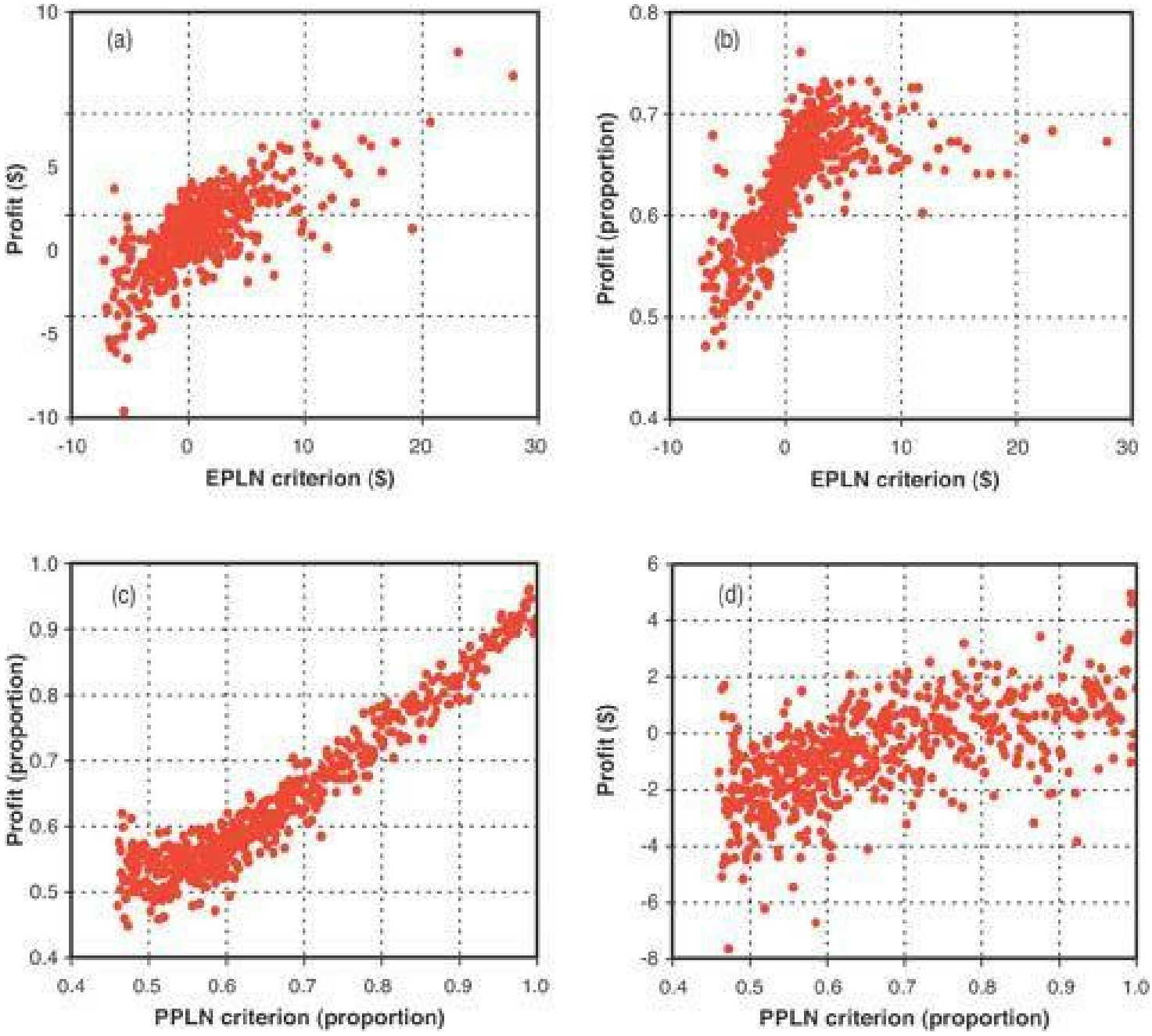
- Criterion in U.S. dollars (EPLN)—profit in U.S. dollars
- Criterion in U.S. dollars (EPLN)—profit as proportion
- Criterion as proportion (PPLN)—profit as proportion

- Criterion as proportion (PPLN)—profit in U.S. dollars

The first and third alternatives correspond to situations when the criterion and profit are expressed in the same units (Figure 3.3.2(a) and 3.3.2(c)). The second and fourth alternatives depict the situations when the criterion and profit are expressed in different units (Figure 3.3.2(b) and 3.3.2(d)).

Figure 3.3.2. Relationships between values of two criteria (EPLN and PPLN)

and profit expressed in two ways (in U.S. dollars and as proportion of profitable combinations) for the short strangle/straddle strategy. (a) and (c)—criterion and profit are expressed in the same units. (b) and (d)—criterion and profit are expressed in different units.



The analysis of four regression models leads to the following conclusion: When profit and the criterion are measured in the same units, the effectiveness indicator is significantly higher. For

example, the determination coefficient of EPLN, when both the criterion and profit are expressed in US dollars, is $R^2 = 0.66$ (Figure 3.3.2[a]). However, expressing profit as the proportion of profitable combinations leads to a substantial drop in the effectiveness indicator ($R^2 = 0.36$, Figure 3.3.2[b]). Similarly, the correlation of the PPLN criterion and the profit is very high, when both values are expressed as proportions ($R^2 = 0.92$, Figure 3.3.2[c]). Still, expressing profit in U.S. dollars decreases the effectiveness indicator to $R^2 = 0.37$ (Figure 3.3.2[d]).

The results of this investigation suggest that expressing profit in the same units as

forecast by criterion substantially increases the effectiveness indicator. Besides, such approach seems to be more correct from the methodological point of view and increases the reliability of the whole process of criteria effectiveness evaluation.

We propose the following approach to profit expression when evaluating the effectiveness of the main criteria described in Chapter 2.

- For criteria forecasting future profit (expected profit on the basis of different distributions), we recommend to express profit in U.S. dollars with the same normalization as for criteria

values.

- For criteria forecasting profit probability on the basis of different distributions, it is convenient to express profit as a proportion of profitable combinations. The same approach can be used for the BEVR criterion. Although profit cannot be expressed in the same units as forecast by this criterion, the notion of the "positive profits area" is close to probability and can be approximated by the proportion of profitable combinations.
- For criteria based on the ratio of

expected profit to loss, we recommend to express profit as the ratio of the difference between profitable and unprofitable combinations to the total number of combinations.

- The *IV/HV* ratio criterion does not forecast profit. Therefore, to evaluate its effectiveness we can calculate (instead of estimating realized profits) the ratio of historical volatility realized on a certain date in the future (the forecast date) to historical volatility used to obtain the criterion value. Likewise, for all other specific (not universal) criteria, we should establish

special forms of profit expression that correspond to the criterion nature as much as possible.

3.3.3 Expressing Effectiveness Indicators

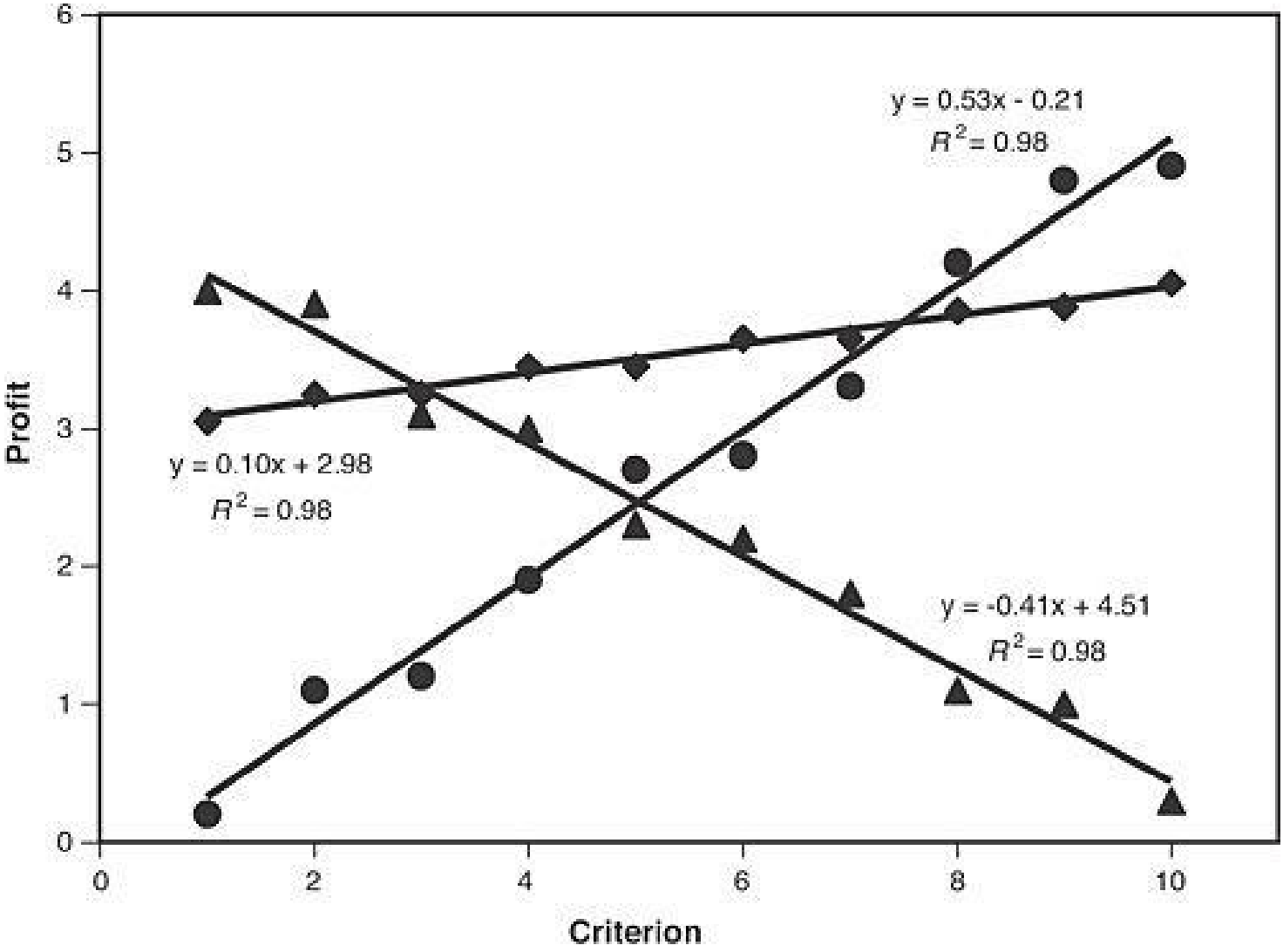
In the previous sections we evaluated the criterion effectiveness using a regression model between criterion and realized profit values. The determination coefficient that reflects the variability of data around the regression line was accepted as the effectiveness indicator. Though the determination coefficient adequately reflects the degree of interdependence between the criterion

and the profit, it says nothing about the shape of their relationship. Consider the following hypothetical situation. Suppose that we have to evaluate three different samples, each consisting of ten combinations created within a certain option strategy. We have to assess the criterion effectiveness in selecting combinations for each strategy.

We purposely generated data for this simulation in such a way that in each case the coefficient of determination is very high and equal for all three strategies. It seems that the criterion effectiveness (as it was defined in the previous sections) is the same for each strategy. However, the regression chart points at the prematurity of this

conclusion (Figure 3.3.3).

Figure 3.3.3. Hypothetical model simulating three different types of relationships between criterion and profit values.



In one case the combination profit grows steeply as the criterion value increases. In the second case the growth is more gradual. And finally, in the third case the increase in the criterion value results in the drop in the combination profit. Given these forms of {criterion × profit}

relationships, we should conclude that this criterion is more effective for the first strategy, less effective when applied to the second one, and unsuitable for the third strategy. However, our indicator (the coefficient of determination) indicates equal effectiveness of this criterion in all three situations!

This simulation experiment indicates the necessity to establish an additional indicator of criterion effectiveness, which takes into account not the {criterion × profit} correlation strength but rather the shape of this relationship. The shape can be described by the regression equation with the regression line slope being the most important

coefficient for our purposes. In the models presented in Figure 3.3.3, the greatest slope is attributed to the first strategy (0.53); it is lower for the second one (0.1) and negative for the third strategy (–0.41).

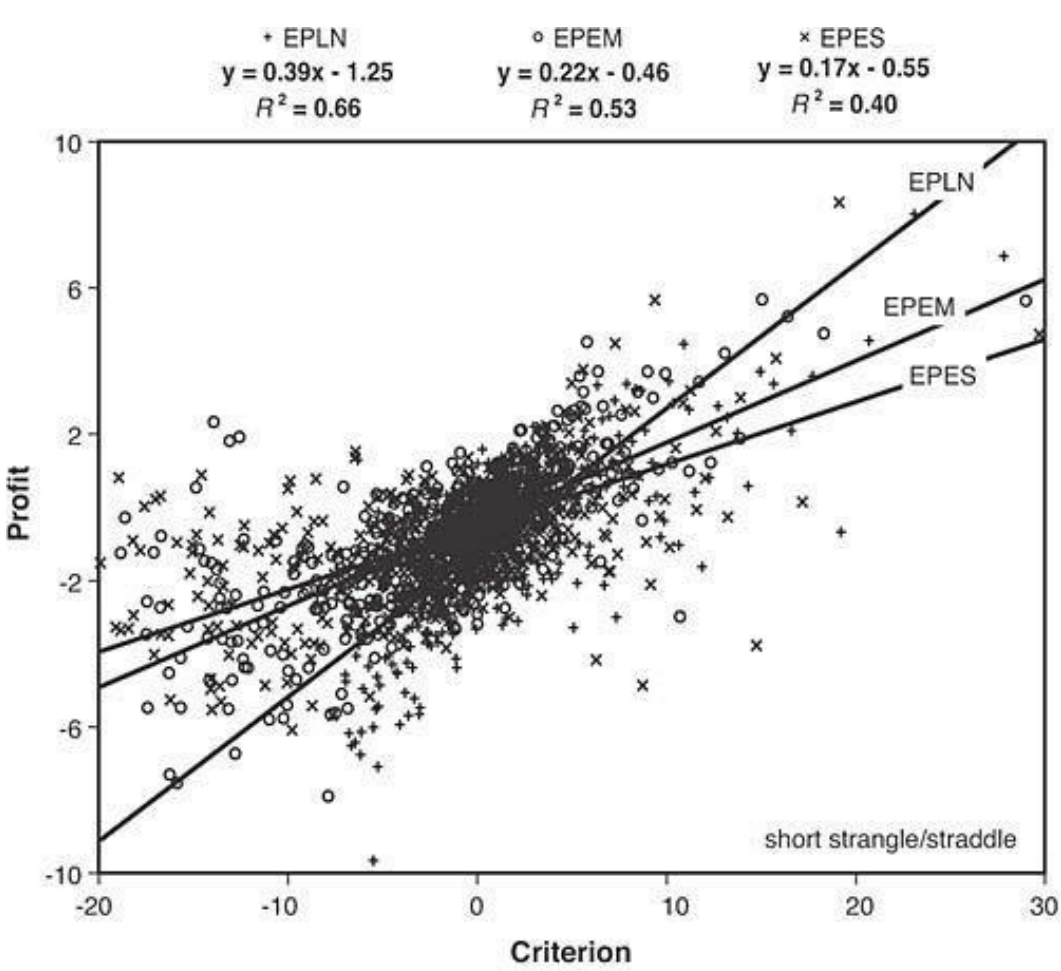
It seems reasonable to express the criterion effectiveness indicator in two supplemental forms: as the determination coefficient (which reflects the correlation strength between the criterion and the profit) and as the slope coefficient (which reflects the shape of the relationship). At the same time we should mention that simulating the hypothetical situation presented in Figure 3.3.3 we intentionally generated data in the way that made determination

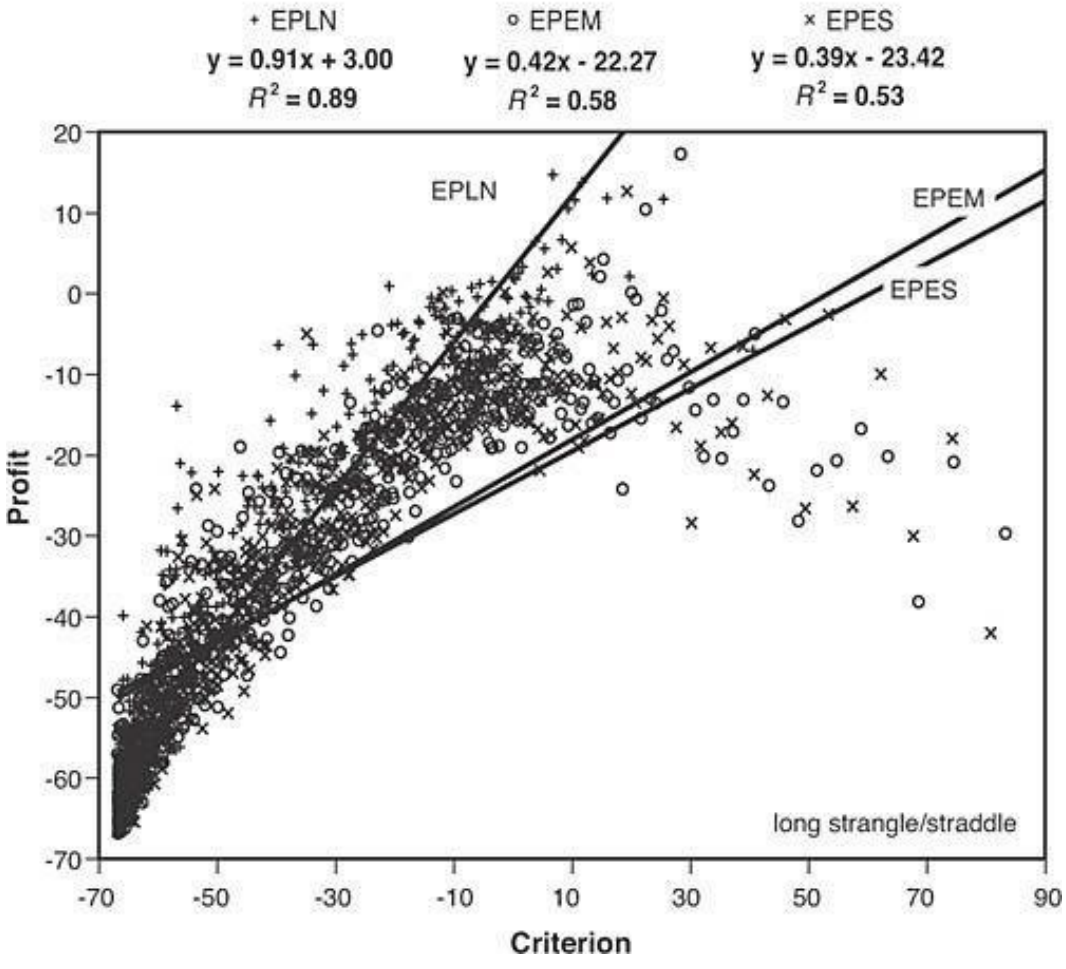
coefficients equal and independent of regression slopes. Therefore, it would be useful to analyze the real relationship existing between these two criterion effectiveness indicators using the market data.

Consider the {criterion × profit} regression models for three criteria (expected profit on the basis of lognormal (EPLN), empirical (EPEM), and symmetrized empirical (EPES) distributions) applied to two opposite option strategies (short and long strangle/straddle). The parameter values are equal to those fixed in the previous studies: a 100-day averaging period and 500 combinations used in the analysis. For the EPLN criterion the effectiveness

indicator expressed as a determination coefficient is maximal for both short and long strategies (Figure 3.3.4). If criterion effectiveness is measured by regression slope, the result is exactly the same—for both strategies the highest coefficient is detected for the EPLN criterion (Figure 3.3.4). EPEM and EPES criteria take the second and the third positions for both the long and the short strategies. In these cases two forms of the effectiveness indicator again produced consistent valuations—both the determination coefficient and the slope were higher for EPEM relative to EPES (Figure 3.3.4).

Figure 3.3.4. Regression analysis of {criterion × profit} relationships for three criteria and two option strategies. Criteria abbreviations are the same as in Figure 3.2.4.


+ EPLN
y = 0.39x - 1.25
R^2 = 0.66
○ EPEM
y = 0.22x - 0.46
R^2 = 0.53
× EPES
y = 0.17x - 0.55
R^2 = 0.40
EPLN
EPEM
EPES
short strangle/straddle
Profit
Criterion
10
6
2
-2
-6
-10
-20
-10
0
10
20
30


+ EPLN
y = 0.91x + 3.00
R^2 = 0.89
○ EPEM
y = 0.42x - 22.27
R^2 = 0.58
× EPES
y = 0.39x - 23.42
R^2 = 0.53
EPLN
EPEM
EPES
long strangle/straddle
Profit
Criterion
20
10
0
-10
-20
-30
-40
-50
-60
-70
-70
-50
-30
-10
10
30
50
70
90

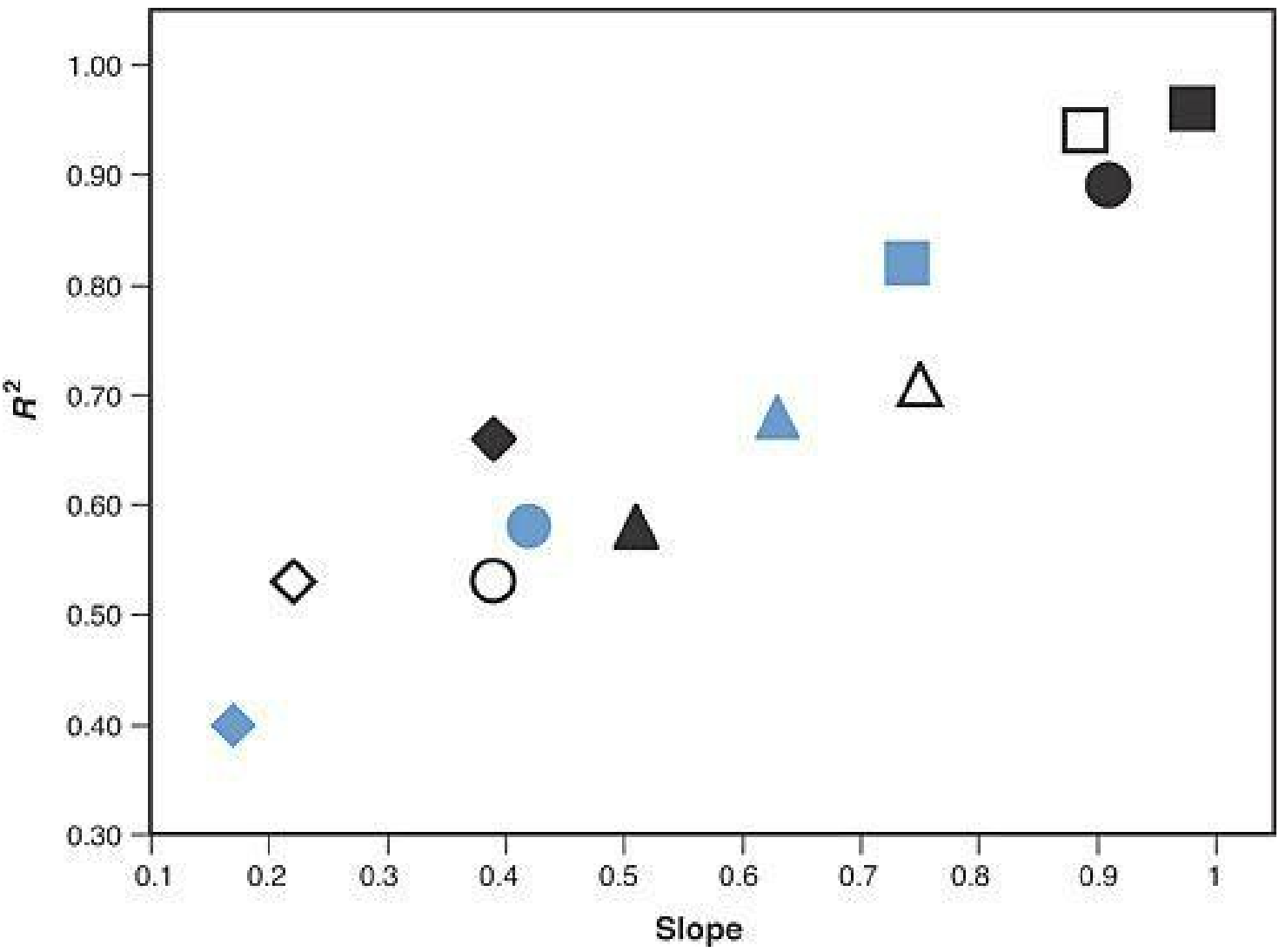
Summarizing the aforesaid, we conclude that usually the determination coefficient is positively correlated with the slope coefficient. This suggests that both forms of the effectiveness indicator evaluate the forecasting abilities of criteria quite similarly. To provide more ground for this suggestion, we build regression models analogous to those presented in Figure 3.3.3 for two more option strategies: short and long calendar spread. The values of both coefficients were computed for each regression, and the relationship between determination and slope coefficients were analyzed.

Our expectation proved to be true. There is a strong correlation between two

forms of this effectiveness indicator (Figure 3.3.5). Hence, evaluation of the criterion effectiveness using the {criterion × profit} regression can be limited to only one of them. At the same time one should remember that, when evaluating other criteria and using them with other strategies, we can face other shapes of the relationship between different forms of a certain effectiveness indicator. (There may even be no relationship at all.) In such cases it would be reasonable to use several forms of the effectiveness indicator because information contained in each of them could be supplemental rather than overlapping.

Figure 3.3.5. Relationship between the determination and the slope coefficients of the {criterion × profit} regression models estimated for three criteria and four option strategies. The EPLN criterion is in black, EPEM is in gray and EPES is denoted by contour signs. Diamonds stand for the short strangle/straddle strategy, circles for the long

strangle/straddle strategy, triangles for the short calendar spread, and squares for the long calendar spread.



3.4 Review of Criteria Effectiveness Indicators

In this section we review the main effectiveness indicators intended to evaluate the criteria forecasting abilities. The indicators described here do not constitute a complete set of criteria effectiveness analysis tools. The development of new effectiveness indicators and elaboration of existing ones is a continuous process progressing in parallel with creating and upgrading of criteria as such.

Properties of different effectiveness indicators are illustrated by examples based on real market data. Using September 10, 2007, as the starting point, we calculate indicators on the basis of a 100-day averaging period. (The grounds for this choice were discussed in section 3.2.5.) The number of combinations used in the analysis will be 500. (This choice was discussed in section 3.3.1.)

Four criteria (EPLN, EPEM, PPLN, EPLL) and four option strategies (long and short strangle/straddle, long and short calendar spread) serve as the examples for each effectiveness indicator. Wherever it is possible, profit is expressed in the same form as

criterion (as described in section 3.3.2).

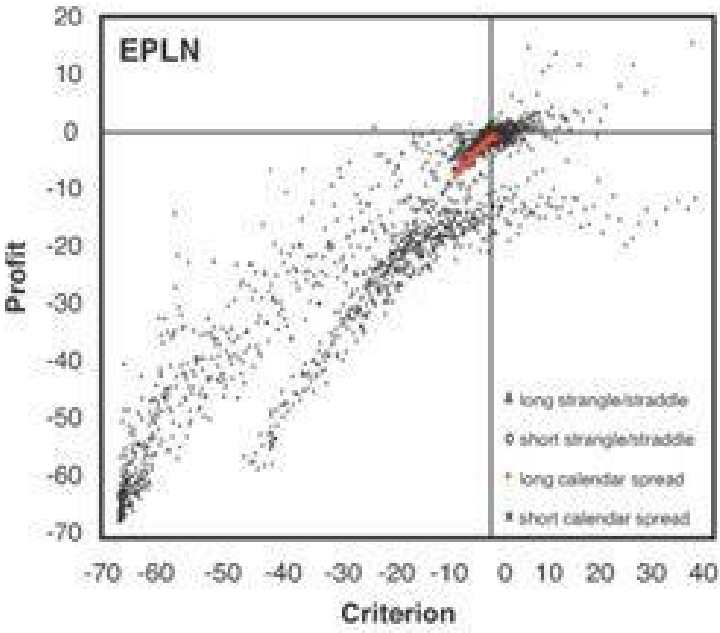
3.4.1 Correlation Between Criterion and Profit Values

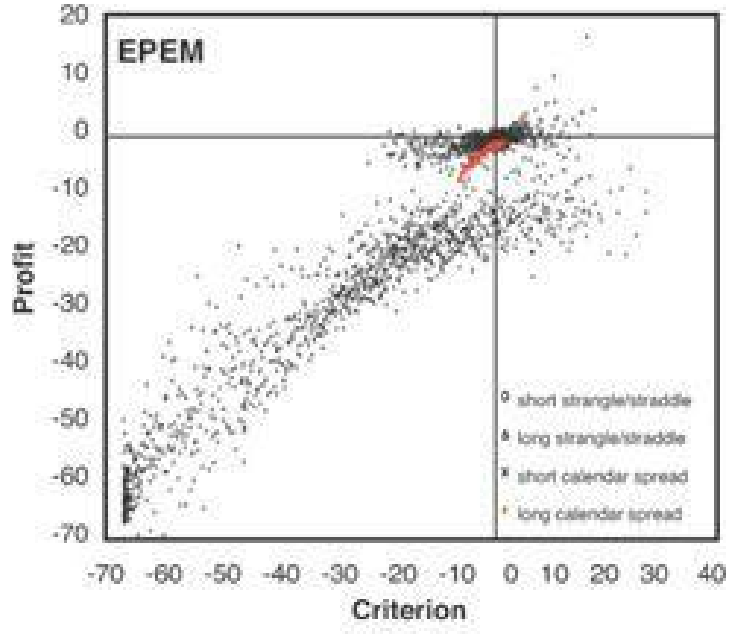
This effectiveness indicator was previously discussed in detail. Hence, we will not return to the calculation method and will proceed directly to its application to different criteria and option strategies.

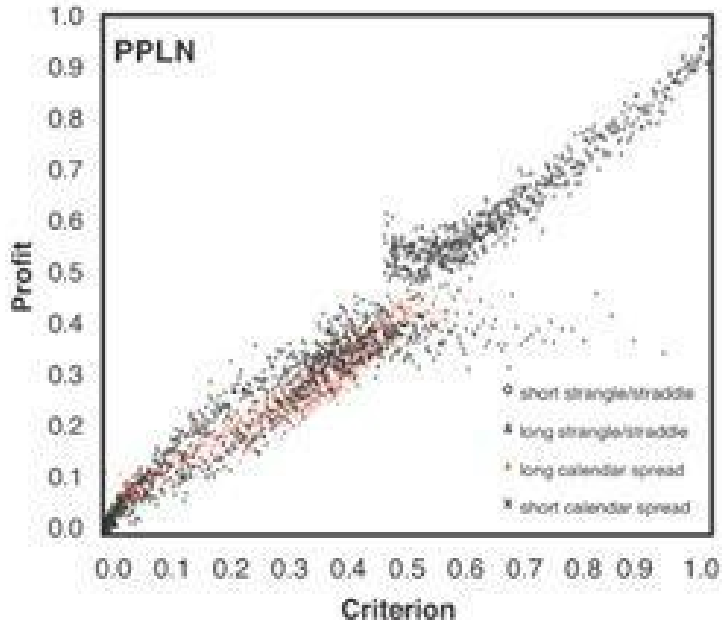
Obviously, this indicator evaluates the qualities of different criteria differently (Figure 3.4.1). The values of the criteria, which forecast profit probability (PPLN) and the profit to loss ratio (RPLL), demonstrate a much stronger

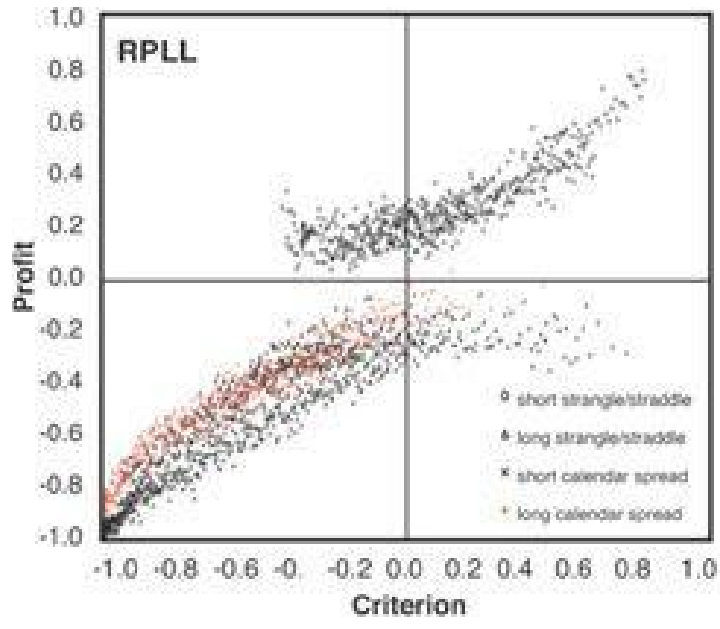
correlation with realized profit than the criteria forecasting expected profit (EPLN and EPEM). This is true for all strategies except long calendar spread where the highest determination coefficient was detected for the EPLN criterion ($R^2 = 0{,}96$).

Figure 3.4.1. Regression analysis of {criterion × profit} relationships for four option strategies and four criteria.









Concerning the differences between strategies, the following conclusions can be drawn. The PPLN and RPLL criteria show quite similar effectiveness for all four option strategies (Figure 3.4.1). The

only exception is the short calendar spread for which both criteria showed lower effectiveness than for the other strategies. In contrast to PPLN and RPLL, the effectiveness of the other two criteria turned out to be heterogeneous with respect to different strategies (Figure 3.4.1). EPLN and EPEM were the most effective when applied to the long calendar spread strategy. EPLN was the least effective for the short calendar spread and EPEM for the short strangle/straddle strategy.

Summarizing the analysis of data presented in Figure 3.4.1, we conclude that this effectiveness indicator allows ranking of criteria according to their appropriateness for selecting potentially

profitable option combinations. It is noteworthy that a certain criterion is highly effective when applied to one strategy yet displays inferior effectiveness with others. Similarly, a certain criterion may effectively evaluate option combinations created within a given strategy while other criteria fail in accomplishing this task.

This means that the quality of every criterion should be thoroughly evaluated with regard to different option strategies. This evaluation results in classifying of criteria and option strategies according to their mutual compatibility. Such approach establishes a correspondence between strategies and their most suitable criteria.

3.4.2 Correlation Between a Criterion and Profit Indexes

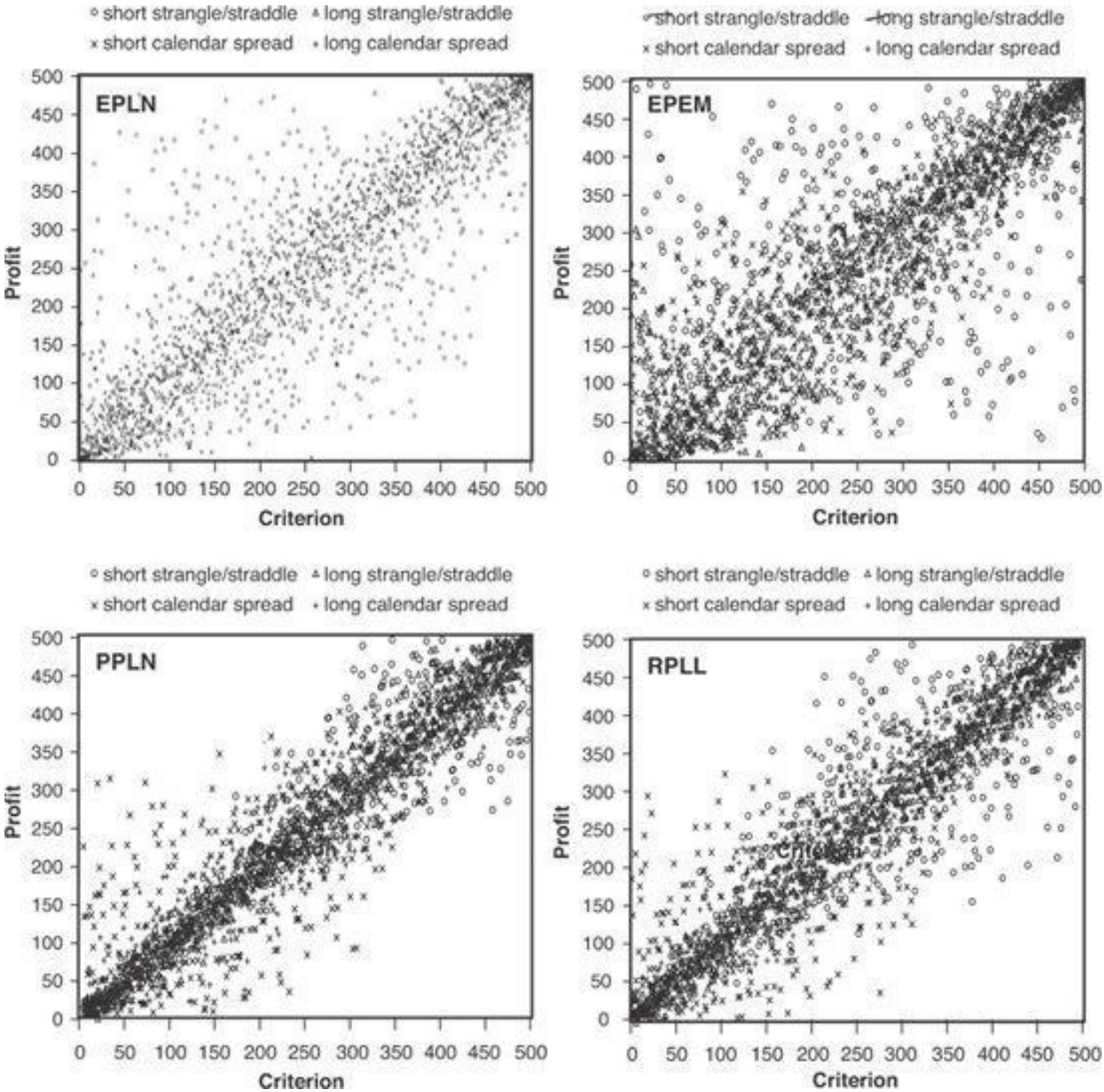
This indicator is similar to the previous one in its nature and calculation methodology. The difference is that instead of measuring the correlation between the absolute values of criterion and profit we analyze their relative values. All combinations are sorted by criterion values and get indexes corresponding to their positions in the ordering. The first combination, having the highest criterion value, is assigned index 1; the second combination receives index 2, the last combination,

having the lowest criterion value, is given index 500. Then all combinations are sorted once again according to their profit values and receive second indexes according to their positions in this ordering. As a result, each combination possesses two indexes corresponding to its criterion and profit values. The criterion effectiveness is determined by the regression of two indexes, and the determination coefficient is used as an indicator of effectiveness.

The advantage of this effectiveness indicator over the previous one (section 3.4.1) is in smoothing extreme criterion and profit values. Apart from that, the criterion and the profit variables always have the same scales ranging from 1 to

500. This allows distributing data evenly within a two-dimensional coordinate system that makes visualization easier (compare Figures 3.4.1 and 3.4.2).

Figure 3.4.2. Regression analysis of criterion and profit indexes for four option strategies and four criteria.



Visual observation in Figure 3.4.2 reveals that the EPLN criterion gives better results for the long strategies than

for the short ones. This criterion is the least effective for the short strangle/straddle strategy as evidenced by the fuzzy cloud of points broadly scattered on the regression plane. The same conclusion relates to the EPEM criterion. Moreover, this criterion is characterized by greater fuzziness of points in the bottom-left corner of the graph in comparison with the top-right corner (Figure 3.4.2). It means that EPEM erroneously more frequently identifies combinations as “good” (the bottom-left corner of the chart corresponding to high criterion values) than identifying them as “bad” (the top-right corner corresponding to low criterion values).

With regard to all strategies, PPLN and RPLL are more effective than the other two criteria. (It complies with the results obtained on the basis of another effectiveness indicator described in section 3.4.1.) Charts corresponding to PPLN and RPLL (Figure 3.4.2) clearly show that when applied to the short calendar spread strategy these criteria effectively reveal bad combinations but often make mistakes identifying good ones. This feature is proved by the fuzziness of points in the bottom-left corners of these charts and their concentration alongside the diagonals of top-right corners. Interestingly, the situation for the short strangle/straddle strategy is the opposite: Points are fuzzy

in top-right corners and are close to the diagonal in the bottom-left corners of the charts (Figure 3.4.2).

The effectiveness indicator based on the correlation between criterion and profit indexes is similar to the one we discussed in section 3.4.1. Furthermore, the numerical values of both indicators (determination coefficients) are so close that the two evaluating methods could be considered as identical. However, their visual characteristics differ. The regression of indexes effectively detects distinctive peculiarities of the relationship between the criterion and the profit that remain hidden if absolute values are analyzed. In particular, index analysis effectively displays the

heterogeneity of forecasting abilities of the criterion within different ranges of its values. Some criteria can be highly effective in the area of high values and rather ineffective in the area of low values (or vice versa). This information, being readily obtainable by the visual analysis of index regression, is important for criteria evaluation.

3.4.3 Correlation Between the Sharpe Ratios of Criterion and Profit

This method of criterion effectiveness evaluation is also based on regression analysis. In contrast to the previous methods, here coordinates of points on

the regression plane are determined by the Sharpe ratios of the criterion and profit and not by their raw values.

Because we set a 100-day averaging period, there is a group of 100 elements for each position (ranging from 1 to 500) in 100 orderings of combinations according to the criterion values. These groups are formed in the following way. For a certain date the combination with the maximum criterion value is selected. Similarly, the combination with the maximum criterion value is chosen for the previous date and so on up to 100 preceding days. As a result, we have a group of 100 combinations each of which was the best in one of 100 days. A group of 100 combinations that were at

the second positions for each day is created in an analogous way. Finally, we obtain 500 groups (because we use 500 combinations for each date).

Thus, we have 100 combinations with the maximum criterion values, 100 combinations occupying the second positions in the orderings by criterion values, 100 combinations situated at the third positions, and so on up to the 500th positions. For each of these groups we can compute not only the average criterion and profit value (we used it in sections 3.4.1 and 3.4.2 for the EPLN and EPEM criteria) and the ratio of profitable and unprofitable combinations (we used it in sections 3.4.1 and 3.4.2 for the PPLN and RPLL criteria) but also

the indicator showing the variability of criterion and profit values within each of the 500 groups.

We express the variability by calculating a standard error that is the ratio of a standard deviation to the square root of the number of elements in the sample (in this case, the number of elements is 100). For each group the Sharpe ratios of the criterion and the profit are computed by dividing the average by the standard error. (Traditionally, the Sharpe ratio is defined as the ratio of the excess return [difference between the average profit and the risk-free rate] to standard deviation. In our version of the Sharpe ratio, we assume the risk-free rate to be zero and use standard error instead of

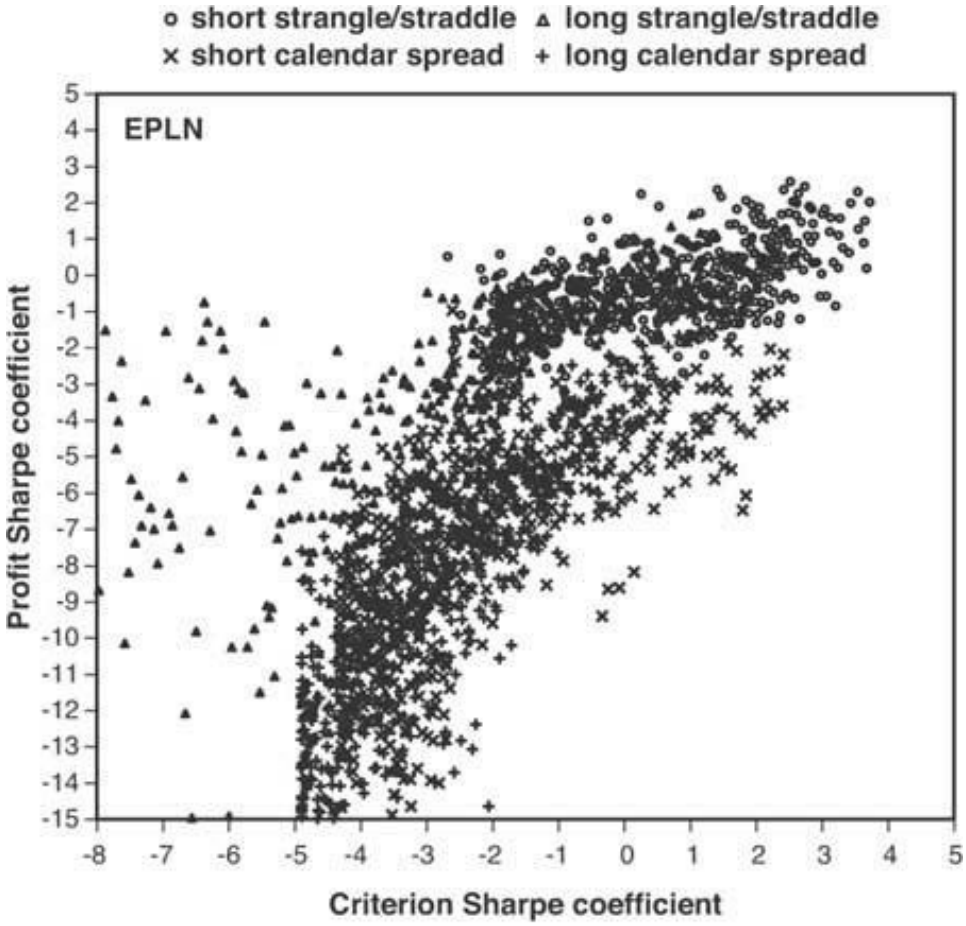
standard deviation.) Next, we apply regression analysis to these data. Point coordinates are the Sharpe ratios of the criterion, and the profit and each point corresponds to one of the 500 groups. The determination coefficient of this regression reflects the strength of the relationship between the profit Sharpe ratio and the criterion Sharpe ratio. This method allows evaluating the {criterion × profit} relationship taking into account not only the averages of these indicators but of their variability as well.

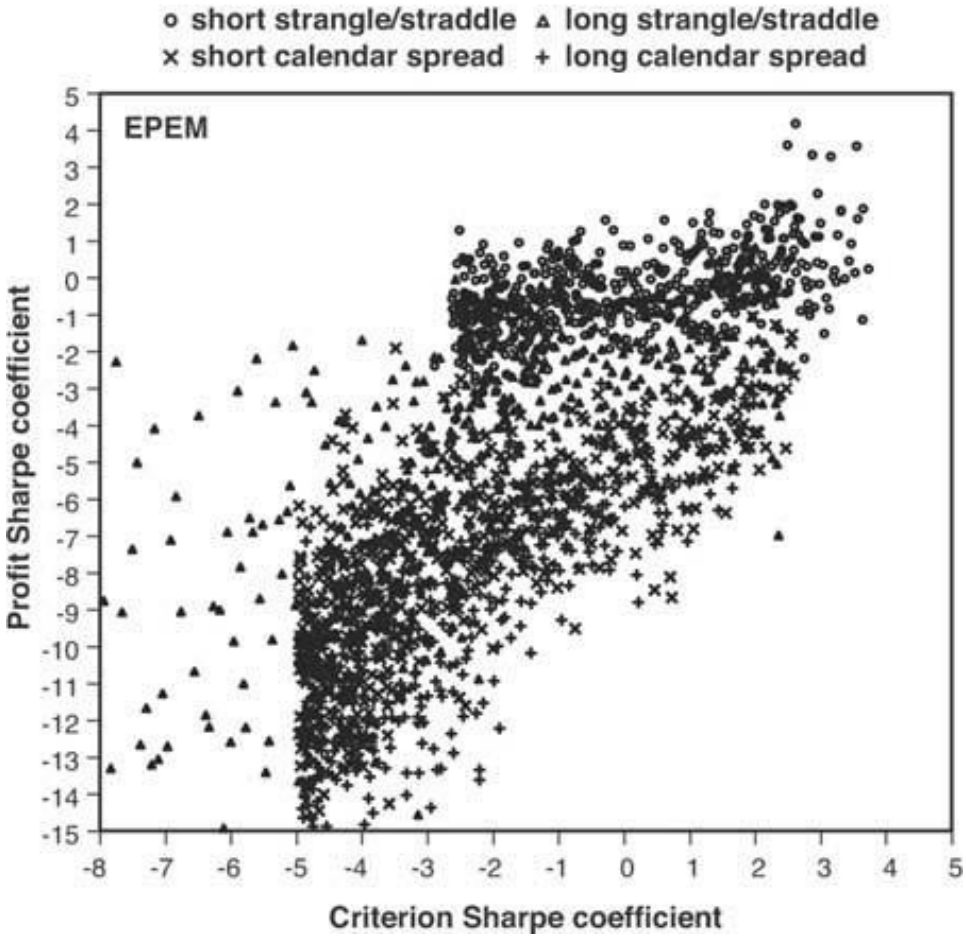
Regression analysis of the Sharpe ratios shows that EPLN is more effective than EPEM for the majority of option strategies (Figure 3.4.3). The only exception is the short calendar spread

strategy for which the first criterion is slightly less effective than the second one (R^2 = 0.50 and R^2 = 0.53 correspondingly). Unfortunately, this method is inapplicable for evaluation of the effectiveness of the other two criteria: PPLN and RPLL. Because values of these criteria correspond to probabilities and ratios, the profits should be expressed as proportions and ratios of profitable combinations within a certain group (see section 3.2.2). Therefore, for each group only one profit indicator is obtainable (in contrast to other criteria in which each of the 100 combinations in a group possessed its own profit value). Hence, we cannot estimate any intra-group variability

measure.

Figure 3.4.3. Regression analysis of the criterion and profit Sharpe ratios for four option strategies and two criteria.


o short strangle/straddle ▲ long strangle/straddle
× short calendar spread + long calendar spread
EPLN
Profit Sharpe coefficient
Criterion Sharpe coefficient


o short strangle/straddle ▲ long strangle/straddle
× short calendar spread + long calendar spread
EPEM
Profit Sharpe coefficient
Criterion Sharpe coefficient

For the long strangle/straddle strategy, the effectiveness of both criteria is extremely low (Figure 3.4.3; $R^2 = 0.13$ for EPLN, $R^2 = 0.01$ for EPEM). This is quite surprising knowing that the determination coefficients obtained using the other two methods were rather high (regression based on relationships between criterion and profit: $R^2 = 0.89$ for EPLN, $R^2 = 0.58$ for EPEM; regression based on criterion and profit index: $R^2 = 0.89$ for EPLN, $R^2 = 0.58$ for EPEM). Apparently, the strong correlation between the average values of the criterion and the profit coincides in this case with the high variability of these indicators hereby leading to the

low correlation of the Sharpe ratios.

The latter example illustrates the interesting possibility of opposite evaluations while estimating a certain criterion using different effectiveness indicators. The awareness about such inconsistency is crucial for comprehensive criteria quality analysis. We consider the effectiveness indicator based on the regression of the Sharpe ratios to be an essential instrument of criterion forecasting abilities evaluation. It contains important information that is not readily disclosed by other indicators.

3.4.4 Areas Ratio

Sorting combinations by criteria values, we assume that each subsequent combination in the ordering is inferior to the previous ones in terms of profit potential or other profit-related characteristics. The effectiveness indicator based on the areas ratio is intended to test this assumption. Unlike all criteria effectiveness evaluation techniques described in the previous sections, this method is not based on regression analysis. The peculiarities of its calculation make it applicable only to criteria forecasting the expected profit of combinations. Next we describe the procedure for calculating this effectiveness indicator using the example of the short strangle/straddle

strategy.

Imagine the relationship between the profit of the portfolio consisting of combinations situated at n top positions in the ordering and the number of combinations included in the portfolio (that is, n that ranges from 1 to 500). If the criterion can indeed identify the best combinations effectively, the ordering would consist of profitable combinations situated mostly at top positions and unprofitable ones positioned mainly at lower positions. In this case, the relationship would have a shape of an ascending convex curve that reaches its maximum at the point corresponding to the combination with a zero profit. Then the curve descends

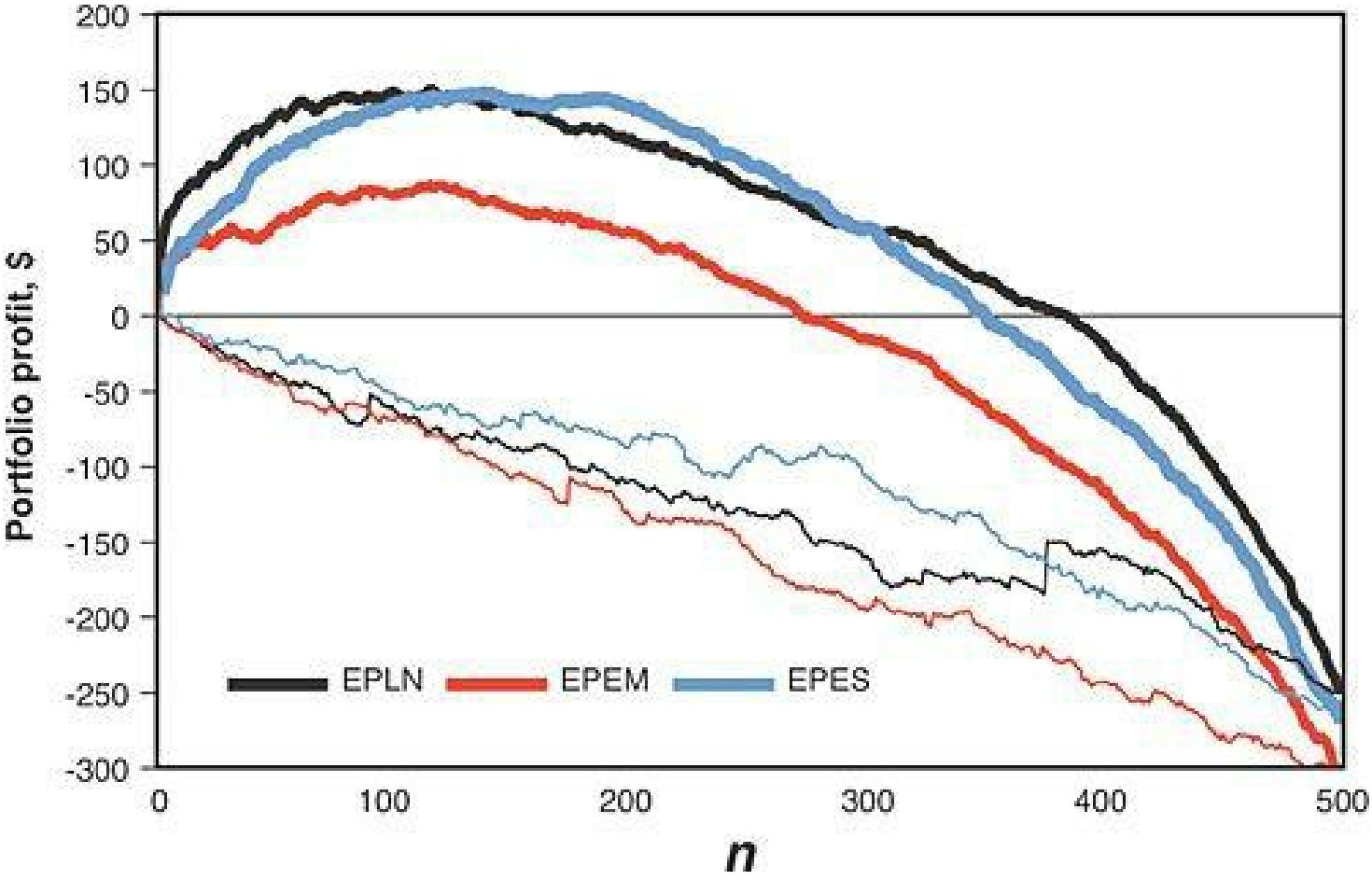
down to its minimum.

The bold curves in Figure 3.4.4 illustrate these relationships for three criteria forecasting expected profit on the basis of different distributions (EPLN, EPEM, and EPES). The shape of these curves corresponds to the theoretical scheme previously described. This suggests that the criteria indeed can sort combinations by their potential profit. An additional confirmation of the criteria effectiveness is the shape of the thin curves in Figure 3.4.4. They are obtained in the same manner except that the combinations were sorted not by the criteria (as it was done in the cases of the bold lines) but randomly. One can easily see that in the

case of sorting combinations by the criteria portfolio profit at first increases (because profitable combinations are situated mostly at the first positions in the ordering) and then decreases. On the other hand, random ordering resulted in a decrease of profits from the beginning (Figure 3.4.4).

Figure 3.4.4. Relationship between the portfolio profit and the number of combinations included in the portfolio (*n*). The bold curves correspond to

portfolios based on combinations sorted by criteria values. (Combinations were sorted by criteria, and top n ones were included in the portfolio.) Thin curves correspond to portfolios of combinations sorted randomly. Criteria abbreviations are the same as in Figure 3.2.4.



Now we proceed from the visual analysis of the data shown in Figure 3.4.4 to the formalization and numerical expression of the criteria's effectiveness indicator. An ideal ordering, which serves as the reference point, should be created for this purpose. Assume that the criterion under consideration is perfect.

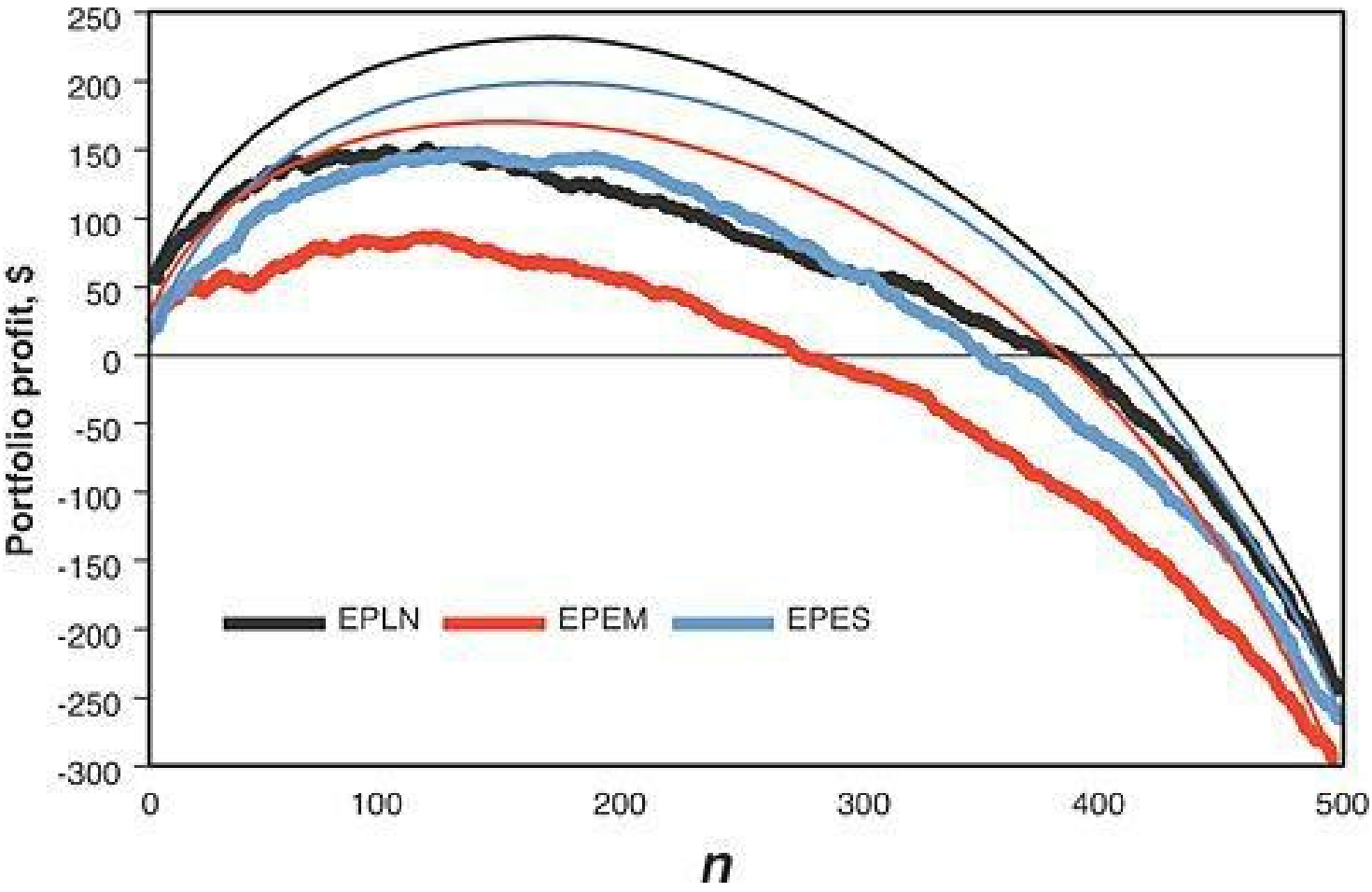
This means that sorting combinations by the criterion values completely coincides with sorting by realized profit. Our effectiveness indicator is based on measuring the degree of coincidence between the two rankings—ordering by the real criterion and ordering by an idealized criterion (that is, by profit).

Idealizations of EPLN, EPEM, and EPES criteria are shown in Figure 3.4.5 by thin curves (obtained by sorting combinations according to their profit values). Obviously, idealizations are better than real orderings obtained by sorting combinations by the criteria values (the bold curves). The degree of this superiority has to be expressed numerically to produce a new criterion

effectiveness indicator.

Figure 3.4.5. Relationship between the portfolio profit and the number of combinations included in the portfolio (*n*). Bold curves correspond to portfolios based on combinations sorted by criteria values. (Combinations were sorted by criteria and the top n ones were included in the

portfolio.) Thin curves correspond to portfolios created by idealized sortings (that is, by future profit). Criteria abbreviations are the same as in Figure 3.2.4.



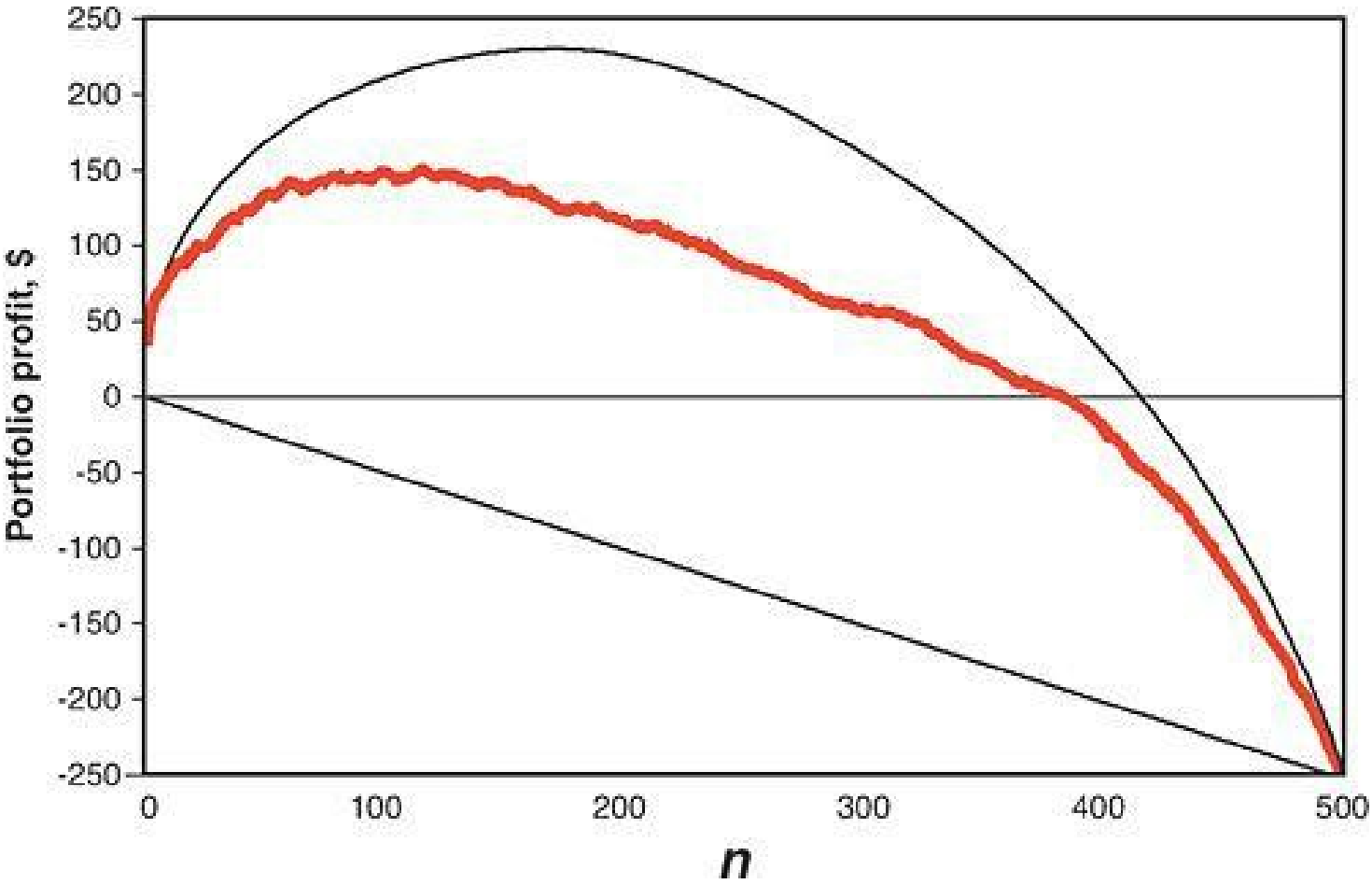
The basic idea of this effectiveness indicator can be easily appreciated by graphical representation. Consider again the relationship between portfolio profit and the number of combinations forming the portfolio (n) in two cases—sorting by values of the EPLN criterion and sorting by future profit (idealization). Let us connect the first and the last points of the two graphs with a straight line (Figure 3.4.6). The starting points of both graphs always coincide; they represent "empty" portfolios (without any combinations) with zero profits. (They are needed for standardization and comparability of the two orderings.) The last points of both graphs also coincide

and correspond to the portfolios consisting of all the combinations (500 in this case). Therefore, regardless of the sorting method, its profit is equal to the total profit of all combinations used in the analysis.

Figure 3.4.6. Graphical representation of two figures used for computing the effectiveness indicator based on the areas ratio. (This example is based on the EPLN criterion.) Both figures are bounded with the

straight line from below. From above each figure is bounded with a curve corresponding to the relationship between the portfolio profit and the number of combinations included in the portfolio (*n*). The bold curve represents the portfolios based on the sorting of combinations by criteria values. The thin curve represents the

portfolios based on the sorting of combinations by future profit.



Connecting the first and the last points with a straight line produces two plane figures with a common base (Figure

3.4.6). The first figure is bounded with a straight line from below and with an idealized curve representing sorting by profit from above. The second figure is bounded with the same straight line from below and with the curve representing sorting by the criterion from above. The area of the first figure is always larger or equal to the area of the second one. The closer the area of the second figure to the area of the first one, the higher the quality of the criterion. In the extreme case the two areas coincide, and this means that the criterion possesses maximum effectiveness because there were no mistakes in its ordering.

The ratio of areas of these two figures stands for numerical expression of the

criterion effectiveness indicator. It ranges from 0 to 1 and can be computed in the following way.

Consider an ordering of combinations sorted by descending profit. The position occupied by a combination in the ordering is denoted as k, $k = 1,...,N$. In our case $N = 500$. Let $p(k)$ be the profit value of combination k. In accordance with the ordering, the following inequality holds: $p(n) \leq p(m)$ for all n and m so that $n > m$. Evaluation of these combinations using a certain criterion results in their descending ordering by its values. We denote the position of a combination situated at the

$$P(n) = \sum_{k=1}^{n} p(k)$$

k-th position as i_k. Sums

$$P^i(n) = \sum_{k=1}^{n} p(i_k)$$

and are profits of two portfolios consisting of n first combinations: $P(n)$ stands for the sorting by profit and $P^i(n)$ stands for the sorting by criterion values. One can easily see that for any n from 1 to N holds $P(n) \geq P^i(n)$, which explains the fact that for any n the curve for the profit of the portfolio corresponding to sorting by the criterion is lower than the idealized curve. We denote the total profit of the portfolio consisting of N combinations as $P^N = P(N) = P^i(N)$. Then the criterion effectiveness indicator representing the

ratio of areas of the two figures described is

$$I = \frac{\sum_{n=1}^{N} \left(P^i(n) - \frac{n}{N} P^N\right)}{\sum_{n=1}^{N} \left(P(n) - \frac{n}{N} P^N\right)},$$

or after a simple transformation:

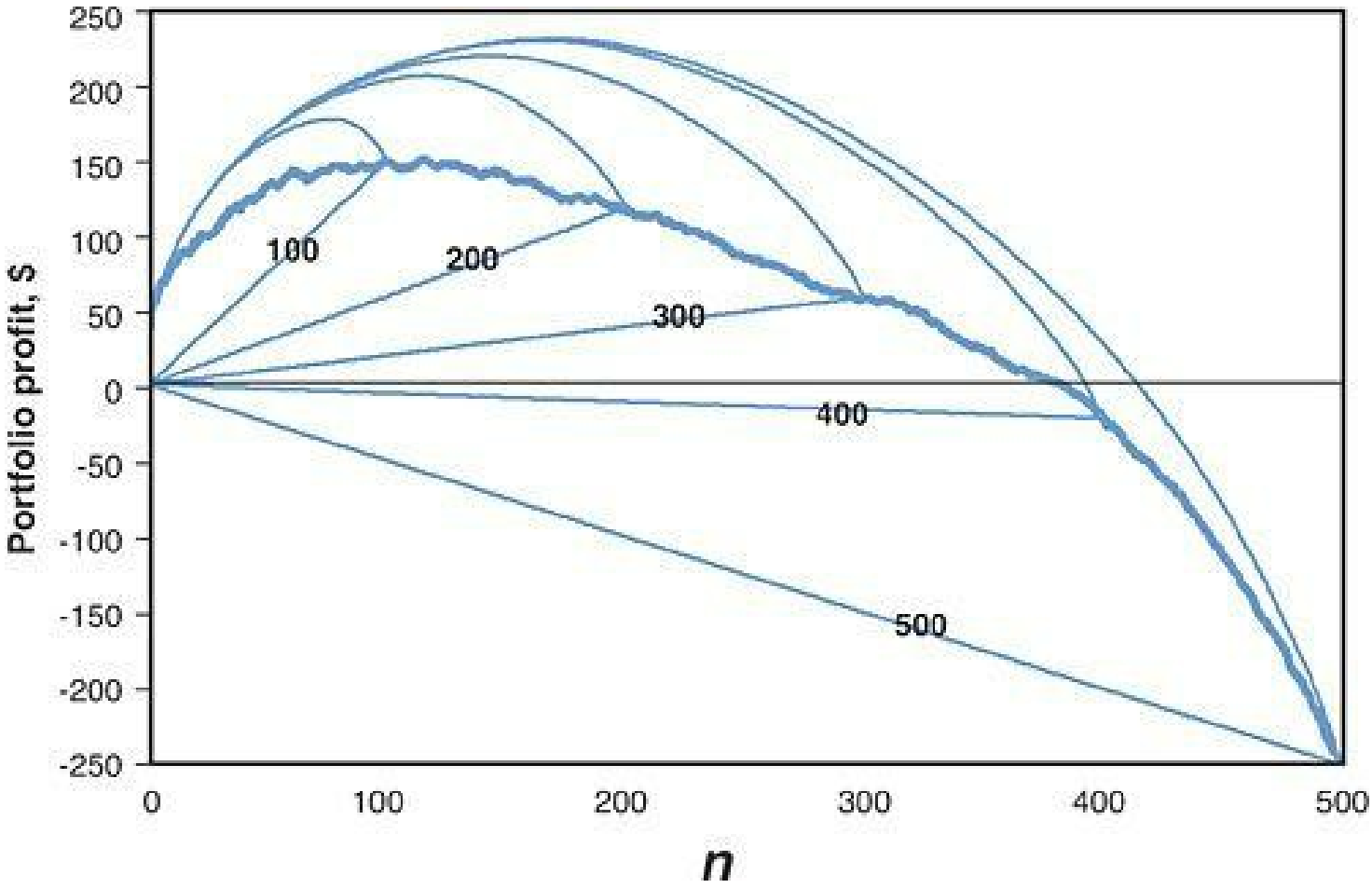
(3.4.1)

$$I = \frac{\sum_{n=1}^{N}\sum_{k=1}^{n} p(i_k) - \frac{(N+1)}{2}P^N}{\sum_{n=1}^{N}\sum_{k=1}^{n} p(k) - \frac{(N+1)}{2}P^N}$$

The indicator of criterion effectiveness computed for the data presented in Figure 3.4.6 using formula 3.4.1 equals *I* = 0.72. Note that the areas ratio is quite sensitive to the total number of combinations used in the analysis. (The similar effect was described in section 3.3.1 for another effectiveness indicator.) The degree of this sensitivity can be estimated by constructing figures based on different numbers of combinations.

Figure 3.4.7 shows pairs of figures corresponding to 100, 200, 300, 400 and 500 combinations used in the analysis of the EPLN criterion effectiveness. As the number of combinations decreases, the ratio of areas changes. It is $I = 0.59$ for 400 and 300 combinations, $I = 0.63$ for 200 combinations and $I = 0.65$ for 100 combinations. Although these changes are not very significant, they can become rather substantial in other circumstances (over other time periods, for other criteria and option strategies). Our statistical investigations and results of real trading point to about 200—300 as the optimal number of combinations to be used in the criterion effectiveness analysis based on the areas ratio.

Figure 3.4.7. Graphical representation of figures used for calculating the effectiveness indicator based on the areas ratio for the EPLN criterion. Each thin curve coupled with a thin line corresponds to a pair of figures with a specific number of combinations used in the analysis.



3.4.5 Other Effectiveness Indicators

The described indicators do not constitute a complete list of evaluation techniques intended for assessment of criteria effectiveness. Analytical

methods based on similar or fundamentally different ideas are constantly developed. Different versions of effectiveness indicators can be produced on the basis of the same principles described in sections 3.4.1–3.4.4. Such indicators, evaluating the forecasting qualities of criteria in a slightly different way, are useful supplemental tools that are indispensable for a comprehensive comparative analysis of many criteria.

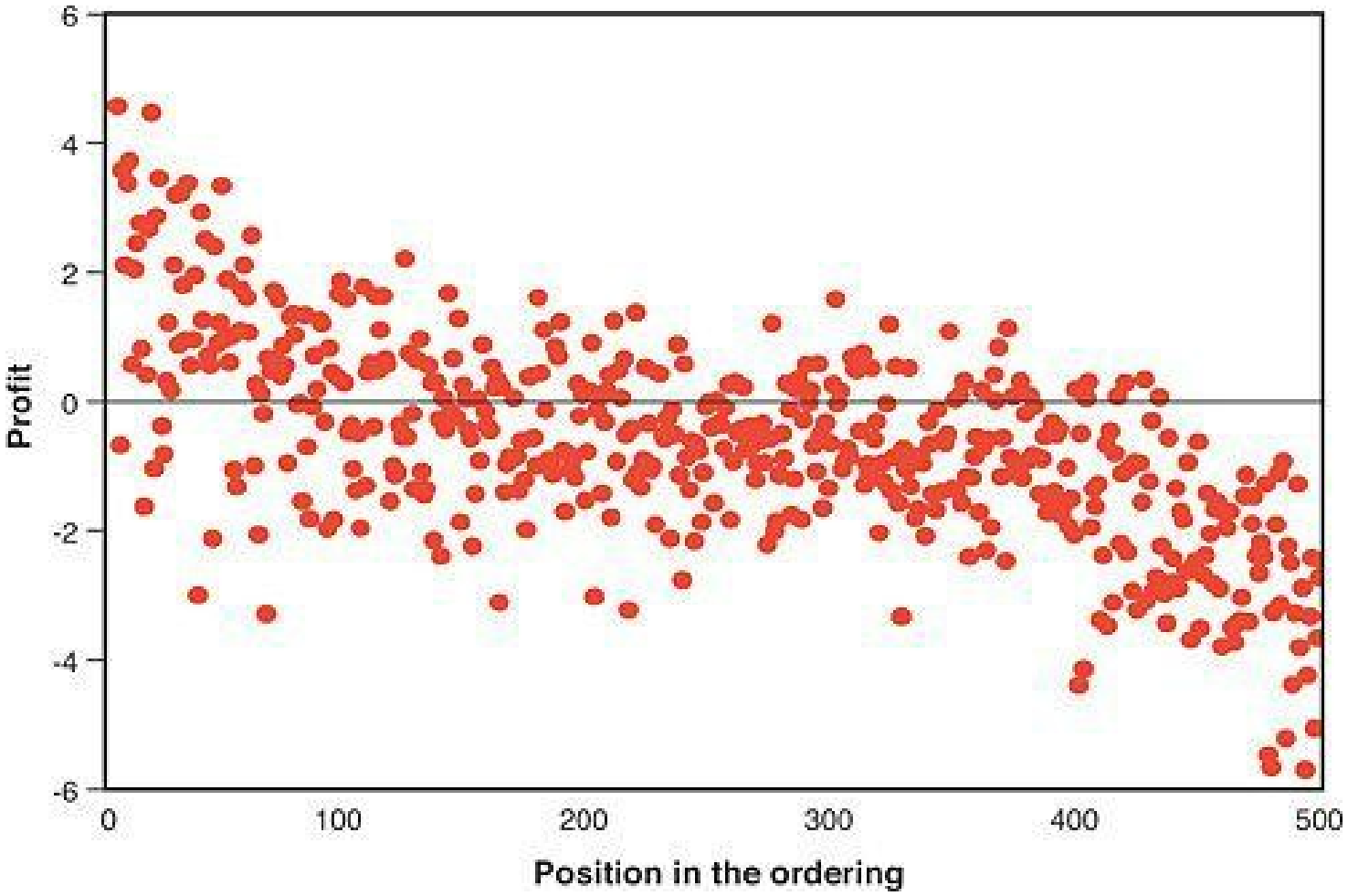
Below we present several examples illustrating such additional versions of criteria effectiveness indicators. (All examples are based on the EPLN criterion and the short strangle/straddle strategy.)

The relationship between the profit and the position of a combination in the ordering. This evaluation method represents a mixture between index regression (section 3.4.2) and {criterion × profit} regression (section 3.4.1). The difference is that here we estimate the relationship between profit expressed in U.S. dollars and the criterion value presented as an index. What is the purpose of such *hybridization*? It turned out that presenting data in this way may reveal the nonlinear nature of a *profit-criterion* relationship (Figure 3.4.8). The increase of the combination's position in the ordering from 1 to 100 leads to a sharp fall in profit. Further promotion from the 100th up to 400th

position does not have much impact on the returns but further increase of the position from the 400th to 500th position again entails an abrupt profit decline. This nonlinearity (which is hardly detectable by means of the other effectiveness indicators) is extremely important for the clear perception of differentiated criteria effectiveness within its different value ranges.

Figure 3.4.8. Relationship between profit of combinations and their position in the ordering. EPLN criterion; short

strangle/straddle strategy.

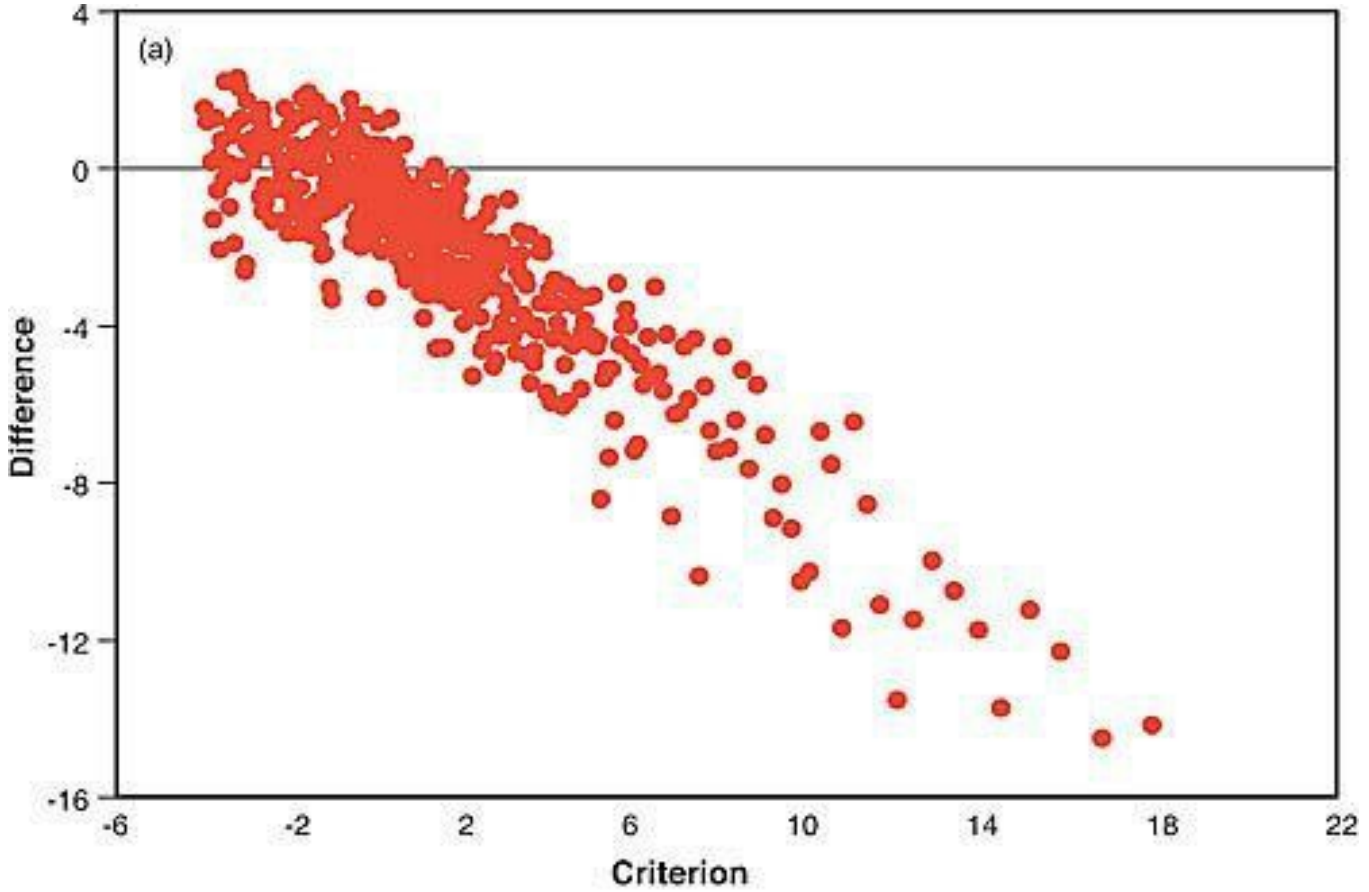


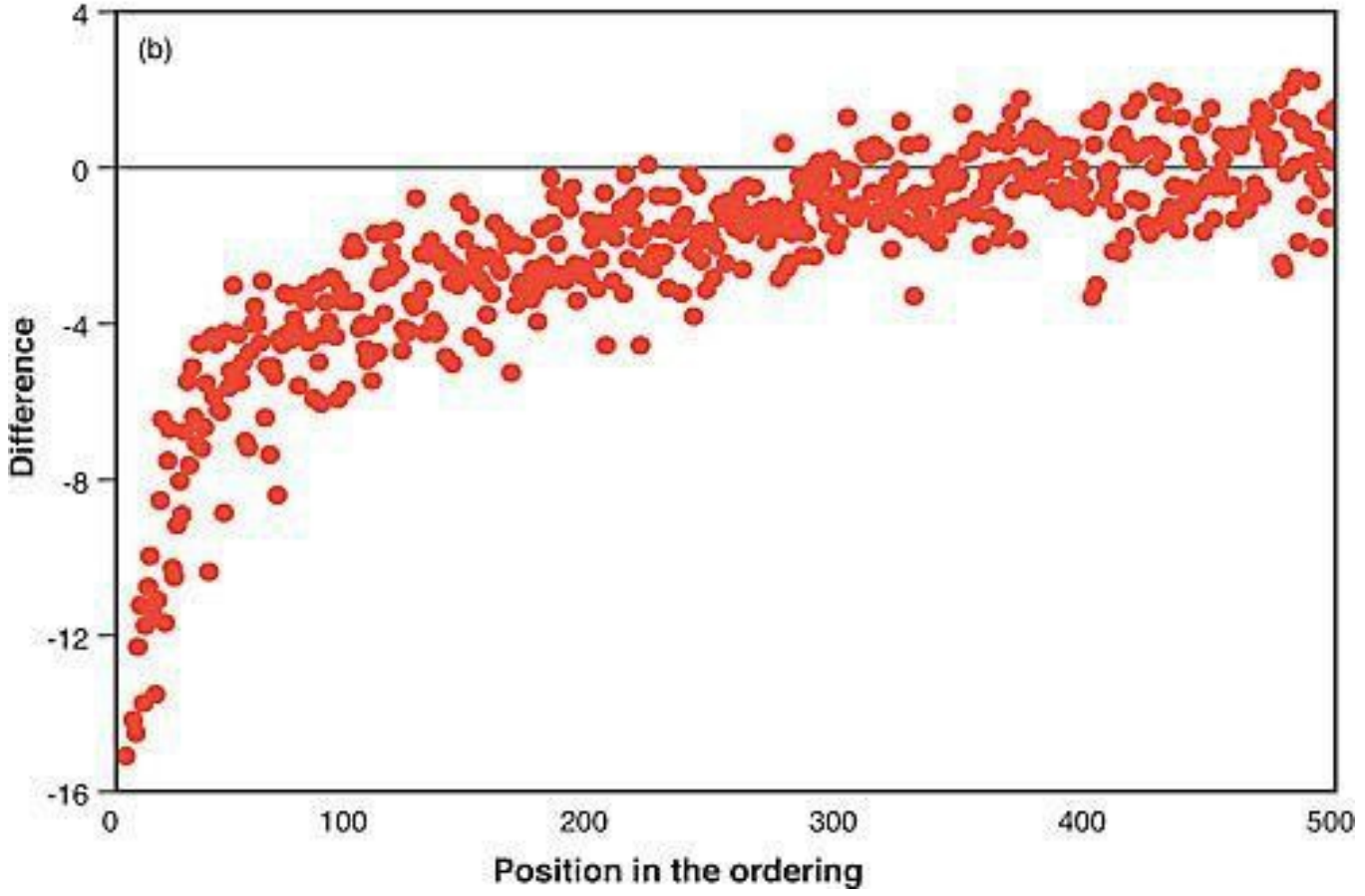
Deviations of profits forecasted by the criterion from realized values. Criteria based on mathematical expectations forecast future profits in the most direct way. Therefore, it would be useful to analyze deviations of forecasted values

from their realizations and possible relationship between these deviations and forecasted values themselves. Figure 3.4.9(a) shows that in most cases forecast values are overestimated. However, it is even more surprising that there is a clear negative relationship between deviations and criterion values. The higher the criterion values, the more overvalued are the forecasts. On the contrary, many negative forecasts turned out to be undervalued because combinations with negative forecast values actually performed better than expected.

Figure 3.4.9. Difference

between profit and criterion values and its relationship with (a) the criterion value and (b) the position of combinations in the ordering. EPLN criterion; short strangle/straddle strategy.


(a)
4
0
-4
-8
-12
-16
Difference
-6
-2
2
6
10
14
18
22
Criterion


(b)
4
0
-4
-8
-12
-16
Difference
0
100
200
300
400
500
Position in the ordering

The relationship between deviations and the positions of combinations in the ordering is shown in Figure 3.4.9(b). Significant overvaluing observed for combinations situated at the first positions in the ordering sharply diminishes as the position number increases. However, this relationship is nonlinear and, therefore, the rate of the overvaluing disappearance slows down after the 100th position. Although after the 200th position a few overvalued forecasts are still observed, after the 300th position profits of most combinations are on average forecasted correctly.

Certainly, deviations of profit from the

forecast values are inevitable. The average value and dispersion of these deviations should be analyzed by statistical methods and must be taken into account in practical application of criteria. It is particularly important not to lose sight of nonlinearity of relationship between deviation and position of combinations in the ordering. As shown in Figure 3.4.9, combinations with the highest criterion values are most likely to be overestimated. Notwithstanding their ultimate profitability, the realized gain value is frequently lower than it was predicted. Therefore, when estimating future return of an option portfolio, we have to consider this feature and try to offset it

by introducing coefficients correcting the absolute profit values forecasted by criteria.

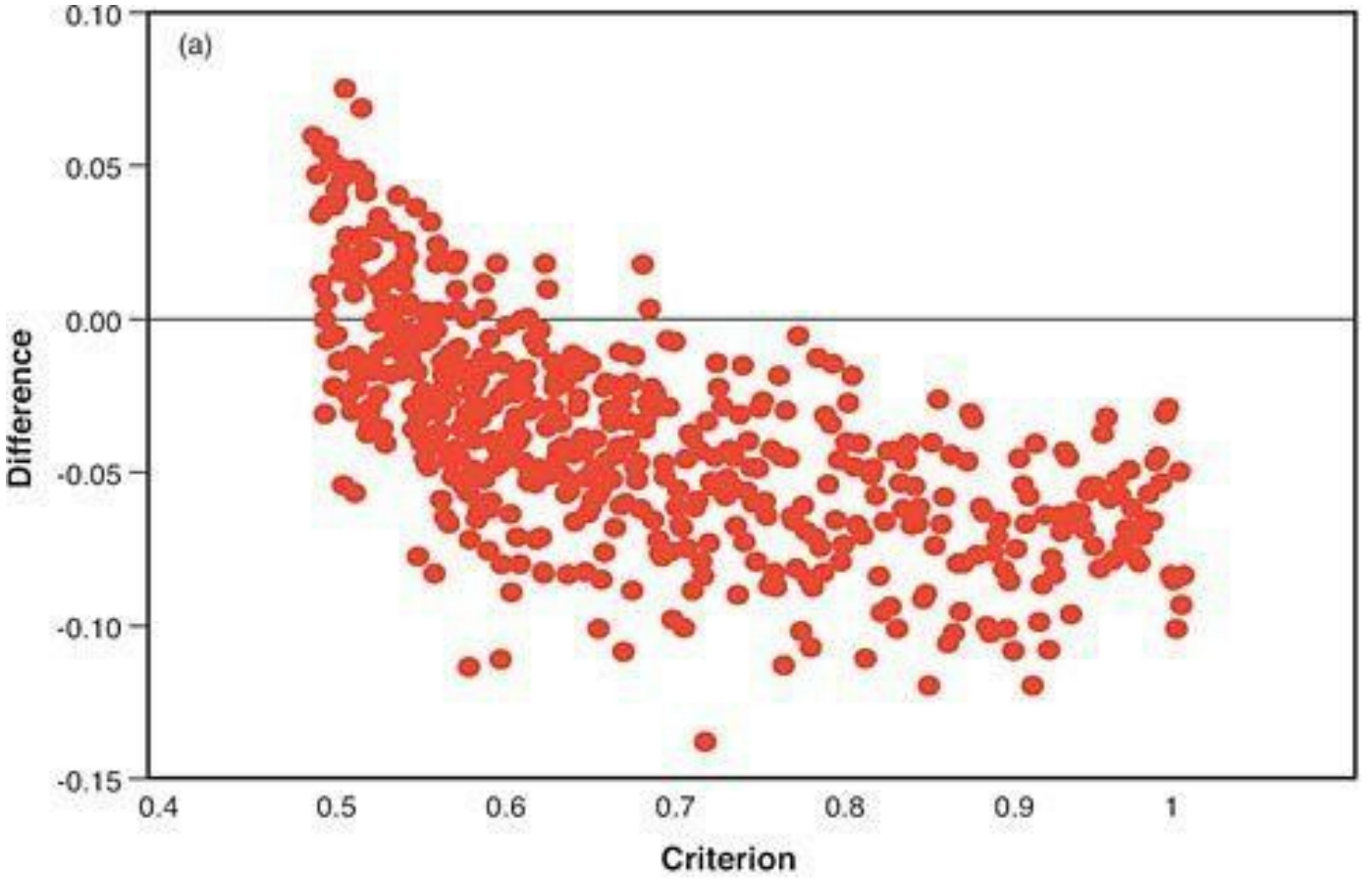
When developing correction coefficients and their application methods, you should take into account specific forms of the relationship between deviations and position of combinations in the ordering. In some cases the relationship may be hardly detectable or even absent; in other cases it may have a different shape not similar to that shown in Figure 3.4.9. The following example is intended to illustrate this. Sticking to the same strategy (short strangle/straddle) we replace the criterion—instead of the criterion forecasting profit value (EPLN) we apply another criterion

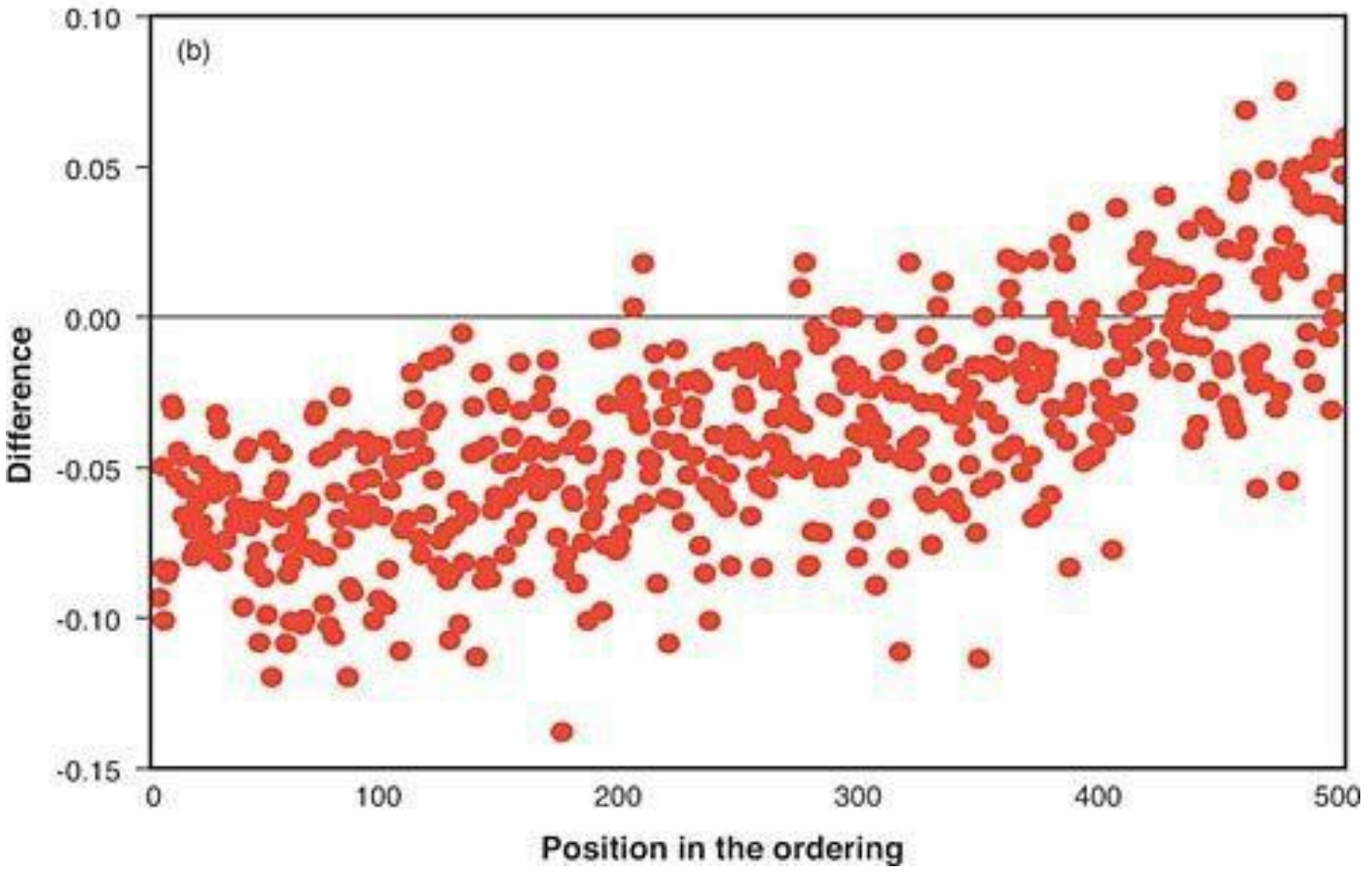
forecasting profit probability (PPLN). Such substitution leads to a different form of relationship (Figure 3.4.10). First, the relationship between the difference and the criterion value changed its shape from linear (Figure 3.4.9(a)) to nonlinear (Figure 3.4.10(a)). Secondly, the relationship between the differences and the position of combinations in the ordering—albeit remained nonlinear—changed its shape from convex (Figure 3.4.9(b)) to concave (Figure 3.4.10(b)). Finally, the degree of these two relationships is also different. There is no need for computing correlation coefficients for nonlinear regressions to see that data dispersion is much higher for the PPLN than for the

EPLN criterion. (Compare Figures 3.4.9 and 3.4.10.) The relationships between deviations and values of other criteria can be even more specific, especially if these criteria are used to evaluate combinations within other option strategies. All peculiarities of such relationships must be taken into account while developing and applying correction coefficients.

Figure 3.4.10. Difference between profit and criterion values and its relationship with (a) the criterion value and (b) the position of the

combination in the ordering. PPLN criterion; short strangle/straddle strategy.


(a)
0.10
0.05
0.00
-0.05
-0.10
-0.15
Difference
0.4
0.5
0.6
0.7
0.8
0.9
1
Criterion


(b)
0.10
0.05
0.00
-0.05
-0.10
-0.15
Difference
0
100
200
300
400
500
Position in the ordering

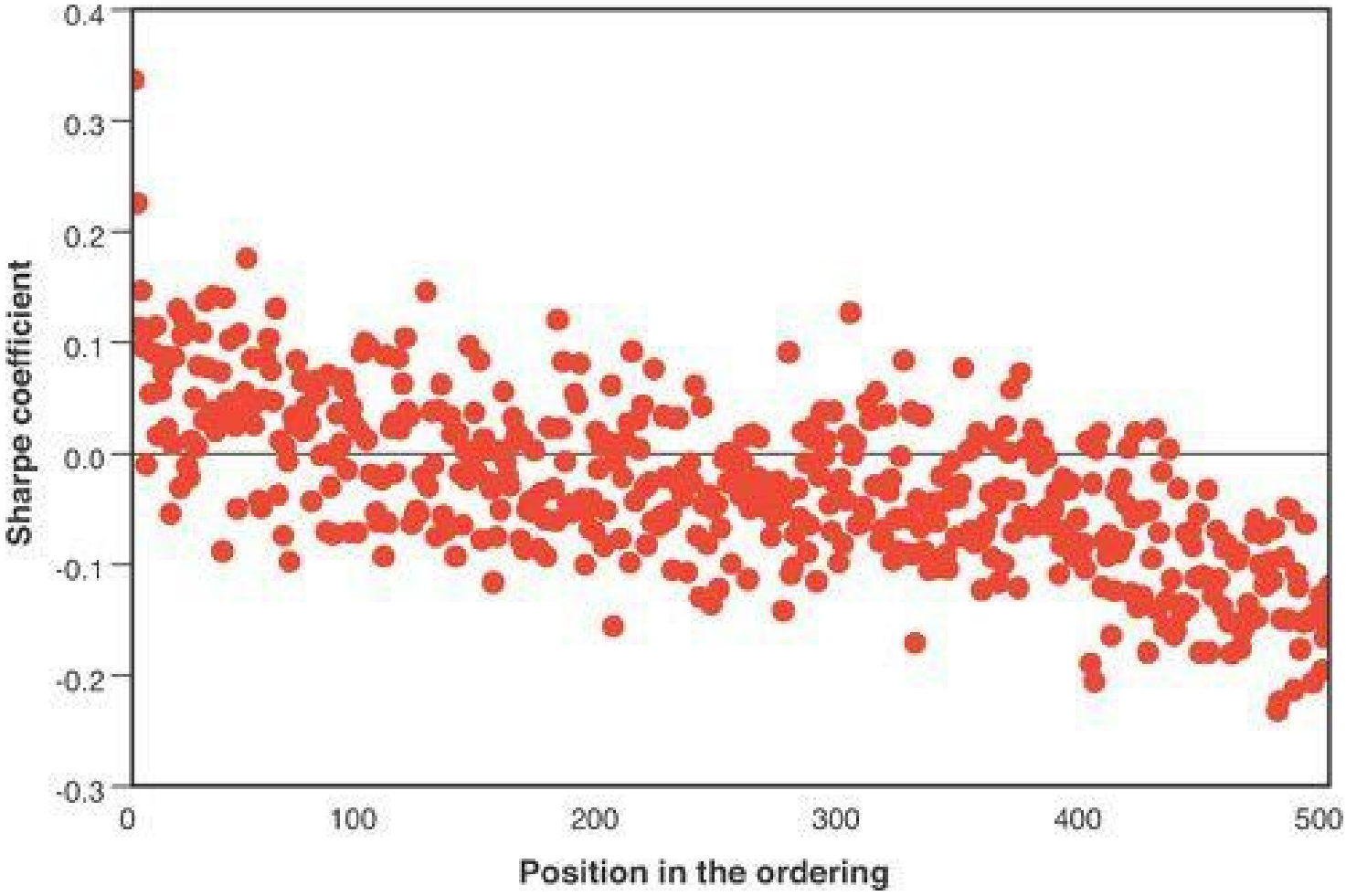
Analysis of the Sharpe ratios. In section 3.4.3 we already used the Sharpe ratios to calculate the effectiveness indicator using the regression model. (This was a regression reflecting the relationship between the Sharpe ratios based on profit values and the Sharpe ratios based on criterion values.) This kind of analysis can easily be expanded. The Sharpe ratio is an extremely important indicator containing information not only on the profitability of combinations but on their variability as well. Thus, it should be applied extensively to obtain a versatile and comprehensive evaluation of criteria forecasting abilities.

The following example demonstrates

extraction of additional information about criteria effectiveness by application of the Sharpe ratio analysis. Figure 3.4.11 illustrates the relationship between the profit Sharpe ratio and the position of the combination in the ordering by criterion values. As we have already noticed in the earlier examples, relationships of this kind are often nonlinear. At first, the increase in the position number leads to a sharp fall in the Sharpe ratio. After the 100th position the decrease of the coefficient slows down and resumes its pace only after the 400th position. The nonlinearity of such relationships is extremely important for estimating the heterogeneity of criteria effectiveness at its different value

ranges.

Figure 3.4.11. Relationship between the Sharpe ratio and the position of the combination in the ordering.



3.5 Summary

The criterion effectiveness analysis is a complex procedure requiring consideration of the exhaustive number

of factors having both direct and indirect influence on the evaluation process.

First, the analysis is complicated by the time-to-time variability of all effectiveness indicators regardless of the option strategies and criteria. In this chapter we illustrated the dynamics of only one effectiveness indicator (to avoid overloading the text with monotonous figures). Nevertheless, our exhaustive research suggests that all effectiveness indicators without exception are to a greater or lesser extent variable in time. We proposed the way to master this problem and described the methods allowing mitigation of the effect of indicators' variability. However, there is no

universal solution suitable for all circumstances. Hence, each situation requires individual and comprehensive analysis.

Furthermore, the evaluation results depend on the averaging procedure necessary to increase the reliability of criteria effectiveness estimation. (This procedure requires optimization of the "averaging period" parameter.) Besides, there are many additional factors each of which should be thoroughly considered and parameterized. We discussed only one of them (the number of combinations involved in the analysis) because its influence is inevitable for almost all criteria, option strategies, and effectiveness indicators. However, the

influence of other specific factors is not so evident and does not always become apparent.

Another difficulty arises from the necessity to express profit in the same form and in the same units as forecast by the criterion. Unfortunately, it is not always possible, and one has to resort to various tricks to bring these values together.

Our research suggests that to ensure the successful evaluation of criterion effectiveness it is not enough to select a correct analytical procedure. Because one procedure can generate several numerical indicators (for example, the determination coefficient and the slope

coefficient in the regression analysis), we must select those that reflect the criterion characteristics better.

As to the number of effectiveness indicators, we can develop as many of them as our imagination and computer capacities allow us to. We described only basic methods of criteria effectiveness evaluation choosing the most universal, understandable, and easily interpreted ones. Undoubtedly, a creative approach allows developing a lot of additional indicators based on various evaluation principles. This clears the way for a most comprehensive criteria effectiveness analysis.

Applying the techniques described in

this chapter in practice requires (1) development and adaptation of transformation procedures, (2) optimization of parameters, and (3) adjustment of evaluation algorithms to the current market conditions and to the option strategies used by an investor.

Part II. The Main Areas of Criteria Application

While describing the concept of systematic approach, we define the main principles for selection of superior option combinations from a large initial set. The sequence of procedures involved in the selection process is presented as a reduction of a three-dimensional matrix of the initial set, realized in the series of consecutive

operations.

The first operation is the selection of the best option combinations for each conjunction of underlying assets and strategies. *The second operation* corresponds to the selection of the best strategies for each underlying. Finally, *the third operation* represents a choice of the optimal number of underlyings for each option strategy.

All the three operations are based on evaluation and analyses of option combinations by means of criteria. Each operation represents an important task relating to a separate area of criteria application and possessing various specific features. It would not be an

exaggeration to state that successful fulfillment of these tasks represents the main component of the investment process and, to a large extent, determines the effectiveness of option trading. Owing to the interrelationships between these operations, a failure to perform one of them reduces the effectiveness of the others or even makes their execution absolutely impossible. All three operations correspond to the problems of selection of option combinations, strategies, and underlying assets.

Basically, all three problems can be solved in many different ways. For instance, the selection of option combinations can be performed by

choosing the desired position for a payoff function relative to the current underlying price. The selection of option strategies can be reduced to visual analyses of payoff functions and choosing those, which comply better with the investor's strategic objectives, his risk tolerance, and market forecasts. And finally, selection of underlying assets can be based on the investor's personal preferences and his inclination to deal with specific underlying assets. The application of many other methods is also possible. However, all of them represent merely subjective solutions that are to a large extent based on nonformalized and, consequently, quite ambiguous decision-making algorithms.

Within the framework of the systematic approach, the only reliable and, in many cases, universal instrument for solving selection problems is the application of strictly formalized criteria.

In the following chapters we describe various methods of criteria application for accomplishing their three main tasks. In practice, these tasks can be executed either simultaneously or successively (by consecutive reduction of the three-dimensional matrix of the initial set). For the clarity of presentation and to avoid interactive and cross effects (when the success in fulfillment of one operation depends on the fulfillment of the others), we study each task separately assuming that the solution of

the others is strictly predetermined. Under such an assumption, the three main problems solved by criteria can be formulated as follows:

1. Selection of option combinations for trading within the certain strategies using *a priori* selected underlying assets
2. Selection of option strategies for trading using *a priori* selected underlying assets and predetermined algorithms for combinations creation
3. Selection of underlying assets for trading within certain option strategies using predetermined algorithms for combinations

creation

A separate chapter is devoted to each of the previously listed problems. Different methods of criteria application are investigated in detail. The main techniques intended to analyze their effectiveness are developed as well. Optimization models that enable maximizing the effect of criteria application are constructed and discussed in Chapter 5, "Selection of Option Strategies," and Chapter 6, "Selection of Underlying Assets."

chapter 4. Selection of Option Combinations

4.1 Introduction

Consider a situation in which underlying assets intended for future trading are predefined. Option strategies have also been preselected and assigned to each underlying asset. Hence, there is only one problem to solve: How to select the best option combination for each

{underlying × strategy} pair.

It was shown in Chapter 1, "General Presentation and Review of Criteria Properties," that even when the conservative algorithm for creation of option combinations is applied, the best variants have to be selected out of quite a large set (21 variants in the case of AAPL and 15 in the case of YHOO). Obviously, the higher the stock price is, the more option strikes are traded, and the larger the number of combinations that can be constructed. For example, for a stock priced around $100, the best combination has to be selected among nearly 50 alternatives. (If the same limitations as in Chapter 1 are applied; otherwise, the number of alternative

variants will increase even further.) Undoubtedly, all attempts to make the selection by visual comparison of such a large number of payoff functions (or using any other heuristic methods) are doomed to fail. The problem of combination selection can be solved only by applying strictly formalized criteria.

4.2 Analysis of Criteria Effectiveness in the Selection of Option Combinations

The procedure of criteria effectiveness evaluation is illustrated using the "expected profit on the basis of lognormal distribution" criterion. (Its meaning and calculation algorithm were described in Chapter 2, "Review of the Main Criteria.") Suppose that two similar option strategies are selected: short strangle and short straddle. The following restrictions are applied to combination creation: Only contracts with the nearest expiration date are allowed; strikes must be distant from the current stock price by no more than 20%; the strike of the Put must be equal to the Call strike (straddle) or be lower (strangle); and the Call to Put ratio within one combination must be equal to

1:1. All combinations that meet these conditions belong to the set from which we have to select the best variants. Even though the above restrictions considerably reduce the set, the number of combinations to be evaluated in the course of the selection process still remains substantial.

To demonstrate the selection process, we use a stock priced over $100, MA. Its close price on February 1, 2007 was $111.92. Applying the preceding listed restrictions, we obtain 45 variants (36 strangles and 9 straddles) constructed using 9 strikes. The structure of all the 45 combinations, their premiums, and margin requirements are shown in Table 4.2.1. (Each combination consists of one

Call contract and one Put contract.) The values of the criterion and realized profits are shown in U.S. dollars and as a percentage of margin requirements.

Table 4.2.1. Values of Criterion and Realized Profits for 45 Option Combinations. Strategies—Short Strangles and Straddles; Underlying Stock—MA; Date of Positions Opening—February 1, 2007; Options Expiration Date—

February 16, 2007.

Combination				Premium	Margin	Criterion. $	Profit. $	Criterion. %	Profit. %
90	Call	90	Put	22.25	44.48	0.33	3.93	0.7	8.8
95	Call	90	Put	17.55	39.78	0.60	4.23	1.5	10.6
95	Call	95	Put	17.80	39.78	0.83	4.48	2.1	11.3
100	Call	90	Put	13.25	35.48	1.15	4.93	3.2	13.9
100	Call	95	Put	13.50	35.48	1.37	5.18	3.9	14.6
100	Call	100	Put	14.05	35.48	1.76	5.73	5.0	16.1
105	Call	90	Put	9.15	31.38	1.45	5.83	4.6	18.6
105	Call	95	Put	9.40	31.38	1.67	6.08	5.3	19.4
105	Call	100	Put	9.95	31.38	2.07	6.63	6.6	21.1
105	Call	105	Put	11.00	31.38	2.52	7.68	8.0	24.5
110	Call	90	Put	5.95	28.18	1.74	5.95	6.2	21.1
110	Call	95	Put	6.20	28.18	1.97	6.20	7.0	22.0
110	Call	100	Put	6.75	28.18	2.36	6.75	8.4	23.9
110	Call	105	Put	7.80	28.18	2.81	7.80	10.0	27.7
110	Call	110	Put	9.50	28.18	3.01	7.82	10.7	27.7
115	Call	90	Put	3.55	22.70	1.63	3.55	7.2	15.6
115	Call	95	Put	3.80	22.70	1.86	3.80	8.2	16.7

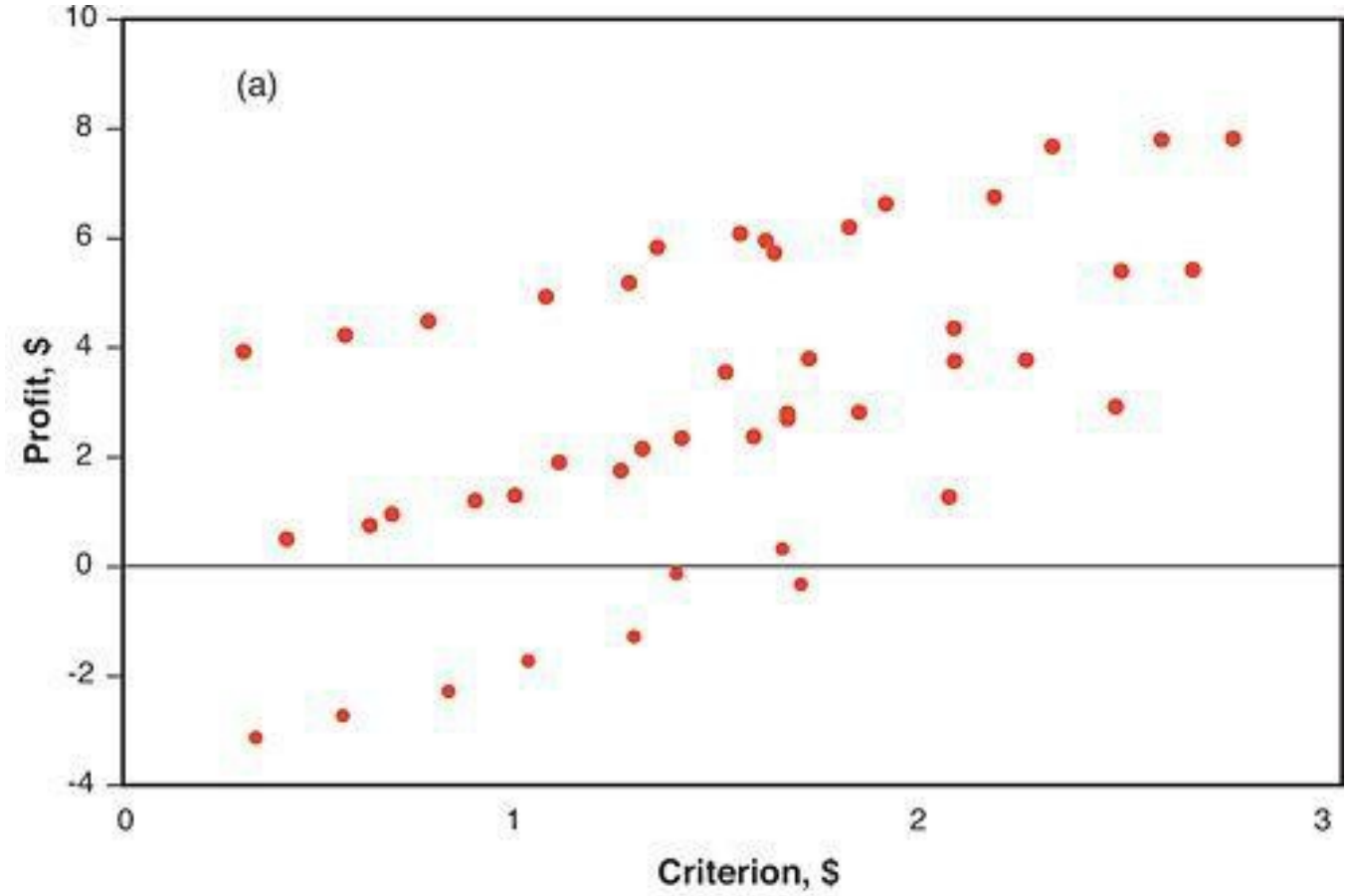
115	Call	100	Put	4.35	22.70	2.25	4.35	9.9	19.2
115	Call	105	Put	5.40	22.70	2.70	5.40	11.9	23.8
115	Call	110	Put	7.10	24.16	2.90	5.42	12.0	22.4
115	Call	115	Put	9.60	28.58	2.69	2.92	9.4	10.2
120	Call	90	Put	1.90	16.05	1.18	1.90	7.4	11.8
120	Call	95	Put	2.15	16.05	1.41	2.15	8.8	13.4
120	Call	100	Put	2.70	16.05	1.80	2.70	11.2	16.8
120	Call	105	Put	3.75	17.46	2.25	3.75	12.9	21.5
120	Call	110	Put	5.45	24.16	2.45	3.77	10.1	15.6
120	Call	115	Put	7.95	28.58	2.24	1.27	7.8	4.4
120	Call	120	Put	11.35	31.98	1.84	–0.33	5.7	–1.0
125	Call	90	Put	0.95	11.99	0.73	0.95	6.1	7.9
125	Call	95	Put	1.20	11.99	0.95	1.20	8.0	10.0
125	Call	100	Put	1.75	11.99	1.35	1.75	11.2	14.6
125	Call	105	Put	2.80	17.46	1.80	2.80	10.3	16.0
125	Call	110	Put	4.50	24.16	1.99	2.82	8.3	11.7
125	Call	115	Put	7.00	28.58	1.79	0.32	6.2	1.1
125	Call	120	Put	10.40	31.98	1.38	–1.28	4.3	–4.0
125	Call	125	Put	14.40	35.98	0.88	–2.28	2.5	–6.3
130	Call	90	Put	0.50	11.54	0.44	0.50	3.8	4.3
130	Call	95	Put	0.75	11.54	0.67	0.75	5.8	6.5
130	Call	100	Put	1.30	11.54	1.06	1.30	9.2	11.3
130	Call	105	Put	2.35	17.46	1.51	2.35	8.7	13.5
130	Call	110	Put	4.05	24.16	1.71	2.37	7.1	9.8
130	Call	115	Put	6.55	28.58	1.50	–0.13	5.2	–0.5
130	Call	120	Put	9.95	31.98	1.10	–1.73	3.4	–5.4
130	Call	125	Put	13.95	35.98	0.60	–2.73	1.7	–7.6
130	Call	130	Put	18.55	40.58	0.36	–3.13	0.9	–7.7

The data presented in the table indicate that the combination with the highest value of the criterion (expressed in U.S. dollars) has the highest value of realized profit (straddle with strike 110). It means that in this particular case the criterion identified the most profitable out of the 45 combinations. Note that the same criterion expressed as a percentage has not succeeded in spotting the best combination, but pointed at strangle "120 Call / 105 Put," that turned out to rank at the 8th position according to the value of realized profit. However, in other cases the situation might be different; expression of the criterion as a percentage may lead to more precise selection.

Next the predictive power of the criterion is evaluated using the most direct and intuitively understandable indicator of criteria effectiveness: the correlation between the realized profit and the criterion value. In Figure 4.2.1(a) we can see that the higher the values of the criterion, the higher the profits. Each point on the figure corresponds to one of the 45 combinations, the regression is statistically significant ($p < 0.0001$), and the determination coefficient is relatively high ($R^2 = 0.36$).

Figure 4.2.1. Relationship

between realized profit and criterion value corresponding to short strangles and straddles constructed for underlying stock MA. Criterion and profit values are expressed in U.S. dollars (a) and in percentage (b).


(a)
Profit, $
Criterion, $
10
8
6
4
2
0
-2
-4
0
1
2
3

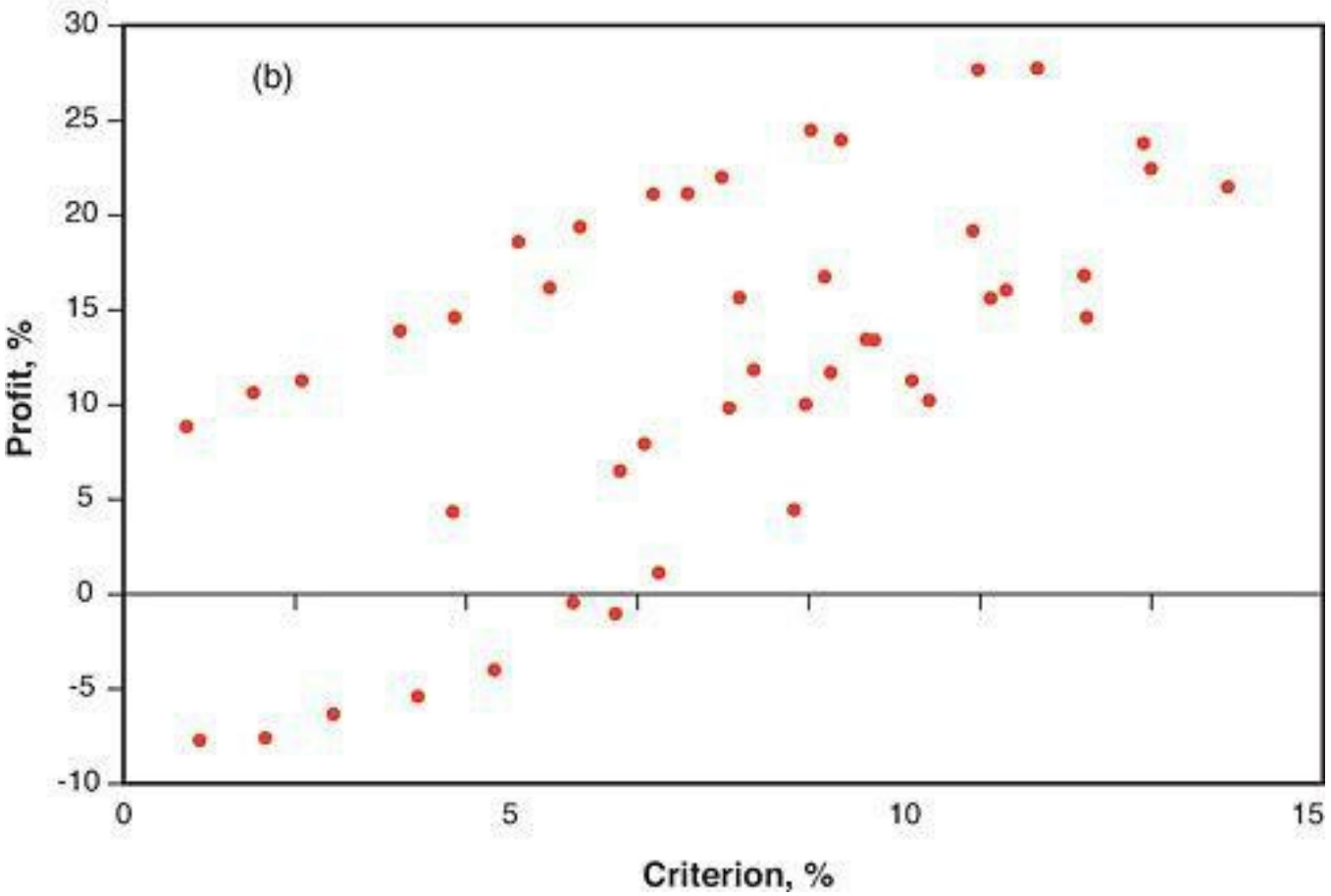

(b)
Profit, %
Criterion, %
30
25
20
15
10
5
0
-5
-10
0
5
10
15

Figure 4.2.1(b) shows the same relationship, but the criterion and profit values are expressed as a percentage of the margin requirements. This regression is also statistically significant ($p < 0.00001$); the determination coefficient is high (R^2=0.40). Comparison of two figures reveals that if the criterion is expressed in U.S. dollars, its highest value coincides with the highest profit value (Figure 4.2.1[a]). In the second case, when the criterion values are expressed as a percentage (Figure 4.2.1 [b]), the combination with the highest criterion value appears to be slightly inferior relative to a few other combinations. However, even not being

the best, this combination still showed quite a decent profit.

4.3 Factors That Affect Option Combinations Selection

4.3.1 Absolute Values of the Criterion

The results presented in the previous section raise an important question: Is the combinations selection process always accompanied by such strong correlation between the criterion and

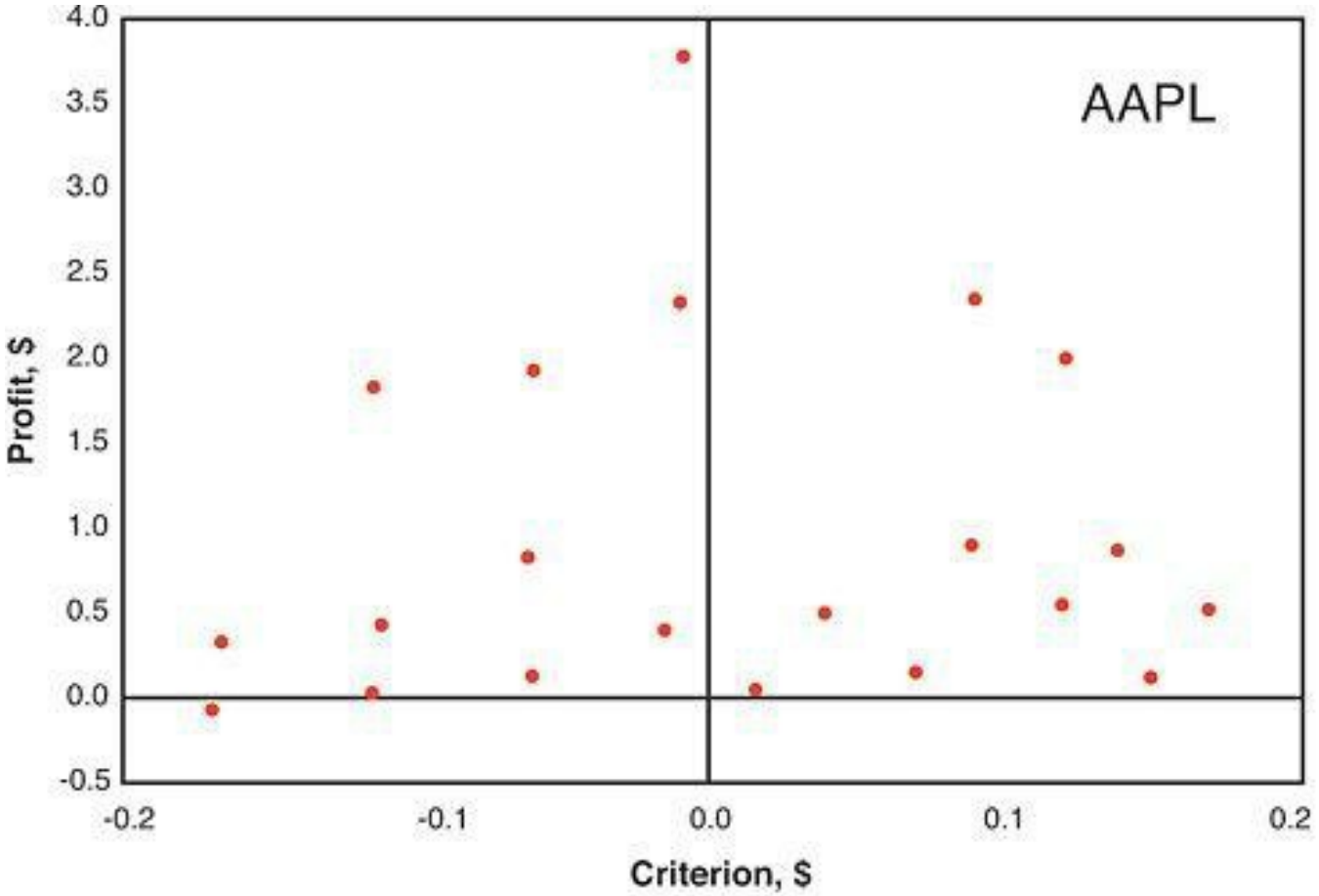
profit values? High determination coefficients detected in the previous section contradict the results presented in Chapter 3, "Evaluation of Criteria Effectiveness," (Figure 3.2.2). Therefore it would be reasonable to verify whether it is always possible to recognize the best combination relying just on the value of a certain criterion. Unfortunately, the answer is negative. We demonstrate this using two other stocks, AAPL and YHOO, and applying the same algorithms for combination formation as it was done earlier for MA. (The criterion values are estimated for the same date, February 1, 2007.) Figure 4.3.1 demonstrates the lack of relationships between the criterion

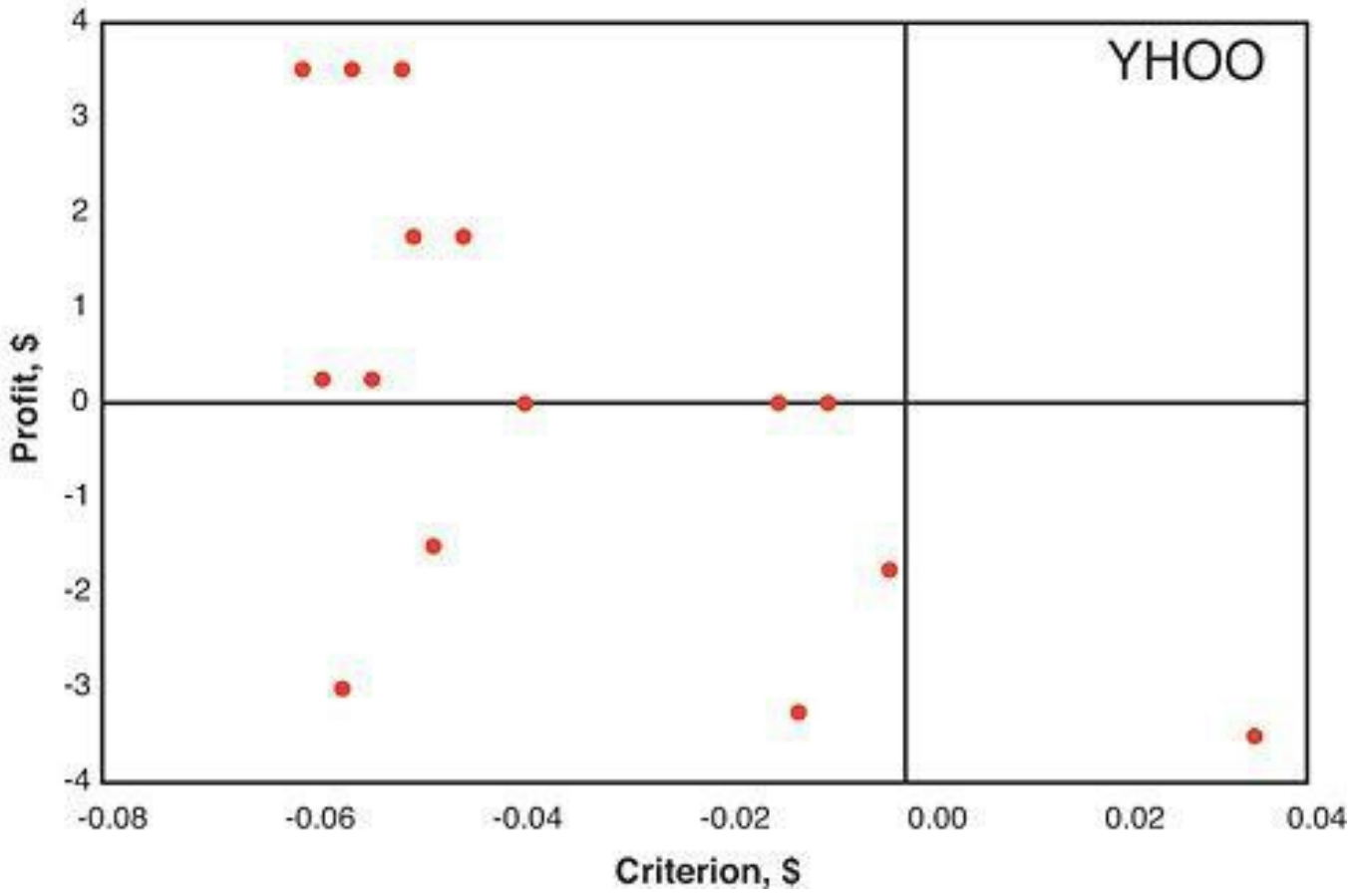
values and realized profits when AAPL was used as the underlying. (The regression is statistically insignificant: $p > 0.6$; the determination coefficient is low: $R^2 = 0.01$.) Furthermore, in the case of YHOO, the relationships between the criterion and profit are negative. (The regression is statistically significant: $p < 0.05$, the determination coefficient is $R^2 = -0.37$.) The absence of the relationships means that the future profit of combinations cannot be forecasted by this criterion. Therefore, in such a case, the criterion does not have any predictive power. The negative relationship points at the possibility of potentially dangerous consequences of combinations selection performed on the

basis of such criterion—the combinations possessing the highest criterion values can turn out to be the most unprofitable! In this case the criterion is not only unable to select the best combinations, but also causes direct loss, identifying the worst combinations as the best ones.

Figure 4.3.1. Relationship between realized profit and the criterion value corresponding to short strangles and straddles constructed for underlying

stocks AAPL and YHOO.


AAPL
Profit, $
Criterion, $
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-0.2
-0.1
0.0
0.1
0.2


YHOO
Profit, $
Criterion, $
4
3
2
1
0
-1
-2
-3
-4
-0.08
-0.06
-0.04
-0.02
0.00
0.02
0.04

The heterogeneity of {criterion × profit} relationships obtained for different underlying stocks can be explained by the absolute criterion values. When MA was used as the underlying stock, the criterion values for most combinations were high (7% on average, maximum 13%). On the contrary, in the two other cases the absolute criterion values were low: For AAPL the average was 0.1%, the maximum was 1%; for YHOO the average was –0.6%, the maximum was 0%. These observations suggest that if values of criteria are relatively high for most estimated combinations (or, at least, they are quite high for several combinations), then such criteria can be

used for selection purposes. Otherwise, when the criterion gives a poor forecast for most combinations, it should be considered as inappropriate.

Summarizing the data presented in Figures 4.2.1 and 4.3.1, we conclude that application of the criterion through simple ordering of combinations by its values produces unstable results. The absolute values of the criterion should be imperatively taken into account. For the "expected profit on the basis of lognormal distribution" criterion, this seems to be quite obvious and intuitively understandable: If the profit is expected to be negative for all combinations, it is unlikely that any of them will be profitable. In other cases, when criterion

values represent some abstract figure that could not be referred directly to profit forecasting (such criteria were classified in Chapter 1 as "nonforecasting universal criteria"), its threshold absolute value may be difficult to establish. In any case, if the absolute values of a certain criterion are below some predetermined level, the application of such criterion by the simple ordering of combinations by its values may be dangerous. This phenomenon illustrates that the criterion effectiveness depends to a large extent on the thoroughly adjusted rules for its application.

4.3.2 Strategy and Underlying Assets

The procedure for option combinations selection considered in this chapter assumes that strategies and underlying assets are predefined and fixed. Therefore, the criterion cannot be "responsible" for unsuccessful selection of strategies and underlyings. For example, the short strategies used in the previous section are profitable if the underlying stock price does not change substantially until the expiration day. If considerable price move happens, the short strategies yield a loss. Hence, the successful selection of the combination is possible only if the strategy and the underlying asset were selected correctly. If the criteria for selection of the pair {underlying × strategy} were chosen or

applied erroneously, the criterion intended to select the best combination for this pair will fail as well. This means that if by the expiration date the underlying stock shows a large price move (that is, the strategy or underlying were selected incorrectly), the relationship between the criterion values and realized profits will be weak or even negative. On the other hand, if the strategy and the underlying are selected successfully (that is, there is no significant price move of the underlying), this relationship will be strong and positive.

4.3.3 Simultaneous Analysis of Factors Affecting Combinations Selection

At least in some cases the criterion that forecasts expected profit on the basis of lognormal distribution is an effective tool in dealing with the problem of option combinations selection. However, in other cases the influence of certain factors may significantly complicate the accomplishment of this task. We identified two such factors: The low absolute criterion value for most combinations and significant price move of the underlying after trading position has been established.

The simultaneous effect of these factors on the selection results can be quite substantial. To demonstrate this, we constructed a multitude of regressions

similar to those presented in Figures 4.2.1 and 4.3.1. For each regression we registered three parameters:

- The absolute criterion value of the best combination
- The absolute magnitude of the underlying stock price change
- The correlation coefficient for the relationship between the criterion value and the realized profit.

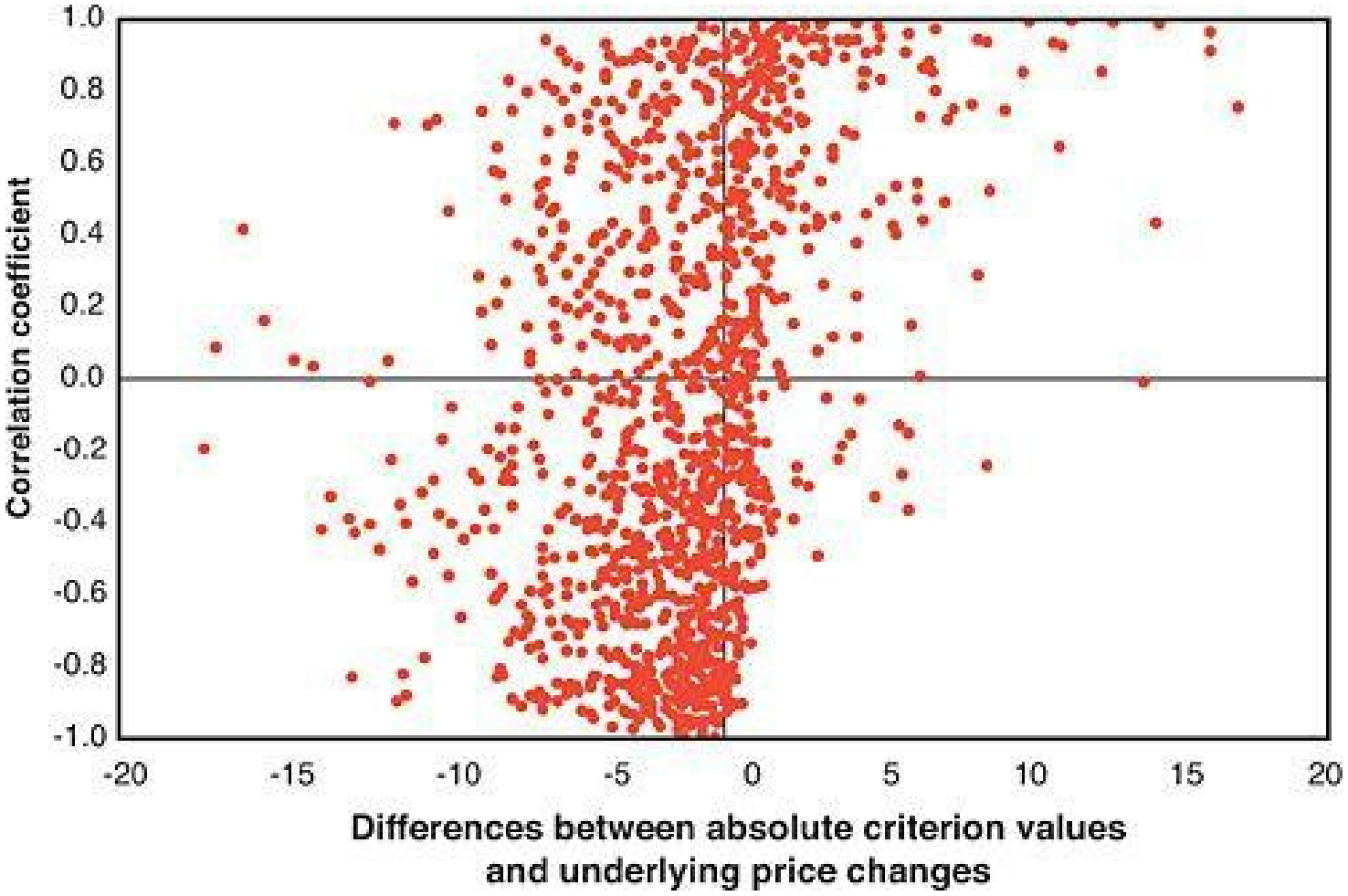
Option strategies and the algorithm for creating combinations were the same as in the previous examples. Altogether we examined the {criterion × profit} relationships for 500 underlying stocks and three dates (February 1, 2007,

February 2, 2007 and February 5, 2007). Thus, 1,500 regression models were created and analyzed.

To illustrate the combined effect of the absolute criterion value and the magnitude of the underlying price change, we combine these two parameters into a single one. Because the relationship between the correlation coefficient and the criterion value is expected to be direct, whereas the relationship between the correlation coefficient and the underlying asset price change should be negative, we can obtain the single parameter by subtracting the values of the price change from the criterion values. Relationship between the resultant

differences and the correlation coefficients corresponding to them are shown in Figure 4.3.2.

Figure 4.3.2. Relationship between the correlation coefficients of regressions {criterion × profit} and the differences of the absolute criterion values and underlying price changes.



High values of the combined parameter (horizontal axis) correspond to high correlation coefficients (vertical axis) that in many cases almost achieves the maximum $r = +1$ (Figure 4.3.2). On the other hand, low values of the combined parameter correspond to low correlation coefficients (frequently almost falling to

its minimum $r = -1$).

Figure 4.3.2 shows a strong positive dependence between two variables, even though it is more S-shaped than linear. However, whatever the shape of the relationship is, it indicates clearly that the criterion can accomplish its mission successfully (as indicated by high positive correlation coefficient) only when the absolute criterion value is high and the pair {strategy × underlying} was selected properly. (That is, no substantial underlying price change has happened.) Therefore, the effectiveness of criteria depends to a large extent on the simultaneous impact of these two factors. If their influence cannot be managed and mitigated, the result of

criteria application may become contrary to our expectations.

4.4 Multistrategy, Long-Term Evaluation of Criteria Effectiveness

The preceding investigation indicates the applicability of the criterion “expected profit on the basis of lognormal distribution” for selection of superior option combinations. However, these results are not sufficient to fully support the general conclusion of the criteria usefulness as long as we tested only one

criterion, only one option strategy, and a few dates. To prove our general thesis stating that in most cases criteria are an effective tool for solving the problem of combinations selection, we have to expand the time horizon of the study and consider option strategies opposite to those that have been utilized in the previous sections. Besides, it is necessary to test the effectiveness of other criteria described in Chapter 2.

4.4.1 Methods

The long strangle/straddle strategy is used as an opposite to the short strangle/straddle strategy. We call these

strategies *opposite* due to the fact that any market changes (of the underlying price or implied volatility) that are favorable to one of them are adverse to another one (and vice versa). In addition to the criterion tested in the previous sections (expected profit on the basis of lognormal distribution [EPLN]), we investigate the effectiveness of the following four criteria: profit probability on the basis of lognormal distribution (PPLN), expected profit on the basis of empirical distribution (EPEM), ratio of implied to historical volatility (*IV/HV*), and degree of symmetry (DSYM). The first three criteria were described in detail in Chapter 2. The fourth criterion is

intended to select combinations with the most symmetric payoff functions relative to the current underlying price. Its value is calculated as

$$Symmetry = \left| \frac{c(S)}{p(S)} - 1 \right|,$$

where $c(S)$ and $p(S)$ are premiums for Call and Put options with strike S.

The effectiveness of each criterion will be tested over the time period starting at the beginning of 2000 and ending in August 2007. We have to specify and fix the algorithms for selection of option strategies and underlying assets. These procedures should be designed in such a manner that their effectiveness will not

affect our estimate of the criteria effectiveness in the selection of option combinations. To achieve this goal we divided the initial set of 2,500 underlying stocks into two subsets. The first subset consists of stocks that are more suitable for the short option strategies, and the second one consists of stocks that are more suitable for the long strategies. The allocation of stocks between the subsets is executed by application of the *IV/HV* criterion (according to the methods described in Chapter 5, "Selection of Option Strategies"). Then in each subset we select the 50 best underlying stocks. This selection is performed by applying the same criterion, the *IV/HV* ratio, in

accordance with the methods described in Chapter 6, "Selection of Underlying Assets." All these selection procedures are repeated consecutively for each date.

Why should the selection of option strategies and underlying assets be affected by the criterion based on the *IV/HV* ratio? In Chapter 2 it was emphasized that this criterion is not only suitable for evaluation of options and their combinations, but is also an effective tool for estimating the suitability of underlying assets for certain strategies. This feature allows the selection of strategies and underlying assets, while avoiding the evaluation of particular combinations. This property of the *IV/HV* ratio enables the estimation

of the effectiveness of option combinations selection neutralizing the interactive effects associated with selection of strategies and underlyings.

After the strategies and underlying assets have been selected, we create all possible combinations according to the algorithm described in section 4.2. For each of the 50 stocks, the best combination is selected by one of the five criteria. (Each criterion is analyzed separately.) Portfolios consisting of 50 combinations selected in such a way are modeled. Every combination consists of one Call contract and one Put contract. The total profit or loss of the 50 combinations is calculated at the expiration date.

In addition to the selection of combinations according to the criterion values, one random combination for each of the 50 underlyings is also selected. (These 50 combinations represent the control group.) At the expiration date the total profit or loss of the 50 combinations selected by the criterion and those constituting the control group are calculated.

The difference between the profit of combinations selected by the criterion and the profit of randomly selected combinations reflects an advantage acquired due to the criterion application over one trading day. Summing up those differences as we advance along the

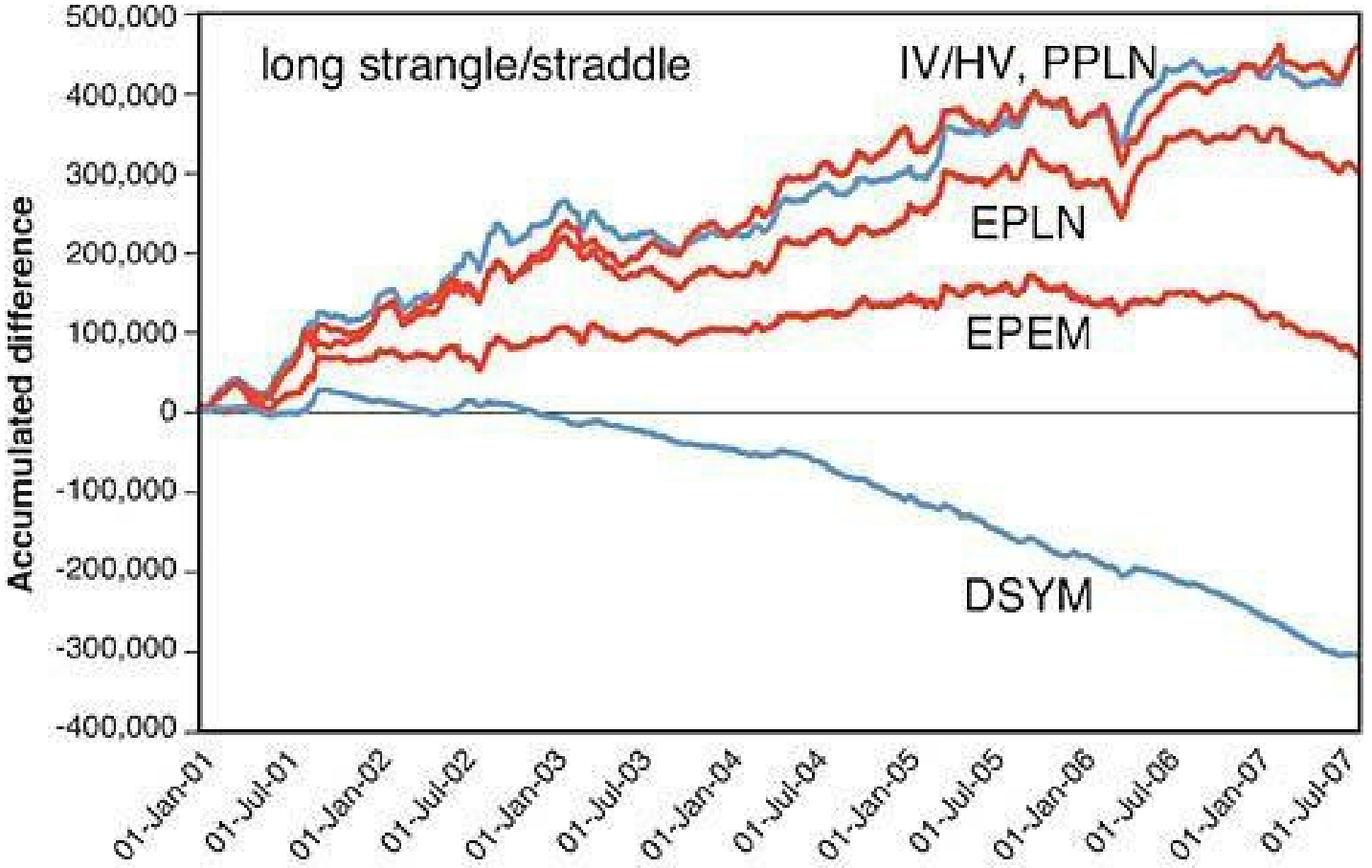
time series (that is, calculating the cumulative difference for every date) generates an estimate of the profit accumulated due to the consistent application of the criterion.

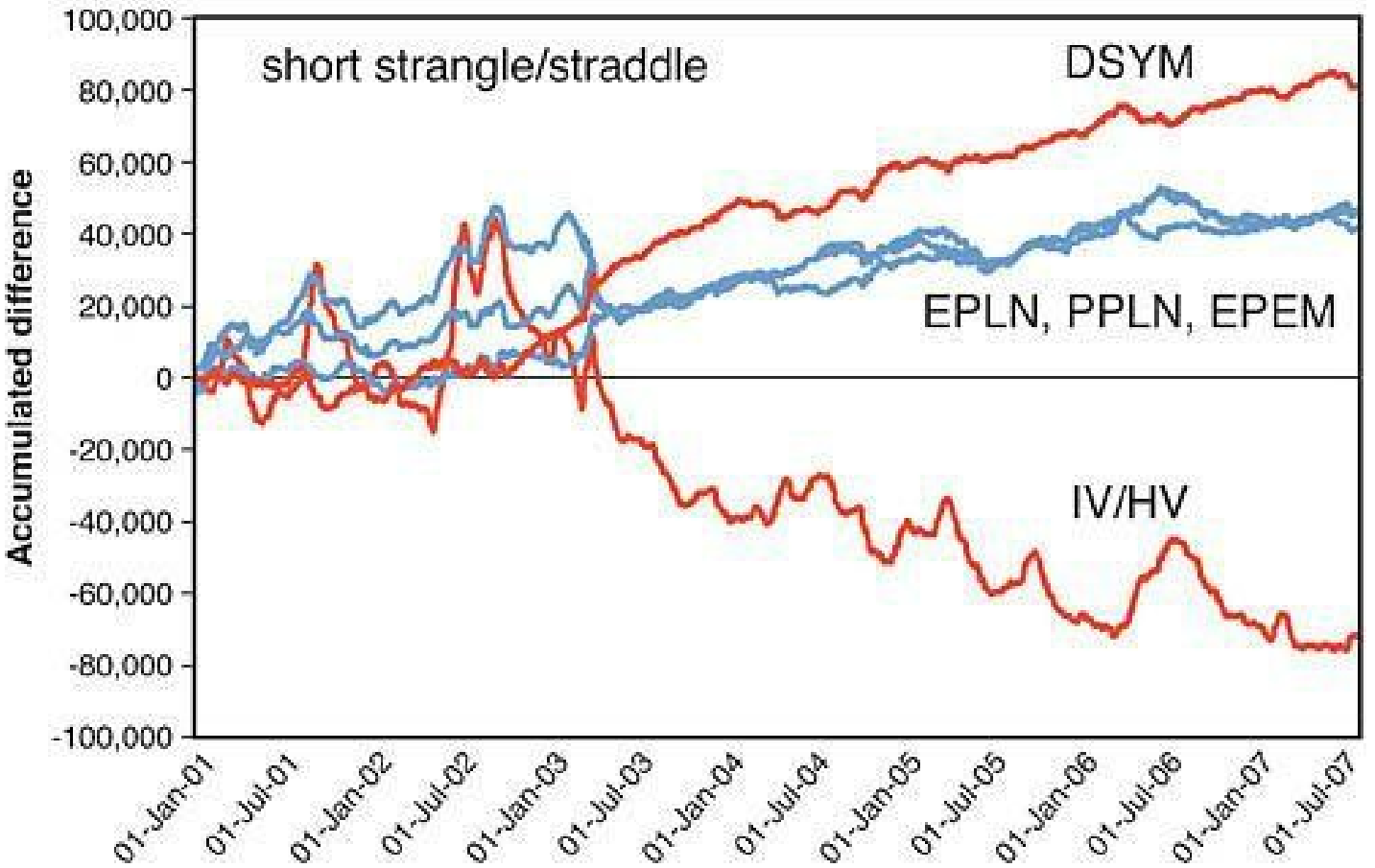
4.4.2 Results

Applying the described methods, we analyzed and tested the effectiveness of five criteria. The results are presented in the diagrams of the accumulated difference between profit of combinations selected by the criteria and profit of randomly selected combinations (Figure 4.4.1).

Figure 4.4.1. Dynamics of the accumulated differences between profits of combinations selected by criteria and profits of randomly selected combinations. The following criteria were applied: expected profit on the basis of lognormal distribution (EPLN), expected profit on the basis of empirical distribution (EPEM), profit

probability on the basis of lognormal distribution (PPLN), ratio of implied to historical volatility (IV/HV), and degree of symmetry (DSYM).


long strangle/straddle
IV/HV, PPLN
EPLN
EPEM
DSYM
Accumulated difference
500,000
400,000
300,000
200,000
100,000
0
-100,000
-200,000
-300,000
-400,000
01-Jan-01
01-Jul-01
01-Jan-02
01-Jul-02
01-Jan-03
01-Jul-03
01-Jan-04
01-Jul-04
01-Jan-05
01-Jul-05
01-Jan-06
01-Jul-06
01-Jan-07
01-Jul-07


short strangle/straddle
DSYM
EPLN, PPLN, EPEM
IV/HV
Accumulated difference
100,000
80,000
60,000
40,000
20,000
0
-20,000
-40,000
-60,000
-80,000
-100,000
01-Jan-01
01-Jul-01
01-Jan-02
01-Jul-02
01-Jan-03
01-Jul-03
01-Jan-04
01-Jul-04
01-Jan-05
01-Jul-05
01-Jan-06
01-Jul-06
01-Jan-07
01-Jul-07

Note that for the long strategy the effect of criteria application was stronger than for the short strategy. The total amount accumulated by the former is about $400,000, whereas the amount gained during more than 7 years by the latter strategy is just $80,000 (Figure 4.4.1). This divergence can be explained by the difference in payoff functions of the two strategies; the profit potential of the short strategy is limited whereas the potential of the long strategy is unlimited. The values shown in Figure 4.4.1 depend on capital allocation and other specific methods used in this study. Therefore, the absolute values of accumulated profits are meaningless.

However, they are useful for comparison of different criteria and strategies.

For the short strategy four out of the five criteria prove to select combinations more effectively relatively to random choice. The diagrams of accumulated differences for these criteria demonstrate permanent and smooth growth (Figure 4.4.1). The best performance is attributed to the criterion estimating the degree of symmetry. Expected profits on the basis of lognormal and empirical distributions and the criterion estimating profit probability show quite similar degree of effectiveness. A remarkably negative result is recorded for the *IV/HV* ratio; this criterion obviously tended to select

the worst rather than the best combinations. This phenomenon cannot be explained by the influence of external forces (like it was done in section 4.3) because the impact of such factors as the absolute criterion value and the algorithm for selection of strategies and underlyings had been taken into account in the course of developing the technique for this analysis (see section 4.4.1).

The long strategy shows quite similar results (Figure 4.4.1). Three out of the five criteria demonstrate high performance in selecting the best combinations; their diagrams of accumulated profit differences grow stably and evenly. The effectiveness of one criterion (expected profit on the

basis of empirical distribution) does not appear to exceed the random choice. Surprisingly, the selection of combinations according to their degree of symmetry shows negative results; this criterion chooses combinations inferior to those selected randomly (Figure 4.4.1).

The results of this study bring us to a number of important conclusions and allow classifying the criteria according to their applicability to solving problems of option combinations selection.

- Some criteria are effective regardless of the strategies they are used for. In the previous

example, the criteria belonging to this class are the expected profit and the profit probability on the basis of lognormal distribution.

- Certain criteria may be highly effective when applied to some strategies but are ineffective for others. The example of such a criterion is the expected profit on the basis of empirical distribution.
- Some criteria may show the highest effectiveness when selecting combinations for a certain strategy but become the worst instrument if applied to the opposite strategy. Being the worst, these criteria not only show weak

performance, but may also cause significant damage by selecting the worst possible combinations. In our example such criteria are the degree of symmetry and *IV/HV* ratio.

4.5 Summary

The general conclusion drawn from the data discussed in this chapter is that criteria represent a useful tool allowing the successful selection of option combinations within the predefined framework of strategies and underlying assets. This conclusion is based on the

correlation between the criterion and realized profit values. The identification of the best combinations is achieved by sorting all possible variants according to the criterion values.

However, if the absolute criteria values are low for all estimated combinations, their application may not produce the desired result. If the value of the criterion evaluating expected profit is negative or close to zero for most combinations, then the relationship between the criterion value and the future profit is either negative or nonexistent.

The application of the criterion to the selection of option combinations will be

successful only if the proper strategy has been selected. The quality of the strategy selection is determined among other factors by our ability to estimate the potential of the underlying price move. For short strategies, the more the price changes, the bigger the loss incurred by the strategy. Our research confirms that in cases when short strategy has been chosen and this choice proved to be correct (that is, the underlying price has not changed significantly), the correlation between the criterion value and the profit is strong and positive. If the price move is significant, then the correlation is either weak or negative. For long strategies the opposite is true.

Despite the fact that our conclusions are

based on the analysis of only five criteria and just two strategies, we have good reasons to expect similar results when the other criteria and strategies are applied. According to our research (not presented here), if strategies and underlying assets are selected properly, most of the criteria described in Chapter 2 demonstrate impressive effectiveness in combinations selection. However, the last assertion must be accompanied by one important note.

Some criteria may turn out to be ineffective and even detrimental. In some circumstances, they may select the worst rather than the best combinations. At first glance, it seems to be natural to identify and exclude such hazardous

criteria from the trading system. However, paradoxically the same criteria might turn out to be the best tools when applied with the other strategies. Therefore, simply giving them up is not the wisest decision. Quite the contrary, such criteria should be thoroughly and comprehensively investigated to use their potential in those cases in which they allow maximizing the potential profit of certain strategies. This reminds us that the methodology of criteria application is as important as their successful development.

chapter 5. Selection of Option Strategies

5.1 Introduction

Two important issues are examined in this chapter. The first one relates to the analysis of criteria applicability to option strategies selection. The second issue involves designing the optimal technique to select option strategies on the basis of strictly formalized criteria.

The key task of the first issue is to assess the effectiveness of criteria in selecting the best option strategy for a given underlying asset among many available alternatives. Suppose that there is a set of underlying assets for each of which we have to choose one option strategy possessing the highest profit potential. To select the most prospective strategy, we have to construct several option combinations for each underlying (at least one combination for each strategy). Next we have to evaluate all the combinations by means of a certain criterion (or several criteria) and select one combination with the highest criteria value. Will the selected combination correspond to the strategy that is the best

for this underlying asset? In other words, does the criterion have real predictive power in solving the problem of strategy selection?

The search for the answer to this question is hampered by many factors creating interference effects upon the analysis of criteria effectiveness in option strategy selection. The three main complicating factors are briefly described here.

1. *The algorithm of combination selection.* Many combinations with similar payoff functions can be created within the framework of the defined underlying asset and given option strategy. Because only one

of them will be used in further analysis, we have to define the precise algorithm of combination selection. In practice the combinations are selected on the basis of criteria values (see Chapter 4, "Selection of Option Combinations"). However, if we select combinations on the basis of criteria and afterward apply these criteria to selection of strategies (which is our primary task in this chapter), then the indicator of criteria quality will be influenced by two different factors: (i) by the efficiency of combinations selection and (ii) by the efficiency of strategy selection. In other

words, the indicator intended to reflect the effectiveness of criteria in solving the problem of strategy selection will be "interfered" with the effectiveness of criteria in the process of combinations selection. To neutralize this undesirable effect, the combinations in our analysis will be created according to a unified algorithm rather than selected on the basis of criteria.

2. *Domination of certain strategies over others*. Over extended periods of time, certain option strategies may constantly demonstrate their superiority over the other strategies used in the analysis. In many cases such phenomena are caused by

steady trends prevailing in the market of underlying assets. As a result, the dominant strategy may be better than its alternatives regardless of criteria indications. Therefore, the statistical methods used to evaluate the criteria should detect its predictive power regardless of the superiority of certain strategies resulting from long-term market trends.

3. *Comparison of heterogeneous strategies.* Payoff functions of different option strategies may appear in different forms. This feature considerably complicates the comparison of such strategies. A simple comparison of realized

gains and losses is incorrect because risks and profit potentials of heterogeneous strategies are different. Therefore, we have to use special methods of statistical analysis that allow for evaluating the effectiveness of criteria in selecting the best option strategy regardless of the extent to which this strategy may be better or worse than the alternative one.

Taking into account these particular qualities, we developed a special technique of ranking analysis intended to mitigate the effects complicating the evaluation of criteria effectiveness in the course of option combination selection.

In addition, we analyze the effectiveness of the criteria using traditional valuation techniques, such as mean realized profit, its volatility, and the ratio of these two variables (the Sharpe ratio).

The main task of the second issue discussed in this chapter is to develop an effective method of criteria application. Using the results of ranking analysis and the traditional valuations, we have designed a number of utility functions helping to model the effectiveness of criteria application for strategy selection. The utility function is an empirically derived relationship between the value reflecting the effectiveness of modeled trading and the value of the parameter under

investigation. (For example, the relationship between combination profit and volatility of its underlying could be considered as a utility function). The whole set of utility functions is used to create a model intended to optimize the parameter related to selection of option strategies on the basis of criteria. Such optimization can positively influence the criterion effectiveness and the profitability of potential trading.

5.2 Evaluation of Criterion Effectiveness by Ranking Analysis

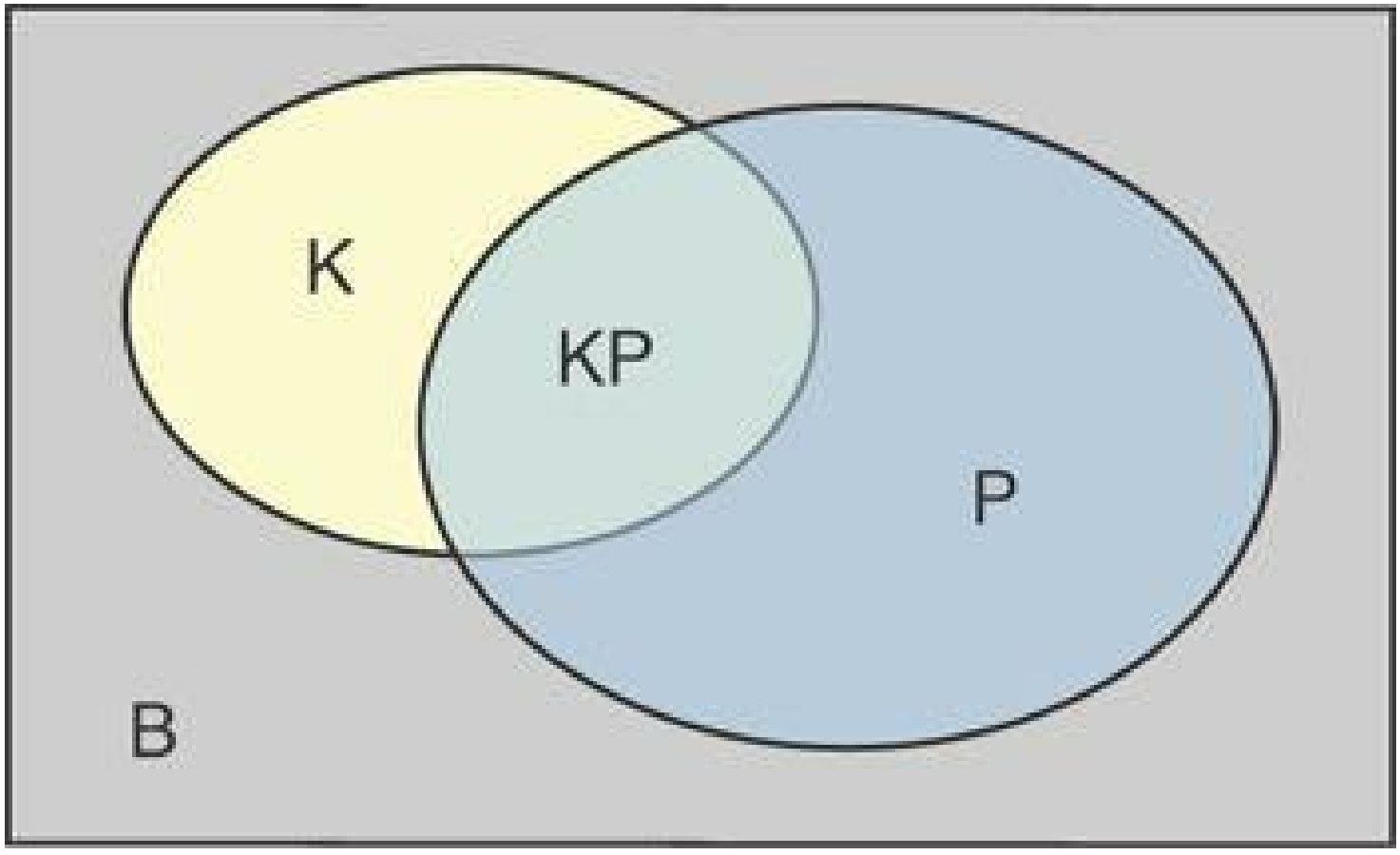
5.2.1 Methods of Ranking Analysis

Suppose that the initial set of underlying assets **B** consists of B stocks. For each stock we open a trading position corresponding to one of the two option strategies. The selection of the strategy is based on the values of one criterion. If the criterion value for the combination built according to strategy 1 is higher than the criterion value for the combination built according to strategy 2, then a trading position for this stock is opened according to strategy 1; otherwise, the position is opened according to strategy 2. Let **K** denote the

subset of stocks in respect of which strategy 1 is preferable to strategy 2. (It means that the criterion values for the combinations corresponding to strategy 1 are higher than its values for the combinations corresponding to strategy 2.) K is the number of stocks in subset **K**. Let **P** denote the subset of P stocks, for which trading results of strategy 1 are better than trading results of strategy 2. (Either profits are higher or losses are lower.) Combinations belonging simultaneously to **K** and to **P** form the subset **KP**. This subset consists of combinations for which strategy 1 is better than strategy 2 according to both criterion and profit values. Figure 5.2.1 shows schematic representation of the

initial set and three subsets and their interrelationships.

Figure 5.2.1. Schematic representation of the initial set and three subsets.



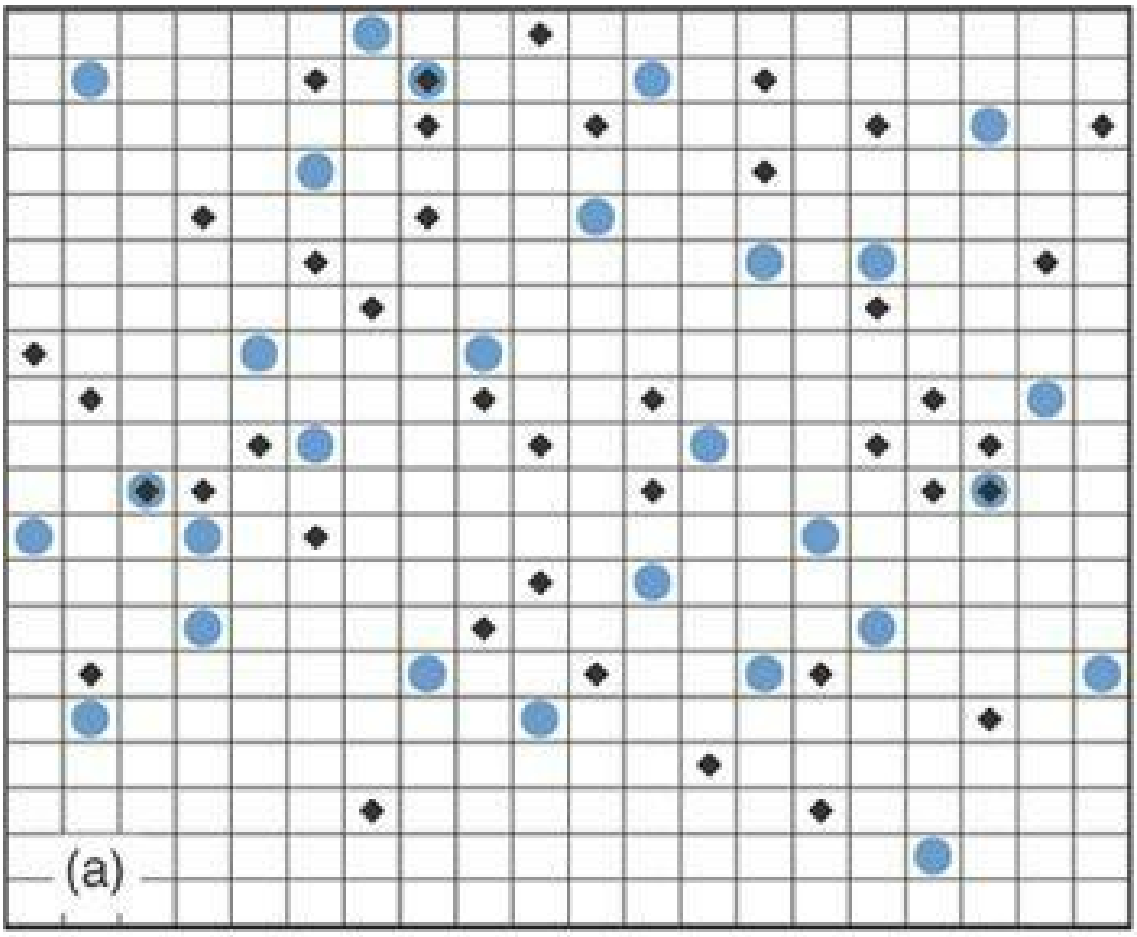
In Figure 5.2.1 the subsets were presented in a simplified and partly misleading form. To make the idea of ranking analysis more precise, let us suggest a more realistic example. Suppose that 400 stocks are used as underlyings ($B = 400$) and option combinations corresponding to these stocks are valued by the criterion. Strategy 1 appears to be better than strategy 2 for 30 out of 400 stocks ($K = 30$). The initial set **B** is depicted as a field divided into 400 cells each of which corresponds to one stock (Figure 5.2.2 [a]). Stocks, for which combinations created according to strategy 1 have higher criterion value than combinations created according to

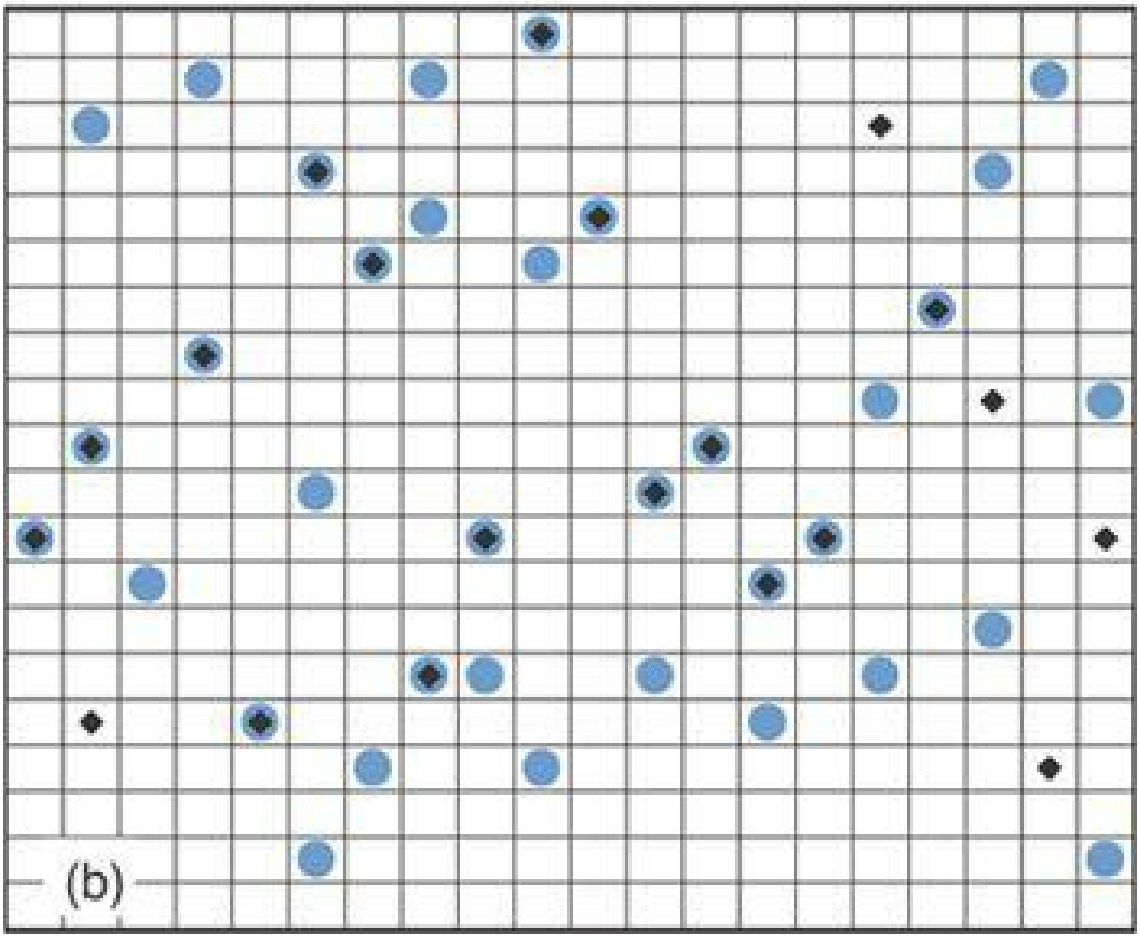
strategy 2, that is, stocks belonging to subset **K** (Figure 5.2.2 [a]), are highlighted by circles. Now suppose that options expired (or positions were closed) and profits and losses were evaluated separately for both strategies and for all the 400 stocks. In Figure 5.2.2 (a) the stocks, for which combinations created according to strategy 1 appeared to be more profitable than combinations created according to strategy 2, that is, stocks belonging to subset **P**, are marked by rhombi. In this example $P = 40$. We created a chart illustrating the effectiveness of the criterion in identifying stocks, for which application of strategy 1 is preferable to application

of strategy 2 (Figure 5.2.2 [a]).

Figure 5.2.2. Schematic representation of the initial set where each cell corresponds to one of 400 stocks. Cells highlighted by circles mark stocks for which the criterion pointed to the superiority of one strategy over another. Cells highlighted by rhombi mark stocks for which this strategy turned out to be

more profitable than the alternative one. (a) Number of matches of circles and rhombi is small, which means that the criterion frequently gives erroneous estimates. (b) Number of matches of circles and rhombi is higher, which means that the criterion is performing effectively.


(a)


(b)

If the criteria were perfect and performed without a single fault, in each case when it indicated the stocks for which strategy 1 should be applied (circles), this strategy would turn out to be better than strategy 2 (rhombi). In this ideal case all the cells highlighted by circles would also be marked by rhombi. However, in reality, absolutely perfect criteria do not exist. Sometimes when the criterion performance commits an error, the strategy selected on the basis of its value appears to be less profitable than the rejected one (cells highlighted only by circles). And vice versa; the strategy recognized by the criterion as inferior to its alternative

may turn out to be the most profitable one (cells highlighted only by rhombi). Screening in Figure 5.2.2 (a) reveals that the criterion performed properly only in three cases (when rhombi and circles matched). Is this number of matches large enough? Or maybe it is negligibly small? It would be more appropriate to put this question in a different way: What number of matches is sufficient to judge on the effectiveness of a particular criterion?

To answer this question, suppose that the criterion does not "work" and all matches of rhombi and circles are random events. In this case the probability of any circle-containing cell being randomly marked by rhombi is

equivalent to the *K/B* ratio. Thus, if rhombi are randomly distributed (that is, they match circles by chance and not because the criterion is effective), then the expected number of matches of circles and rhombi (*KP*) can be calculated as follows:

(5.2.1)

$$KP\ mathematical\ expectation = P \times (K/B)$$

Putting the figures from our example into formula 5.2.1 gives us 40 × 30 / 400 = 3, meaning that under the assumption of criterion ineffectiveness (that is, randomly distributed rhombi), the number of cells containing both circles

and rhombi is expected to be around this figure. In the example illustrated by Figure 5.2.2 (a), we observed exactly this number of matches of circles and rhombi. Therefore, these matches are probably of a random nature. We used the word "probably" on purpose to emphasize that formula 5.2.1 estimates the number of matches that is expected and is inappropriate for calculation of its exact value in every case. It means that if there is a sufficient number of observations, we should obtain an average number of matches that approximates the value calculated by formula 5.2.1. (The example shown in Figure 5.2.2 (a) represents an individual observation.)

Consider another example. Assume that another criterion was applied to the same 400 stocks and to the same two option strategies. The following results were obtained: $K = 35$, $P = 20$, $KP = 15$ (Figure 5.2.2 [b)]). A substantial difference between Figure 5.2.2 (a) and Figure 5.2.2 (b) is easily observable. Although in the first case the number of cases when the criterion performed effectively (that is, when circles and rhombi matched) was low, in the second case it was high. Again, the same question arises: Is the number of matches sufficient to draw the conclusion that the criterion is effective?

By analogy with the first example,

formula 5.2.1 is used to calculate the number of matches expected under assumption that all matches are random. The mathematical expectation is 1.75 (35 × 20 / 400), which means that if the criteria were inefficient and all matches were obtained by pure accident, the value of *KP* should be around 1.75. However, in reality, the value of *KP* is substantially higher (equals 15). Therefore, in this example we have serious reasons to claim that 15 matches occurred not due to coincidence but because of the effective selection of the preferred strategy by the criterion.

Formalizing the previously mentioned reasoning, we conclude that the criterion can be considered as effective, if *KP* >

$P \times (K/B)$. Transforming this inequality we obtain a formula to calculate the coefficient of criterion effectiveness k (the criterion has predictive power if k is greater than 1):

(5.2.2)

$$k = \frac{KP \times B}{K \times P} > 1,$$

where B is the number of stocks (initial set of underlying assets); K is the number of stocks for which the criterion judged a certain strategy as superior relative to all the other alternatives; P is the number of stocks for which this strategy turned out to be more profitable than the other

strategies; *KP* is the number of stocks that belong to both sets **K** and **P** (in our examples *KP* is the number of matches of circles and rhombi; and in real trading a stock belongs to *KP* if the criterion selects a certain strategy for the stock and this strategy is confirmed to be the best one according to the realized profit).

Using formula 5.2.2, we can calculate k values for many past trading days and in this way get a sufficient number of observations to make valid conclusions concerning the deviation of k from 1. If $k > 1$ and the Student's test indicates that the difference is statistically significant, we can state that the criterion may effectively be used for selection of the

best option strategy from a set of alternatives.

To test this statistical approach, we formed an initial set of underlying assets consisting of 500 U.S. stocks with high option trading volume in March 2007 (estimated for contracts with the nearest expiration date). Four option strategies were tested: long straddle, short straddle, long calendar spread (buying a Call with the nearest expiration date and selling short a Call with the next expiration date), and short calendar spread (selling short a Call with the nearest expiration date and buying a Call with the next expiration date). For all the strategies, combinations were built according to the following principles:

- Only contracts with the nearest expiration date were used (except spreads that consisted of the nearest and the next month contracts).
- Only strikes closest to the current stock price were utilized
- Numbers of Puts and Calls in each combination were equal.

Thereby, we strictly fixed the algorithm of combinations construction that was necessary to avoid the influence of this algorithm on the assessment of the effectiveness of strategy selection. (The effectiveness of combination creation should not affect the valuation of the

criterion effectiveness.) For each trading day and for each stock, we evaluated four combinations (one for each strategy) according to the criterion "expected profit on the basis of lognormal distribution." For each combination we also estimated its profit or loss at the expiration date. (The criterion values and profit/loss values were expressed as a percentage of margin requirements.) For the purpose of this analysis, we selected trading dates belonging to the third week prior to the option expiration date (for example, for the expiration of February 16, 2007 we used the following dates: January 29, January 30, January 31, February 1, and February 2). Therefore the entire time

series ranging from November 2000 to March 2007 included 347 trading dates belonging to the third weeks. Thus 347 k values were calculated, providing a sufficient number of observations for reliable statistical analysis.

The strategies were compared by pairs in the following conjunctions:

- Long straddle—short straddle
- Long straddle—long calendar spread
- Short straddle—short calendar spread
- Long calendar spread—short calendar spread

Although testing four strategies allows comparison of six pairs, we limited our analysis to the preceding four listed pairs because the strategies in each pair are either competitors (short straddle and short calendar spread) or antipodes (long straddle and short straddle). Choosing between such individual strategies represents an important routine task, which is truly challenging for any investor dealing with options.

Two separate comparative analyses have to be carried out for each pair. For example, for the pair long straddle—short straddle, we have to analyze separately the effectiveness of the criterion in selection of stocks, for which the long straddle strategy is

preferable to the short straddle strategy. Then we have to assess separately the criterion effectiveness in selecting stocks, for which the short straddle strategy is superior to the long straddle strategy. Even though, at first glance, it looks like a superfluous activity, in practice it might appear that a certain criterion successfully selects stocks, for which strategy 1 is preferable to strategy 2, yet poorly accomplishes the opposite task. Accordingly, the evaluation of four strategy pairs requires a comparative analysis of eight conjunctions (for each of which we have 347 k values; this number corresponds to the number of trading dates in the sample).

Next we describe the technique of

ranking analysis developed specifically for evaluation of criteria capability to select the best option strategies. To calculate the coefficient of criterion effectiveness *k*, the values of *K*, *P*, and *KP* have to be established (see equation 5.2.2). For that purpose we need to determine to which rank each of the stocks belongs. The ranks are determined by the criterion value and by realized profit. The ranking procedure is illustrated in the following example (using the data for March 1, 2007). On this date we selected the first 30 stocks out of the 500 underlyings assessed (in alphabetical order) and estimated the criterion values and the realized profits for the short straddle and the long

straddle strategies. These data were used to analyze the criterion effectiveness in the selection of combinations for which the former strategy is superior to the latter one. The results are presented in Table 5.2.1.

Table 5.2.1. Selection of Stocks, for Which Strategy Short Straddle Is Superior to Strategy Long Straddle. The Data Required to Determine the Coefficient of Criterion Effectiveness *k*.

Stock ticker	Criterion Value, %		Difference Between Criterion Values	Realized Profit, %		Difference Between Profit Values	Rank		Belonging to Subsets		
	Short Straddle	Long Straddle		Short Straddle	Long Straddle		According to the Criterion	According to the Profit	KP	K	P
AAPL	–0.8	0.2	–1.0	–91.4	26.6	–118.1	b	a			
ABT	–8.7	1.4	–10.1	22.9	–3.8	26.7	b	a			+
ABX	2.9	–0.8	3.7	26.6	–7.5	34.0	a	a	+	+	+
ACE	–1.7	0.7	–2.4	17.7	–6.8	24.4	b	a			+
ACI	2.7	–1.7	4.4	37.8	–23.5	61.3	a	a	+	+	+
ACN	–11.9	2.4	–14.3	–80.0	15.9	–95.9	b	b			
ACS	–16.7	4.0	–20.7	–59.6	14.1	–73.7	b	b			
ADI	–4.8	2.7	–7.,5	15.3	–8.6	24.0	b	a			+
ADM	–4.1	1.2	–5.3	–18.4	5.4	–23.8	b	b			
ADSK	–11.,3	3.0	–14.3	33.3	–8.9	42.2	b	a			+
ADTN	–14.5	4.3	–18.9	–14.3	4.3	–18.6	b	b			
AEM	5.4	–2.3	7.7	–23.0	9.9	–32.9	a	b		+	
AET	–19.7	5.1	–24.8	–12.8	3.3	–16.1	b	b			
AFFX	–26.4	16.8	–43.2	–72.3	46.2	–118.5	b	b			

AH	−5.7	1.4	−7.1	−40.7	10.1	−50.7	b	b			
AKAM	−10.4	4.4	−14.7	−69.4	29.2	−98.6	b	b			
ALTR	1.8	−0.7	2.4	8.4	−3.1	11.5	a	a	+	+	+
AMAT	−1.0	0.3	−1.3	−41.7	10.4	−52.1	b	b			
AMKR	−10.1	5.6	−15.7	−37.1	20.5	−57.6	b	b			
AMLN	−8.6	3.7	−12.4	−21.6	9.3	−31.0	b	b			
AMZN	1.8	−0.5	2.2	4.9	−1.4	6.3	a	a	+	+	+
APA	1.4	−0.4	1.8	−12.5	3.0	−15.5	a	b		+	
APC	12.8	−3.1	15.9	−67.7	16.1	−83.7	a	b		+	
APOL	2.6	−0.9	3.5	56.9	−19.9	76.8	a	a	+	+	+
ATHR	6.1	−2.5	8.6	−57.1	23.6	−80.8	a	b		+	
BBH	−31.3	4.9	−36.2	6.3	−1.0	7.3	b	a			+
BEAS	2.0	−0.7	2.7	77.0	−26.2	103.2	a	a	+	+	+
BEAV	0.1	0.0	0.1	83.6	−28.2	111.8	a	a	+	+	+
BHI	1.0	−0.3	1.3	−29.4	8.1	−37.5	a	b		+	
BJS	11.5	−3.3	14.8	−64.2	18.4	−82.6	a	b		+	
The number of elements in each subset									7	13	12

To carry out the ranking analysis, we have to calculate the difference between the criterion value for the short straddle strategy and its value for the long straddle strategy. If the difference is positive, the stock belongs to rank *a* according to the criterion. If it is

otherwise, it belongs to rank *b*. Then we calculate the difference between the realized profit values for the short straddle strategy and for the long straddle strategy. Similarly, if the difference is positive, the stock belongs to rank *a* according to the realized profit. If it is otherwise, it belongs to rank *b*. Table 5.2.1 shows to which rank each of the stocks belongs according to the criterion and according to the realized profit values. It would be more appropriate to say that combinations rather than stocks belong to ranks (because all criterion and profit values are calculated for combinations). Nevertheless, as long as only one combination is created for each stock,

we can use stock tickers to denote combinations related to these stocks.

The stock ranked *a* according both to the criterion and the realized profit (when the criterion selected the short straddle strategy, and this strategy indeed turned out to be more profitable than its alternative) belongs to all the three subsets: **K**, **P** and **KP**. The stock ranked *a* according to the criterion and ranked *b* according to the realized profit (when the criterion selected the short straddle strategy but this choice proved to be erroneous) belongs only to subset **K**. If a certain stock is ranked *b* according to the criterion and ranked *a* according to the realized gain (the criterion did not select the short straddle strategy as a superior

one but again performed with an error), this stock belongs only to subset **P**. When the stock is ranked *b* both according to the criterion and according to the realized profit (the criterion did not select the short straddle strategy, and this strategy indeed turned out to be inferior to its alternative), this stock does not belong to any of the subsets (only to the initial set **B**).

The belonging of each stock to subsets is labeled by "+" in Table 5.2.1. Summing up the number of crosses in each column gives us $KP = 7$, $K = 13$, and $P = 12$. By substituting these values into formula 5.2.2, we can calculate the coefficient of criterion effectiveness:

$$k = \frac{KP \times B}{K \times P} = \frac{7 \times 30}{13 \times 12} = 1.35$$

Although this value is apparently greater than 1, it represents only one single observation (based merely on 30 stocks), which is not sufficient for making a valid conclusion about the criterion effectiveness. We need to obtain a considerable number of coefficient *k* values to judge on the effectiveness of the criterion. In the following section we proceed in this direction.

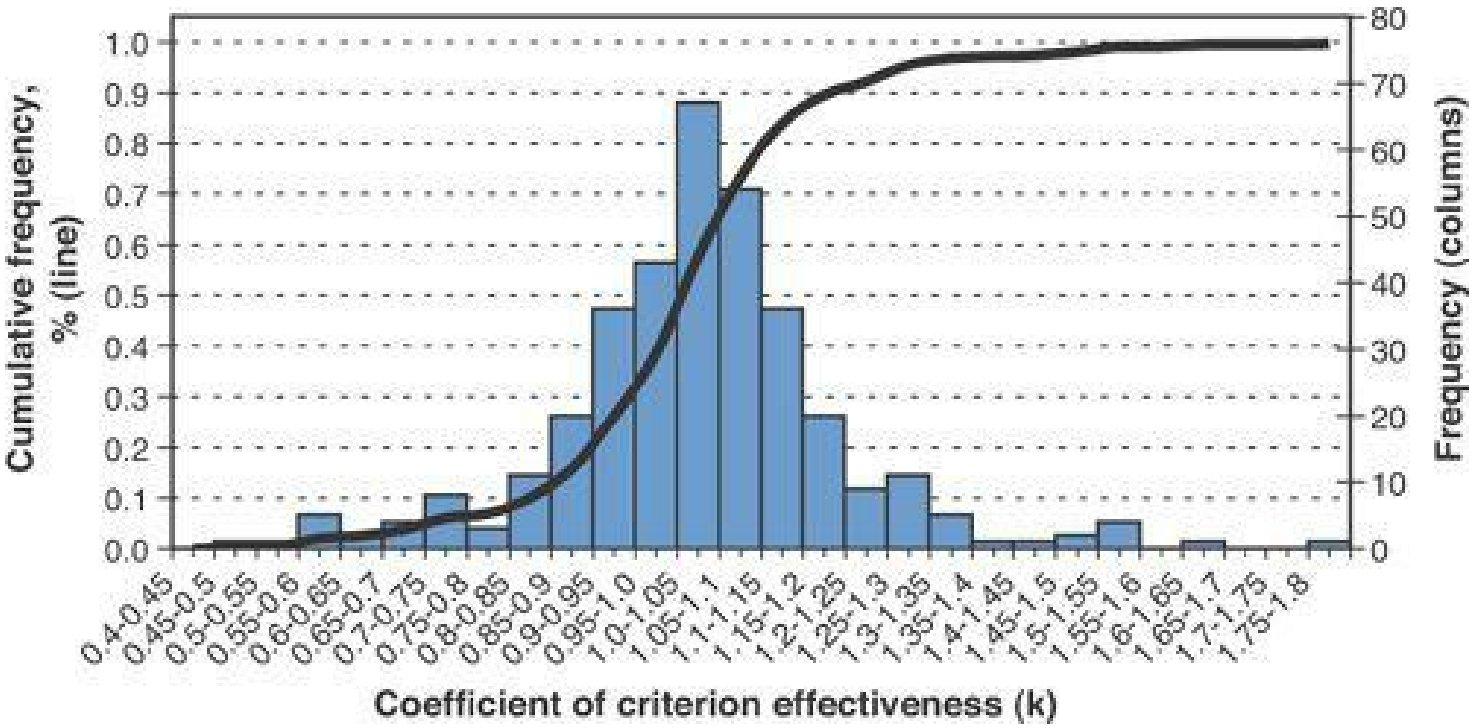
5.2.2 Results of Ranking Analysis

Consider 347 coefficients of criterion effectiveness obtained by comparing the long straddle and the short straddle strategies. The frequency distribution of coefficient values is presented in Figure 5.2.3. The most frequent k value falls within the range between 1 and 1.05 (occurred 67 times). The range 1.05 to 1.1 occupies the second position by the frequency of occurrence (54 k values). The range below 1 (from 0.95 to 1) occupies the third position only (43 k values). The cumulative percentage frequency of different k values is shown as a line in Figure 5.2.3. We see that only in 10% of the cases k value was less than 0.9 and only in 40% of cases k value was less than 1. The average k

value is 1.029. Is this deviation from 1 sufficient to make a conclusion that the criterion may be appropriately used for selection of stocks for which the long straddle strategy is superior to the short straddle one? The answer to this question depends on the results of the statistical analysis.

Figure 5.2.3. Frequency distribution of the coefficient of criterion effectiveness (*k*). The long straddle strategy is compared with the short

straddle strategy by the "expected profit on the basis of lognormal distribution" criterion.



According to Student's t test, the deviation of $k = 1.029$ from 1 is statistically significant ($t = 3.22$, $p = 0.14\%$). For all other pairs of strategies

the coefficient of criterion effectiveness is also slightly higher than 1 (Table 5.2.2). Although the difference is small, the variability of this coefficient (expressed as standard error) is also negligibly small. It means that, despite the fact that *k* only slightly exceeds 1, the difference is quite stable and persistent. Student's *t* test proves that for all pairs of strategies the difference between *k* and 1 is statistically significant. The value of *p* in Table 5.2.2 reflects the probability of making erroneous conclusion about the statistical significance of the deviation of *k* from 1. In this study the probability of such incorrect conclusion is negligible and does not exceed a small fraction of 1%.

Table 5.2.2. The Average Value of the Coefficient of Criterion Effectiveness (*k*), Standard Error *(SE*), and Statistical Significance of the Difference Between *k* and 1 (t and *p* of Student's *t* Test) for Eight Pairs of Compared Option Strategies. In Each Pair the First Strategy Is Compared to the Second One.

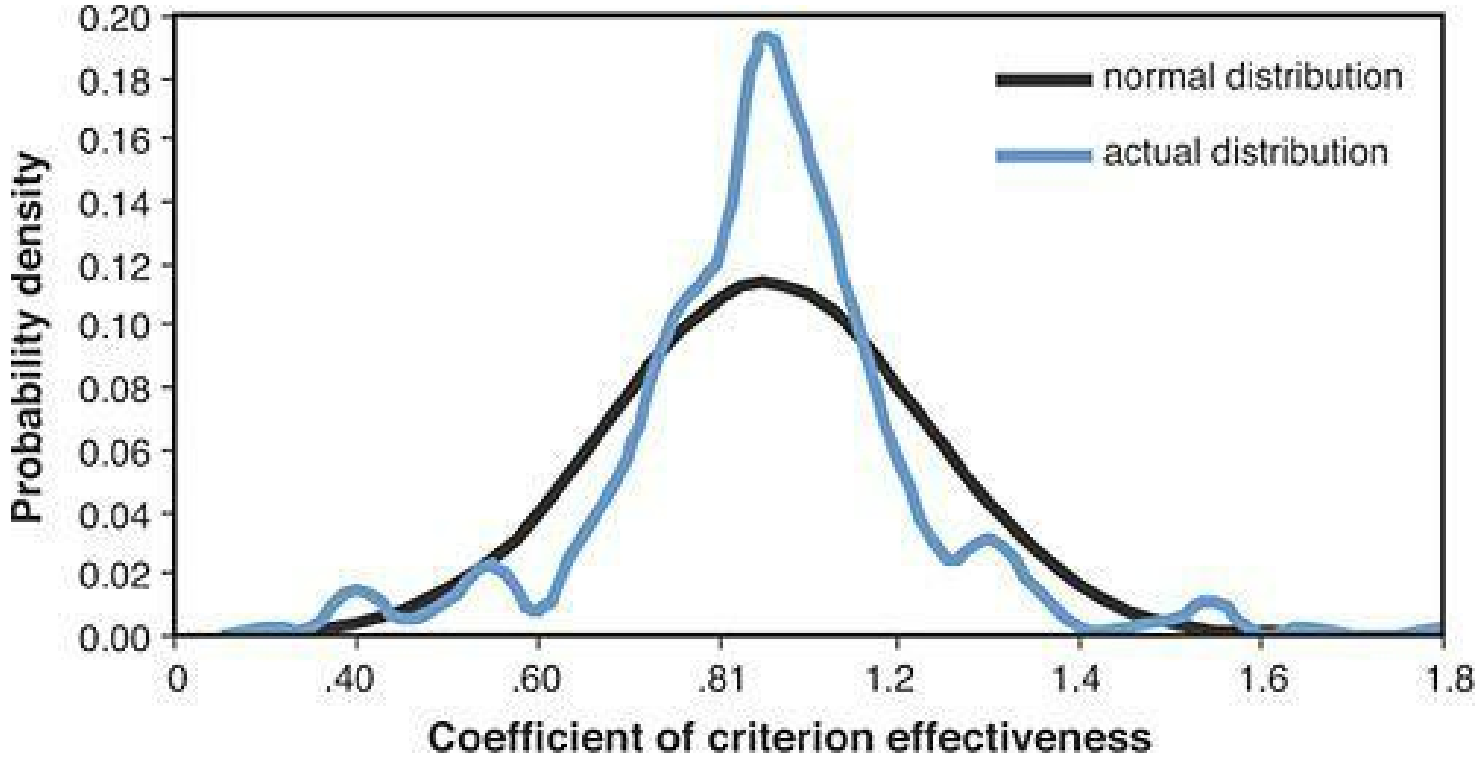
Pair of Strategies	*k*	*SE*	*t*	*p*
Long straddle—Short straddle	1.029	0.009	3.22	0.14%
Short straddle—Long straddle	1.019	0.004	4.75	< 0.001%
Long straddle—Long calendar spread	1.028	0.009	3.11	0.20%
Long calendar spread—Long straddle	1.016	0.003	5.33	< 0.01%
Short straddle—Short calendar spread	1.084	0.006	14.00	< 0.01%
Short calendar spread—Short straddle	1.067	0.004	16.75	< 0.01%
Long calendar spread—Short calendar spread	1.067	0.006	11.17	< 0.01%
Short calendar spread—Long calendar spread	1.053	0.004	13.25	< 0.01%

Nevertheless, we cannot fully rely on the results of the Student's *t* test or similar statistical analysis (for example, *F*-test) because these tests assume normal distribution of investigated variables. The distribution of the criterion effectiveness coefficient values is far from normal. The actual distribution of *k* values, obtained while comparing the long straddle and the short straddle strategies, and the normal distribution of this variable are shown in Figure 5.2.4.

The difference between the two distributions is evident; the actual distribution has a high and narrow peak whereas normal distribution has its classical bell-shaped appearance. It means that *k* values in the range from 1 to 1.1 occur more frequently than expected according to the model of normal distribution. On the other hand, *k* values, which are slightly lower than 1 or slightly higher than 1.1, appear less frequently than they should under assumptions of normal distribution.

Figure 5.2.4. Actual distribution of the coefficient of criterion

effectiveness (*k*) and normal distribution calculated for the same data. Both distributions are based on 347 *k* values obtained while comparing the long straddle and the short straddle strategies by the "expected profit on the basis of lognormal distribution" criterion.



The reason for deviation of the actual *k* distribution from the normal is determined by its nature. Normal distribution has nonzero probability of occurrence of very high and very low variable values. That is to say, the "tails" of normal distribution are unlimited. Contrary to this feature of normal distribution, *k* by definition cannot be less than zero. Therefore its

distribution is limited on the left side. The right side of this distribution is also limited because k cannot exceed a certain possible maximum (in section 5.2.5 we provide the algorithm for calculation of this maximum). The maximum obtainable k values limit the distribution on the right side. Therefore, we cannot fully rely on the results of statistical tests assuming that probability distribution of the analyzed variable is normal. Hence, some additional evidences are needed to prove the significance of the deviation of k from 1.

Analyses that do not require the assumption of normal distribution are nonparametric statistical tests also referred to as distribution-free methods.

These tests allow comparison of the frequency of cases when k was less than 1 with the frequency of cases when k was greater than 1. In our example $k < 1$ occurred 134 times (38.6% of 347 values), whereas $k > 1$ was observed 213 times (61.4% of 347 values). The nonparametric test based on binomial distribution analyzes the difference between the two frequencies observed in the experiment and compares it to the frequency assumed under the null hypothesis. (In our example the null hypothesis states that both $k > 1$ and $k < 1$ occurred in 50% of the cases.) This test proves that the deviation from the equal frequency detected in our example is statistically significant. The

probability that $k > 1$ was randomly observed in 61.4% of the cases (instead of 50% of the cases as assumed under the null hypothesis) is less than 0.003%. Another nonparametric test based on chi-square distribution examines the difference between the frequency obtained in the experiment and any expected frequency. We can once again use the equal frequency as an expectation to test the null hypothesis assuming that values $k > 1$ and $k < 1$ occur with the same frequency and any deviations from these frequencies are of a random nature. In our example the chi-square test showed that the null hypothesis can be rejected with the probability of an error of less than

0.02% (χ^2 = 16.7). Therefore, both nonparametric tests proved that the prevalence of $k > 1$ over $k < 1$ is statistically significant and cannot be attributed to random chance.

Regression analysis can be recommended as an additional technique for evaluation of the effectiveness of the criterion in identifying the preferable option strategy. To perform this analysis, equation 5.2.1 should be rearranged in the following way:

(5.2.3)

$$KP\ mathematical\ expectation\ /\ K = P\ /\ B$$

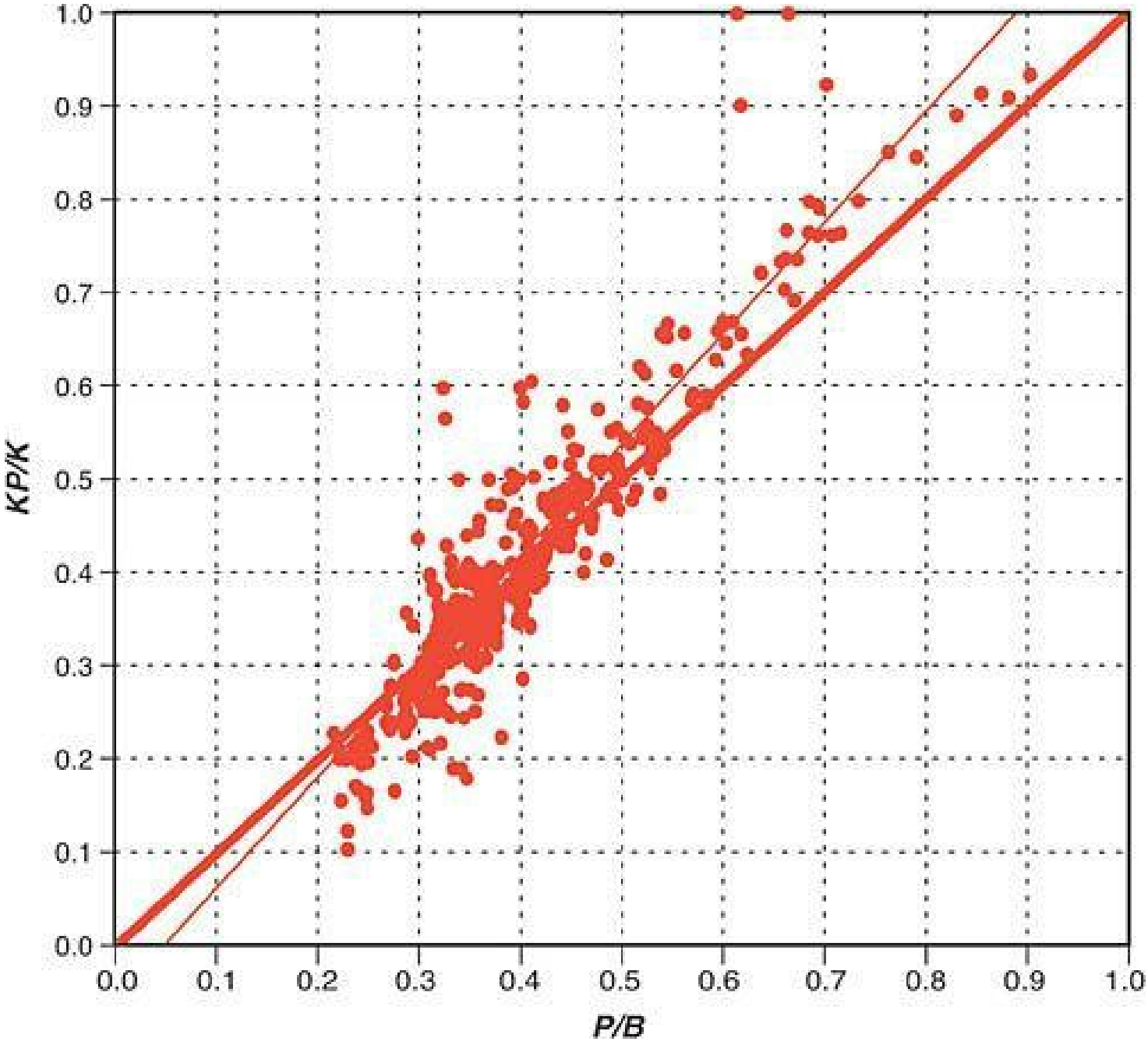
In this form this equation can be interpreted as follows: If the criterion "does not work," the ratio of combinations that turned out to be the best both according to the criterion and the realized profit values to the total number of combinations that were deemed the best according to the criterion (KP/K) will be equivalent to the ratio of combinations that turned out to be the best according to realized profit to the total number of combinations used in the analysis (P/B). Conversely, if the criterion works effectively, we should obtain $KP/K > P/B$. Regression analysis of the relationship between KP/K and P/B using all 347 k values will allow us to

estimate whether *KP/K* is greater than or equal to *P/B*. If the regression slope coefficient is more than 1 at a statistically significant level, then *KP/K* is greater than *P/B*, and, therefore, the examined criterion may be used for effective selection of the best strategy.

The results of such regression analysis performed for the pair of strategies we have used in the previous examples are presented in Figure 5.2.5. The regression slope coefficient equals 1.2, and its difference from 1 is statistically significant ($t = 7.19$, $p < 1\%$). Thus, the regression analysis also confirmed that the criterion can effectively identify superior option strategies.

Figure 5.2.5. Regression relationship between KP/K and P/B. The long straddle strategy is compared to the short straddle strategy according to the "expected profit on the basis of lognormal distribution" criterion. The thin line represents the regression line with the slope coefficient of 1.2, whereas the bold line corresponds to

the slope coefficient of 1.



Summarizing, we can conclude that all the three statistical tests confirmed the predictive power of the criterion in solving the problem of strategy

selection. We found that the technique of ranking analysis provides reliable quantitative evaluation of the criterion capability to select the best option strategies. Besides, we demonstrated that the "expected profit on the basis of lognormal distribution" criterion allows selection of combinations, for which the long straddle strategy is superior to the short straddle strategy. However, we have to note that evaluation of other criteria can give different results, especially when applied to other pairs of strategies.

Generally, we can recommend the following empirical rule: If two out of the three methods described indicate statistically significant predictive power

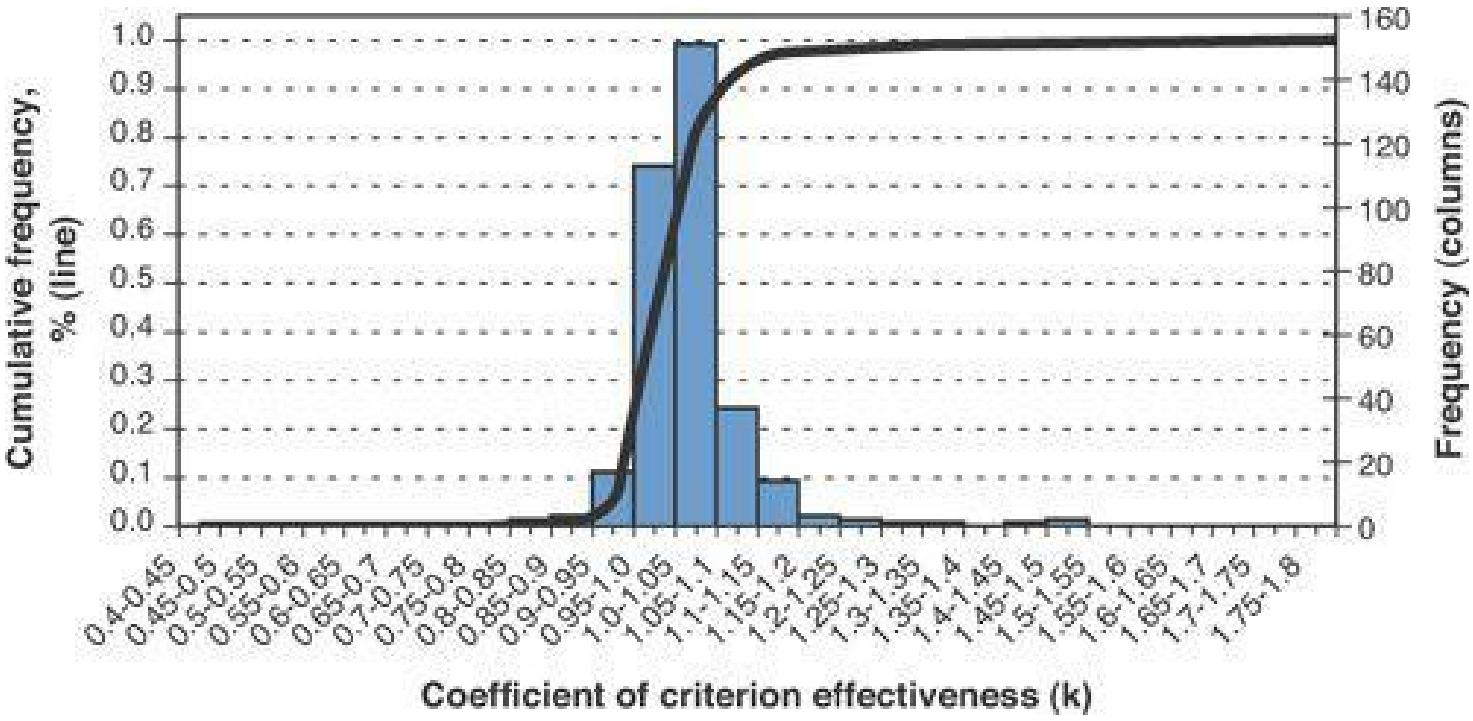
of the criterion, it is possible to rely on the validity of these conclusions and to use the criteria and strategies selected in such a way for practical purposes. In our example, all three techniques showed positive results, but such consistency does not occur in all cases. To demonstrate this, consider another example when the effectiveness of the same criterion is tested in the process of identifying the combinations for which the short straddle strategy is superior to the long straddle strategy.

In this example the average value of the criterion effectiveness coefficient is 1.019. Although this value is slightly lower than in the previous example, its difference from 1 is statistically

significant at a high confidence level (much higher than in the previous example, see Table 5.2.2). The distribution of *k* values demonstrates a considerable skew toward value which exceed 1 (Figure 5.2.6). Only in less than 5% of the cases *k* was less than 0.9, and in less than 40% of the cases its value was less than 1.

Figure 5.2.6. Frequency distribution of the coefficient of criterion effectiveness (*k*). The short straddle strategy is compared with the long

straddle strategy by the "expected profit on the basis of lognormal distribution" criterion.

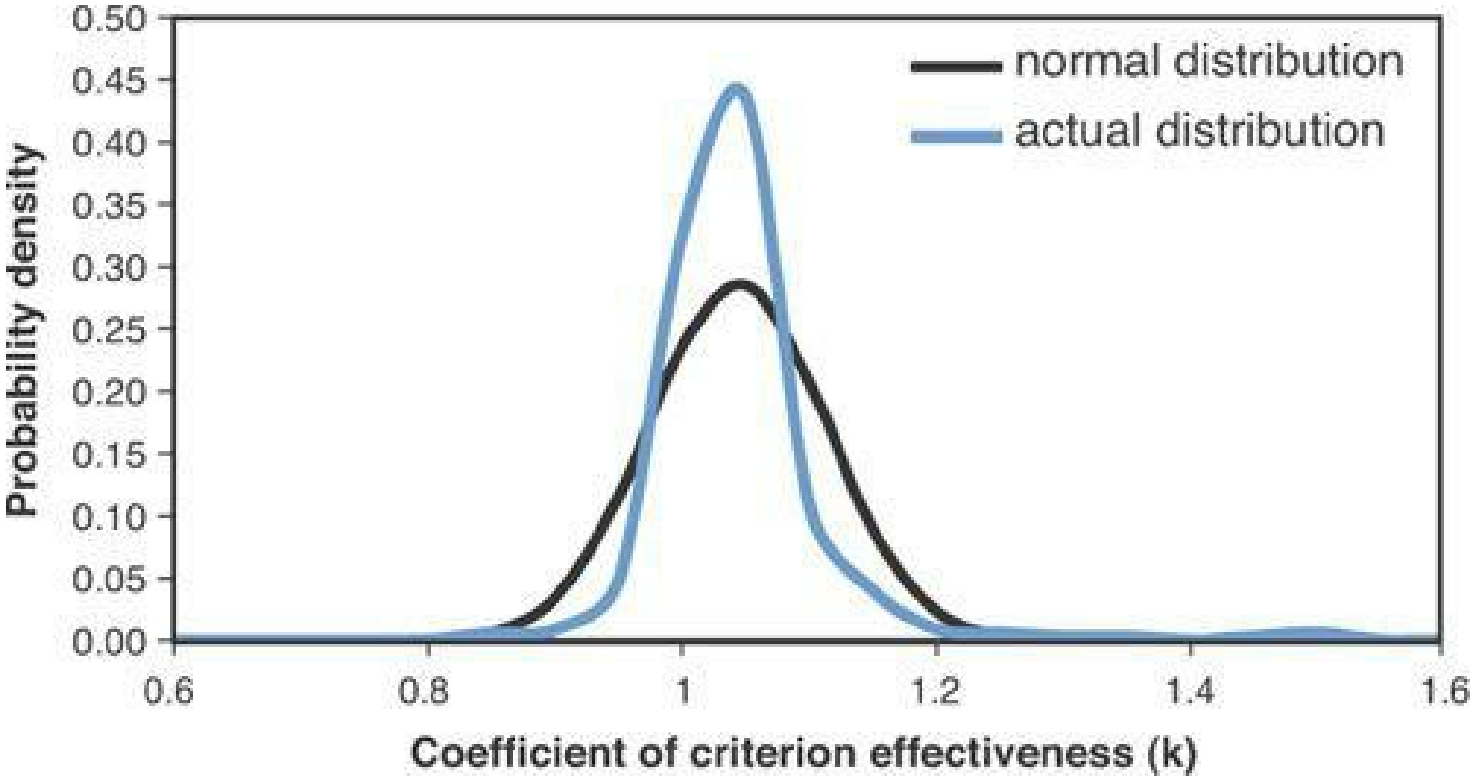


However, the distribution of *k* values in this case is even farther from the normal than in the previous example (Figure 5.2.7). The frequency of values that are

close to the average (somewhat greater than 1) dominates substantially over the frequency of the same values of normal distribution. The neighboring values of moderate deviations from the average are on the contrary represented by lower frequencies than normal distribution would imply (Figure 5.2.7). This puts in doubt the reliability and the statistical significance of the conclusion that *k* value is greater than 1.

Figure 5.2.7. Actual distribution of the coefficient of criterion effectiveness (*k*) and normal

distribution calculated for the same data. Both distributions are based on 347 *k* values obtained while comparing the short straddle and the long straddle strategies by the "expected profit on the basis of lognormal distribution" criterion.



We proceed to the second analysis technique involving an application of nonparametric tests. In this example $k < 1$ occurred in 135 cases (39.6% of 347 values); accordingly, $k > 1$ was observed in 212 cases (61.4% of 347 values; that is close to the results in the first example). The test for the difference between two frequencies based on binomial distribution showed that the

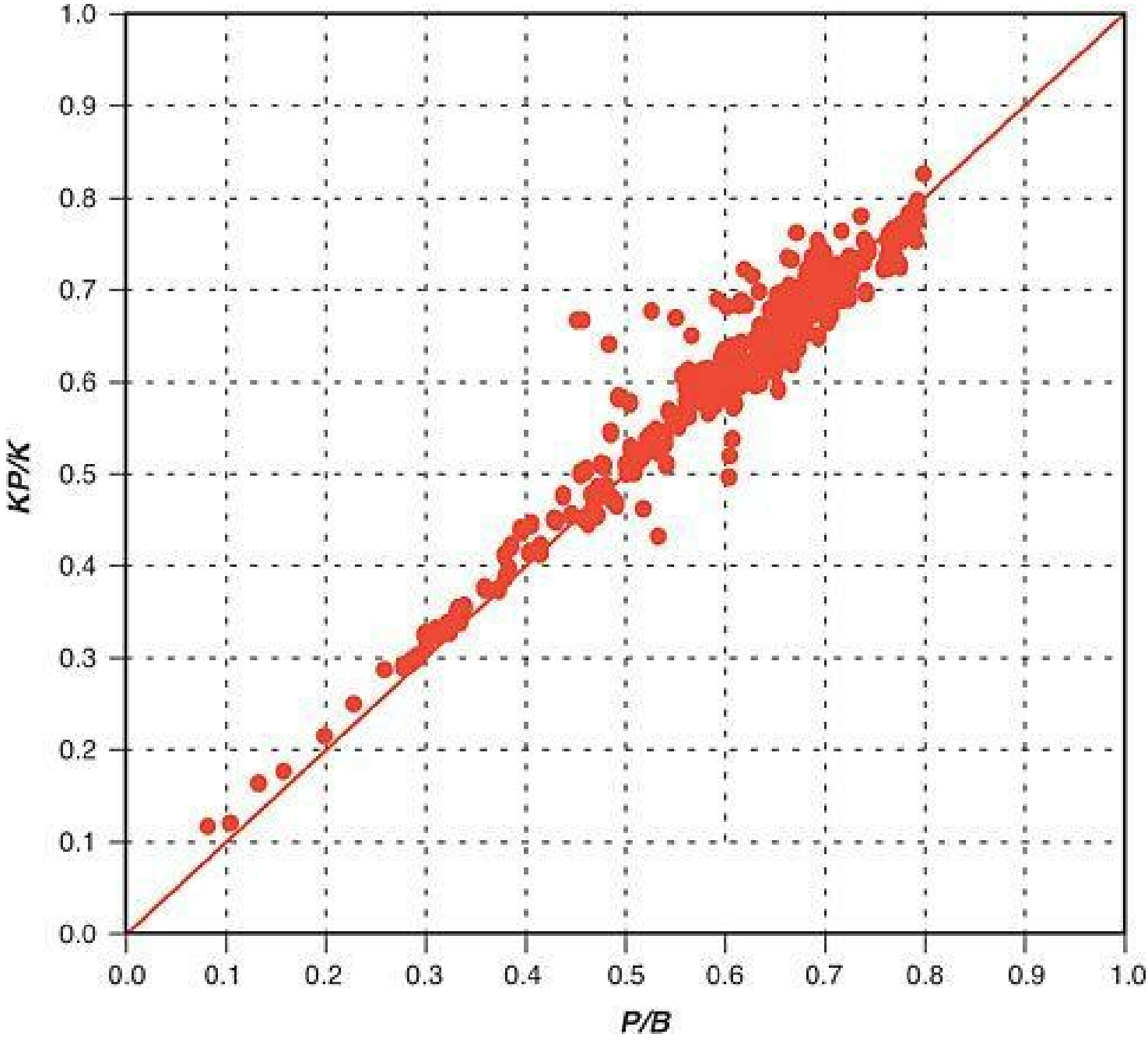
frequencies obtained in this example differ significantly from the equality assumed under the null hypothesis (p = 0.004%). The chi-square test also confirmed the statistical significance of the deviation of observed frequencies from the frequencies you would expect, should the criterion prove to be ineffective in solving the problem of strategy selection (χ^2 = 16.7, p = 0.037%). Hence, just as in the first example, nonparametric tests provide the proof that $k > 1$ appeared more frequently than $k < 1$ not due to random chance.

Finally, we examine the data according to the third method based on the regression analysis. Here we face a

contradictory result: The slope of the regression line almost equals 1. This is easily recognizable in Figure 5.2.8 showing the relationship between *KP/K* and *P/B* in which all points are scattered along the line with the slope coefficient equivalent to 1. It means that, in contrast to the first example, this technique did not prove the effectiveness of the criterion in selecting combinations, for which the short straddle strategy is superior to the long straddle strategy.

Figure 5.2.8. Regression relationship between KP/K and P/B. The short straddle

strategy is compared to the long straddle strategy according to the “expected profit on the basis of lognormal distribution” criterion. The thin line represents the regression line with the slope coefficient of 1.



Summarizing the results of the three analysis techniques applied to two sets of market data, we note that only in one case all three methods confirmed the effectiveness of the criterion in solving

the problem of option strategy selection. In the second case the evidence of the criterion effectiveness was obtained only from application of two techniques, whereas the remaining one showed negative results. Nevertheless, the second case also complies with the empirical rule we proposed: having two confirmations out of three is enough to make a reliable conclusion concerning the criterion eligibility. At the same time we have to emphasize that the same criterion may become useless if applied improperly (to inappropriate pairs of strategies, for example). Therefore, we strongly recommend that investors avoid using newly designed criteria in real trading until they are tested according to

all three techniques for each pair of potential strategies.

5.2.3 Generalized Ranking Analysis and Introduction of the Threshold Parameter

The technique of ranking analysis developed in the previous section is based on the distribution of the initial set of stocks to various subsets and consequent calculation of the coefficient of criterion effectiveness according to equation 5.2.2. Distribution among different subsets is realized according to ranks, which are determined by the

criterion and realized profit values. Illustrating this procedure in Table 5.2.1, we assigned rank *a* to a certain stock according to the criterion (or realized profit) if the value of this criterion (or the value of realized profit) for the given strategy was higher than the corresponding values for an alternative strategy. Formally, a certain stock belongs to subset **K** if $Criterion_1 – Criterion_2 > threshold$, where $Criterion_1$ is the criterion value for the strategy we evaluate, $Criterion_2$ is the criterion value for the alternative strategy, and *threshold* is the threshold value. Similarly, we refer a stock to subset **P** in the same way with the only difference that the criterion values for

each strategy are replaced with realized profit values: $Profit_1 - Profit_2 > threshold$. Thus, we introduce an additional parameter to our valuation system, which is the *threshold*.

In the two examples described we assumed that *threshold* = 0 implying that any difference in the criterion values (or realized profit values) is sufficient to assign a certain rank to a stock and refer it to a corresponding subset. We ignored the fact that the difference can be negligible. As a result, the marginal situation can arise in which a stock is nominally placed in a certain subset while there is little ground justifying such a decision. A stock with a BEAV ticker can be taken as an example

illustrating such a situation. It was assigned rank *a* (according to the criterion) despite the difference between the criterion values for the two alternative strategies was only 0.1% (Table 5.2.1).

It is evident that the zero threshold value used in the two examples previously discussed was only a particular case of a more general approach when the threshold may take on any value. In actual fact, we are not obliged to use such a low threshold value. It would be more appropriate to use a higher threshold. Indeed, if the criterion values estimated for the two alternative strategies are very close to each other, such criterion cannot determine which of

them is best (because the criterion considers both strategies to be similar with regard to potential profit). Below we demonstrate that the maximum effectiveness of the criterion is achieved when the threshold value differs from zero. However, before proceeding to this subject, we must make an important technical specification.

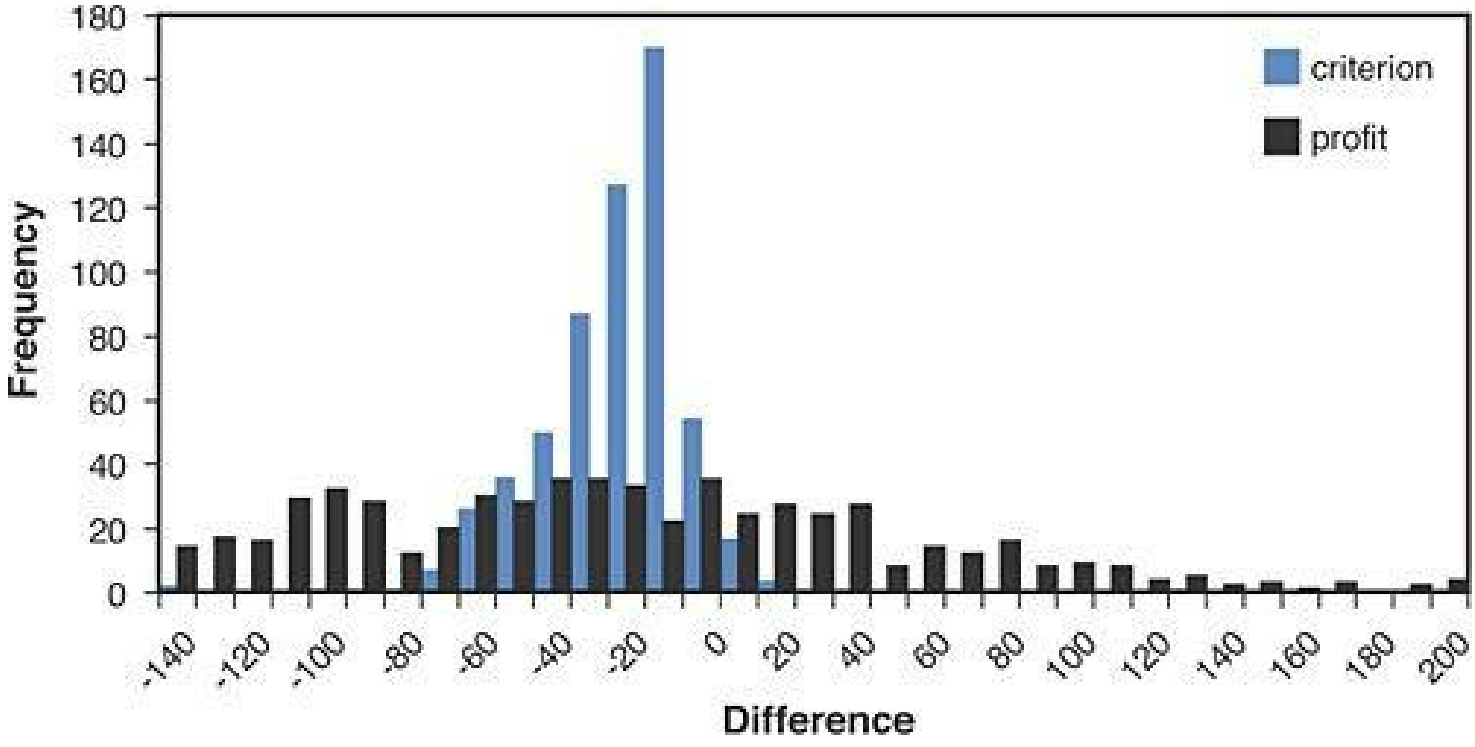
The necessity of a specification arises from the fact that distribution of the differences between criterion values ($Criterion_1 - Criterion_2$) differs substantially from the distribution of the differences between realized profits ($Profit_1 - Profit_2$). In the first case the variability of the difference is

negligible, whereas in the second case it is considerable (Figure 5.2.9). It means that most criterion differences do not differ much from zero, whereas many profit differences have extremely low or high values (Figure 5.2.9). To mitigate this effect and universalize threshold values making them equally suitable for both the criterion and the realized profits, they have to be normalized by the coefficient expressing the variability of corresponding differences. Because the generally accepted measure of variability is the standard deviation, we recommend using it as a normalization coefficient. Application of this normalization results in the stock being attributed to subset K if ($Criterion_1$ –

$Criterion_2$) / σ_c > threshold, where σ_c is a standard deviation of $Criterion_1$ – $Criterion_2$ difference. Similarly, a stock is attributed to subset P if ($Profit_1$ – $Profit_2$) / σ_c > threshold, where σ_c *is* the standard deviation of $Profit_1$ – $Profit_2$ difference.

Figure 5.2.9. Comparison of the frequency distribution of differences between the criterion values with the frequency distribution of differences between the

realized profit values. The data correspond to the example where the long straddle is compared with the short straddle strategy; the criterion is “expected profit on the basis of lognormal distribution.”



5.2.4 Results of Generalized Ranking Analysis

We begin with testing the hypothesis stating that the threshold values different from zero raise the predictive power of the criterion. Values of the coefficient of criterion effectiveness (k) will be calculated for the same pairs of

strategies that have already been examined. However, this time the coefficients of effectiveness will be determined for the whole range of *threshold* values.

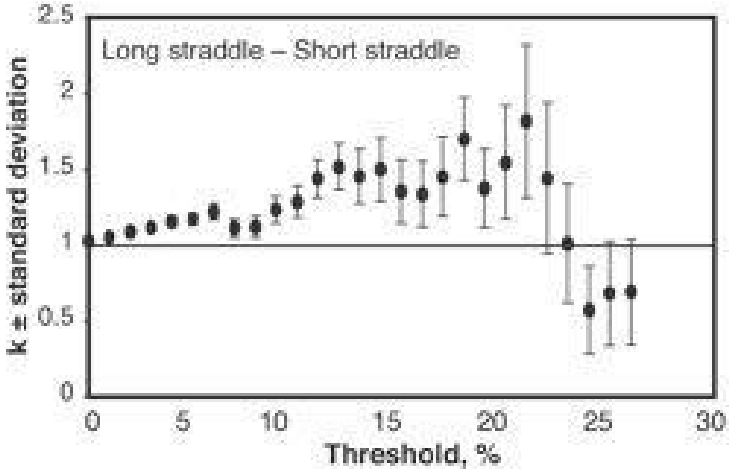
Indeed, coefficients of criterion effectiveness are seriously affected by the *threshold* values. (Some examples are presented in Figure 5.2.10.) The comparison of the long straddle and the short straddle strategies shows that *k* rises as *threshold* rises. However, when the *threshold* values reach a certain level, *k* begins to decrease and finally falls below 1 where the criterion has no predictive power (the top-left diagram in Figure 5.2.10). This is quite a surprising result. We supposed that the

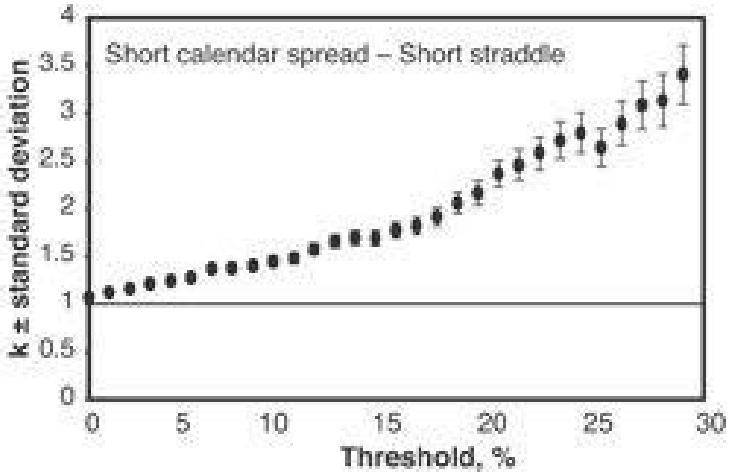
increase of the *threshold* value will raise the criterion predictive power but did not expect that extremely high thresholds would induce an opposite trend. The comparison of the other pairs of strategies indicates that such a pattern does not always exist (Figure 5.2.10). Therefore, it is important in each situation to determine the specific form of relationship between *k* and *threshold*. Otherwise, there is a serious risk that the *threshold* parameter would not enhance the predictive power of the criterion but would rather decrease it. Nevertheless, an evident peak in *k* values allows determining the optimal threshold value for application of this criterion to this particular pair of strategies (21%, see

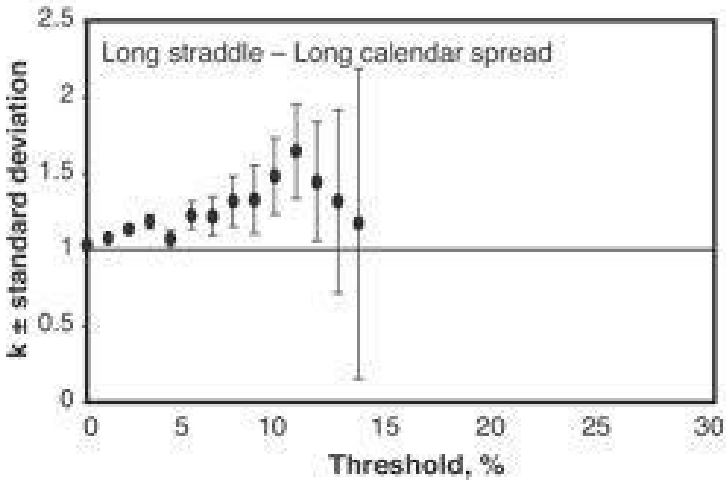
the top-left diagram in Figure 5.2.10). The increase in the *threshold* value also entails an increase in variability of the coefficients of criterion effectiveness (expressed by standard deviation). It can be explained by the fact that application of the *threshold* decreases the size of subsets **K** and **P** and, consequently, the size of their intersection (subset **KP**) also decreases. As a result, the higher the threshold value, the more variable *k* value becomes until we reach some critical *threshold* value when the calculation of *k* becomes meaningless due to data depletion.

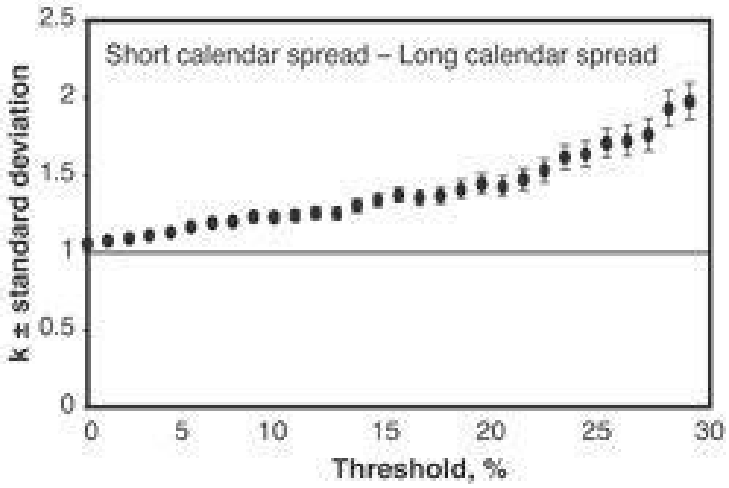
Figure 5.2.10. Relationship

between the coefficient of criterion effectiveness (average k ± standard deviation) and the threshold value for four pairs of alternative strategies.









Similar trends are observed when the long straddle and the long calendar spread strategies are compared (the bottom-left diagram in Figure 5.2.10). Initially, *k* value increases as the *threshold* increases; it reaches the maximum when the threshold value is

10% and afterward sharply decreases. As in the previous example, a strong increase of variability is observed at high threshold levels (due to depletion of **K** and **P** subsets). In this case, the rise of variability is more significant than in the previous example; the calculation of *k* value becomes impossible as soon as the threshold value reaches 13%.

A fundamentally different situation appears when the short calendar spread strategy is compared with the short straddle and the long calendar spread strategies (the top- and bottom-right diagrams of Figure 5.2.10). In these cases the increase of *threshold* value causes permanent and monotonous growth of the criterion effectiveness

coefficient. There is no evident peak in *k* values, and variability does not increase as rapidly as it did in previous examples. Nevertheless, in these cases the depletion of **K** and **P** sets also takes place, which decreases the degree of portfolio diversification. Therefore, even in the cases when *k* constantly increases, it would be preferable to select some intermediate value for the *threshold*. This choice should be a compromise between the intention to maximize the criterion effectiveness coefficient (which is one of the utility functions) and, on the other hand, to minimize the decrease in portfolio diversification (which is another utility function). Hereafter we return to this

issue and show how to use *k*, the number of underlying assets, and other utility functions simultaneously to determine the optimal threshold value.

5.2.5 Maximum Obtainable Values of the Criterion Effectiveness Coefficient

In the previous section we developed the coefficient expressing criterion effectiveness (*k*). It was postulated that the further *k* exceeds 1, the higher the criterion effectiveness is. However, *k* cannot be infinitely high. Its value in each case is limited by a number of factors. Therefore, to evaluate the

quality of the criterion, we should not only estimate whether $k > 1$, but also determine how close k is to its maximum obtainable value. The less the real k value differs from its theoretical maximum, the more effectively the criterion performs.

Using equation 5.2.2 we can determine the algorithm for calculation of the maximum obtainable value for the coefficient of criterion effectiveness k_{max}. Consider the following reasoning. The coefficient value is high when the criterion selects the best option strategy correctly. In the extreme case when the criterion gives faultless performance, KP is equivalent to K. However, it is

possible only if $P \geq K$. We have to consider both cases: $P \geq K$ *and* $P<K$. In the first case the maximum obtainable KP value is equivalent to K, while in the second case it is equivalent to P. Substituting these equalities in equation 5.2.2 we get:

(5.2.4)

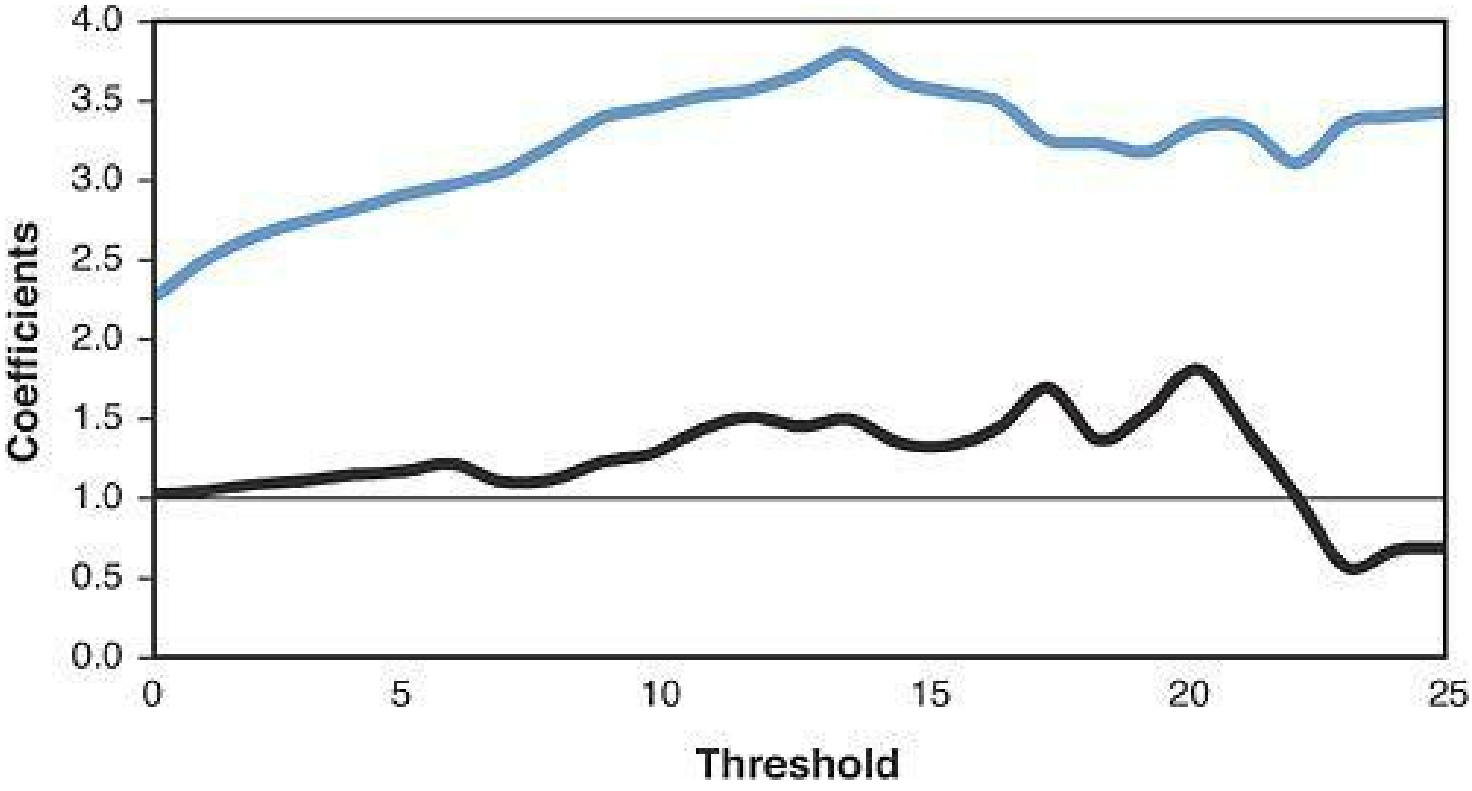
$$for\ P \geq K \quad k_{max} = B/P, \qquad for\ P < K \quad k_{max} = B/K.$$

Using this formula we can calculate the maximum obtainable values for the coefficients of criterion effectiveness and compare them with the real values obtained in the studies described in the previous sections. It would be

interesting to analyze the effect of the *threshold* on these two variables. As an example we consider one pair of strategies: long straddle—short straddle. Not surprisingly k_{max} is greater than k in all the cases (Figure 5.2.11). However, the relationship between each of these two variables and the *threshold* is different. k_{max} reaches its maximum when the *threshold* value is 14%, whereas k peaks when the *threshold* value is 21%. The points of their minimums also do not match. At the same time, the general shape of both relationships is similar. Starting from low values, k and k_{max} increase up to their respective maximums, which are then followed by smooth decreases

(Figure 5.2.11).

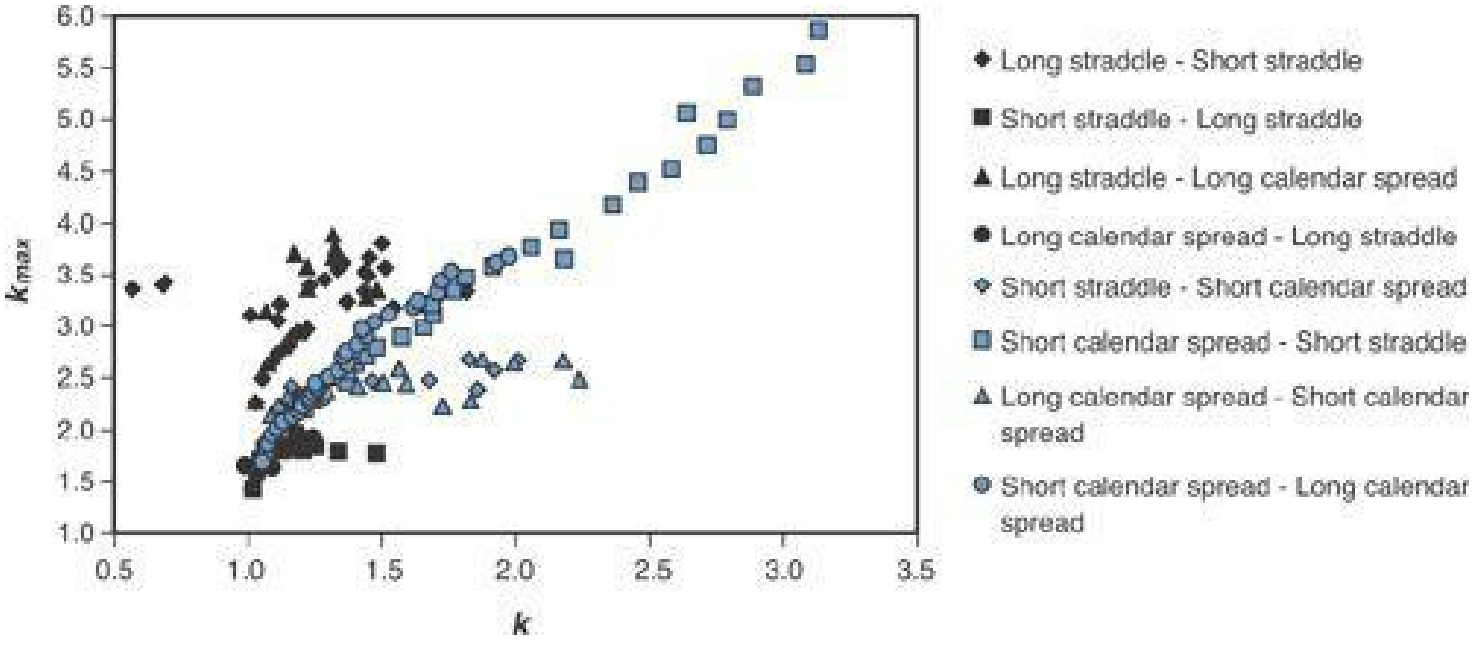
Figure 5.2.11. Fluctuations of the criterion effectiveness coefficient *k* (the black line) and its maximum obtainable value k_{max} (the gray line) as threshold changes. The data corresponds to the example where the long straddle strategy is compared with the short straddle strategy.



Despite the divergence of the extreme values of the two coefficients, it seems that for most *threshold* values greater k values correspond to greater k_{max}. To check this hypothesis, we analyzed the relationships between k_{max} and k for all the pairs of strategies. Each point on the chart representing these relationships (Figure 5.2.12) reflects a certain

threshold value for each pair of strategies. Positive relationships between the criterion effectiveness coefficient and its maximum obtainable value were detected for almost all the pairs of strategies (Figure 5.2.12). It is not evident only in the case of comparing the short straddle with the long straddle strategy. (Probably because in this case the values of both coefficients lie within a very narrow range.) In general, the greater the k value, the greater its maximum obtainable value can be (Figure 5.2.12). Our next study concentrates on deviations of k from k_{max} and on the relationships between these deviations and absolute k values.

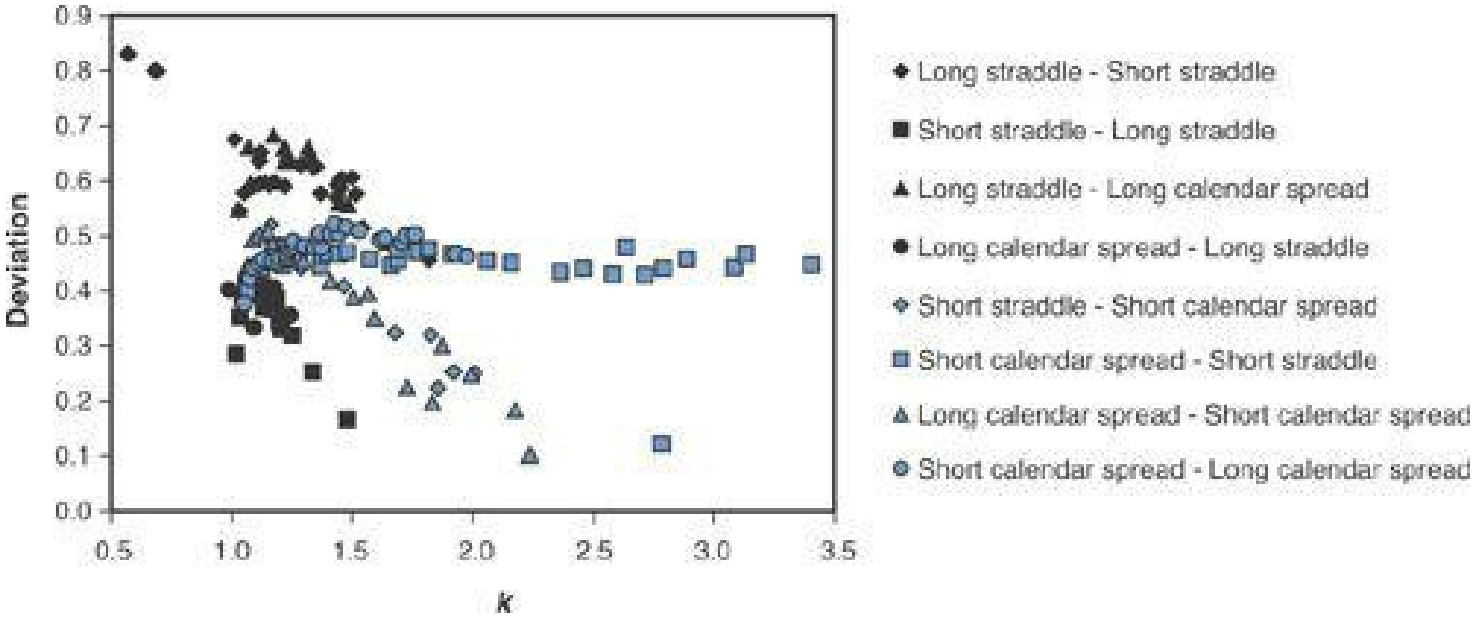
Figure 5.2.12. Relationship between the maximum obtainable value of the criterion effectiveness coefficient (k_{max}) and the actual criterion effectiveness coefficient *(k)* observed for different pairs of strategies.



To measure the deviation of k from k_{max} we use the $(k_{max} - k)/k_{max}$ ratio. This allows expressing the deviations in fractions thereby ensuring their comparability. The relationship between the deviation and the absolute k value will be considered in the coordinate system in which each point represents a certain *threshold* value within a given pair of strategies. Considering the aggregated data (that is, for all the pairs

of strategies and all the *threshold* values) a strong negative relationship is easily detected; the higher the effectiveness coefficient, the lower its deviation from its maximum obtainable value (Figure 5.2.13). The same holds true for most pairs of strategies if considered separately (Figure 5.2.13).

Figure 5.2.13. Relationship between deviation [$(k_{max} - k)/k_{max}$ ratio] and absolute k value for different pairs of strategies.



Thus, increasing *k* by manipulation of the *threshold* values not only improves the criterion forecasting power but also approaches the limits of its predictive force. In other words, the potential contained in the specific criterion is utilized to a much greater extent.

It may seem that regularities established in this section are of theoretical rather than of practical interest. This is not true because the deviations of the coefficient

of criterion effectiveness from its maximum obtainable values will be employed further as another utility function for optimization of *threshold* value. Selection of threshold is of great practical importance because it directly influences trading profitability. (This is demonstrated in the following sections.)

5.3 Traditional Methods of Evaluating the Criterion Effectiveness

Ranking analysis proved the effectiveness of one criterion in selecting option strategies. Analyzing

various aspects of ranking analysis, we avoided expressing results in the form of profit amount. However, because this method is based on the calculation of an abstract coefficient, it provides only an indirect expression of the criterion effectiveness. Nevertheless, we hope that the reasons stated at the beginning of this chapter convinced you that this approach, intended to mitigate the effects of various complicating factors, represents an integral part of the criteria effectiveness evaluation process.

Now it is time to supplement the results of ranking analysis with the direct proofs of the effectiveness of the criteria in solving the problem of option strategies selection. This can be done by

application of traditional evaluation methods. In addition to checking the forecasting properties of criteria, this can help us to derive additional utility functions for *threshold* value optimization.

Certainly, direct evidence of criteria effectiveness comes from comparison of profits of combinations selected by criteria with profits of combinations selected using other principles. In ranking analysis we used two combinations (one for each strategy) for each stock and each date. Stocks, for which the value of the criterion for a combination constructed under strategy 1 is higher than the value of the criterion for a combination constructed under

strategy 2, were considered to belong to subset **K**. Consequently, the average profit of combinations (expressed as a percentage of margin requirements) constituting subset **K** will be a straight and intuitively understandable indicator of the criterion effectiveness. To make it even more convincing we will also compute the average profit for combinations constructed under the alternative strategy 2 for the stocks belonging to subset **K**. This demonstrates what would have happened if we had acted conversely to the criterion recommendations (that is, applied alternative strategies instead of those selected by the criterion). Comparison of these average profits confirm or

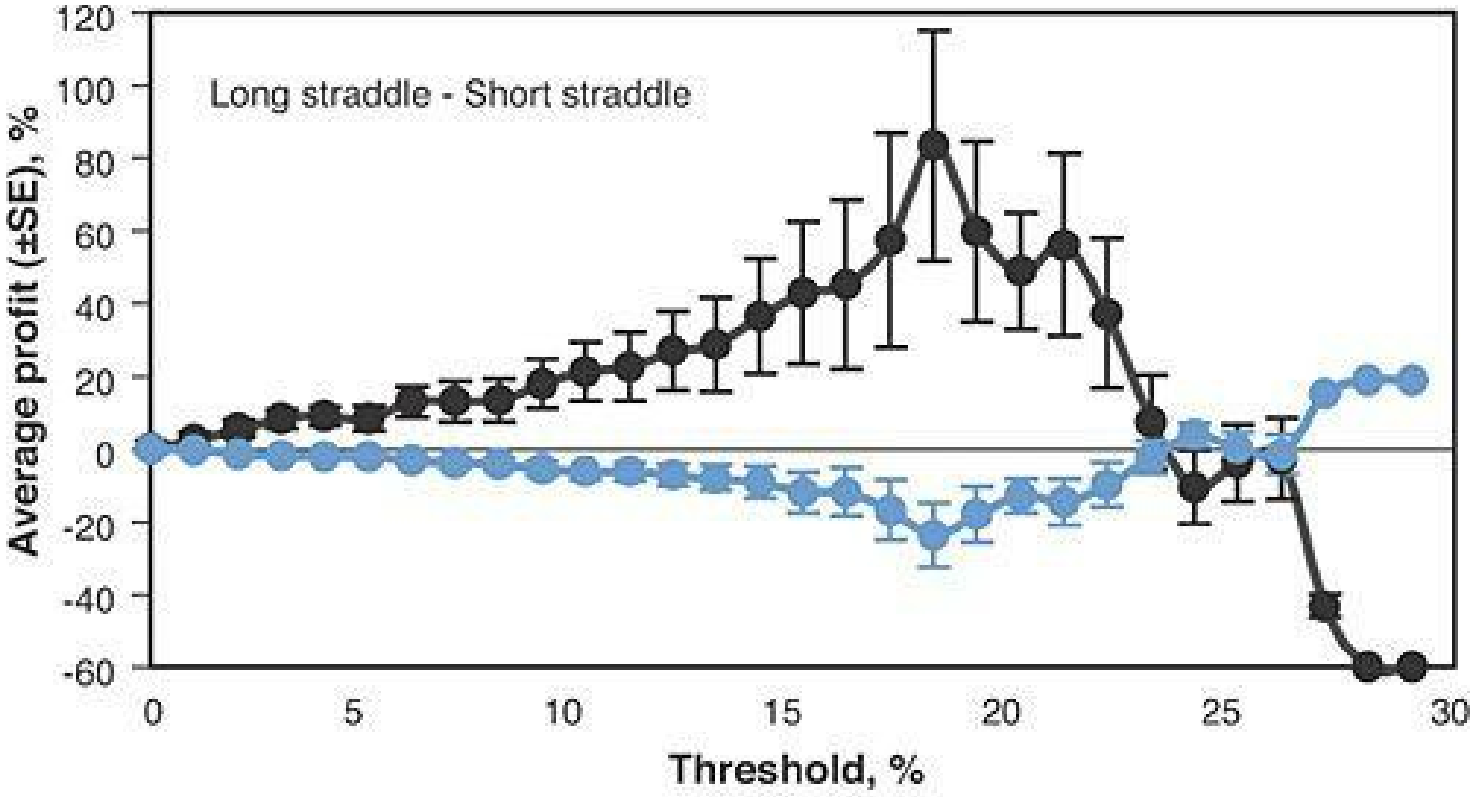
disprove the criterion applicability without employing the indirect concept of ranking analysis.

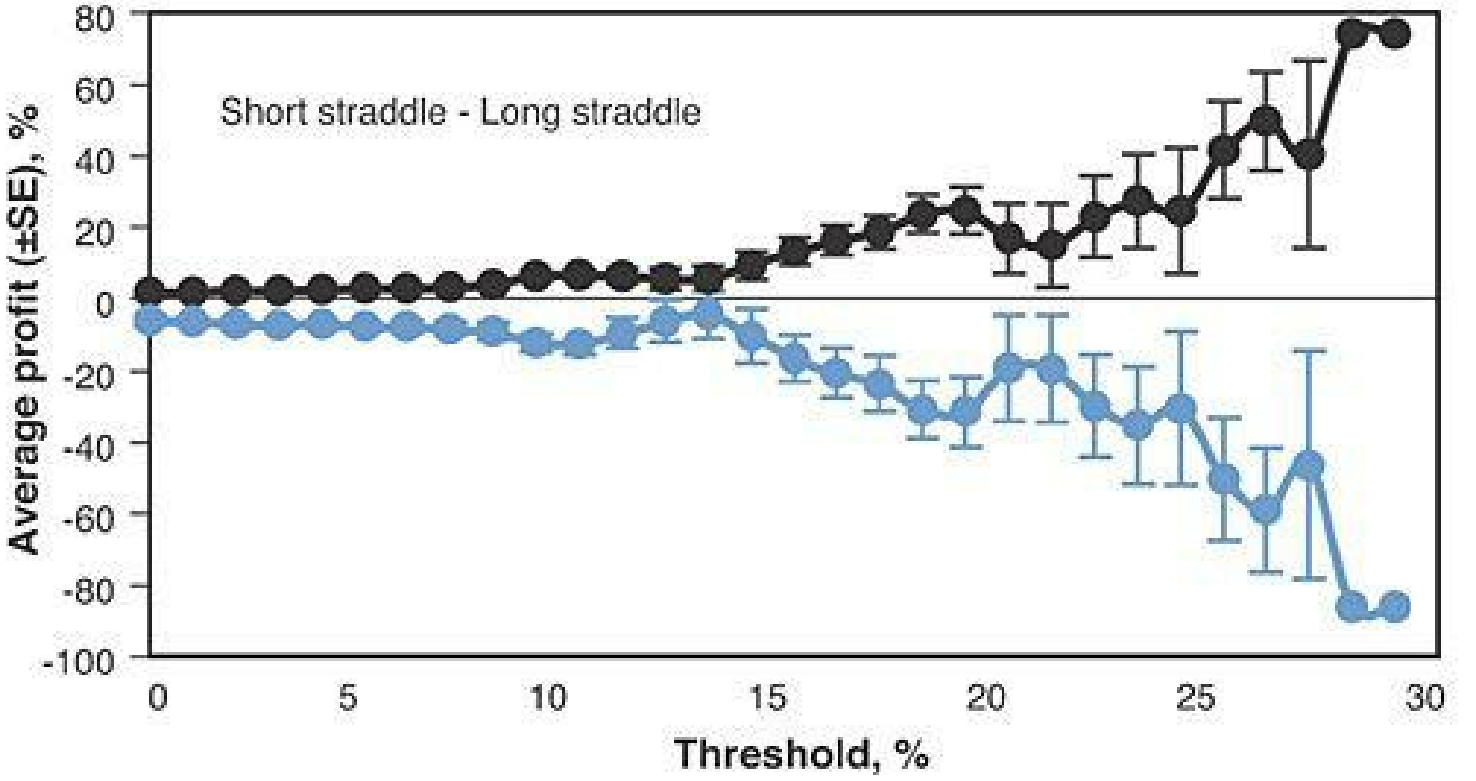
We begin with comparing the long straddle and the short straddle strategies. The upper line on Figure 5.3.1 (a) shows the average profit for combinations belonging to subset **K** (that is, created under the long straddle strategy). The lower line shows profit that would be realized if the alternative strategy (that is, rejected by the criterion, in this case —the short straddle one) was applied to subset **K**. It is obvious that the strategy selected by the criterion dominates over the rejected one. The higher the threshold value, the greater the difference between the profits of

alternative strategies (Figure 5.3.1). The maximum difference between profits corresponds to the *threshold* value of 18%. Right up to the *threshold* value of 23%, the selected strategy shows stable positive profit, and the rejected one demonstrates stable unprofitability. The situation gets inverse when the *threshold* values become higher. This anomaly is caused by depletion of subset **K** (less than four combinations on average in this case).

Figure 5.3.1. Relationship between average profit (± standard error) and

threshold value for two pairs of alternative strategies. Black symbols denote profit of the selected strategy; blue symbols denote profit of the rejected strategy.


Long straddle - Short straddle
Average profit (±SE), %
120
100
80
60
40
20
0
-20
-40
-60
0
5
10
15
20
25
30
Threshold, %


Short straddle - Long straddle
Average profit (±SE), %
80
60
40
20
0
-20
-40
-60
-80
-100
0
5
10
15
20
25
30
Threshold, %

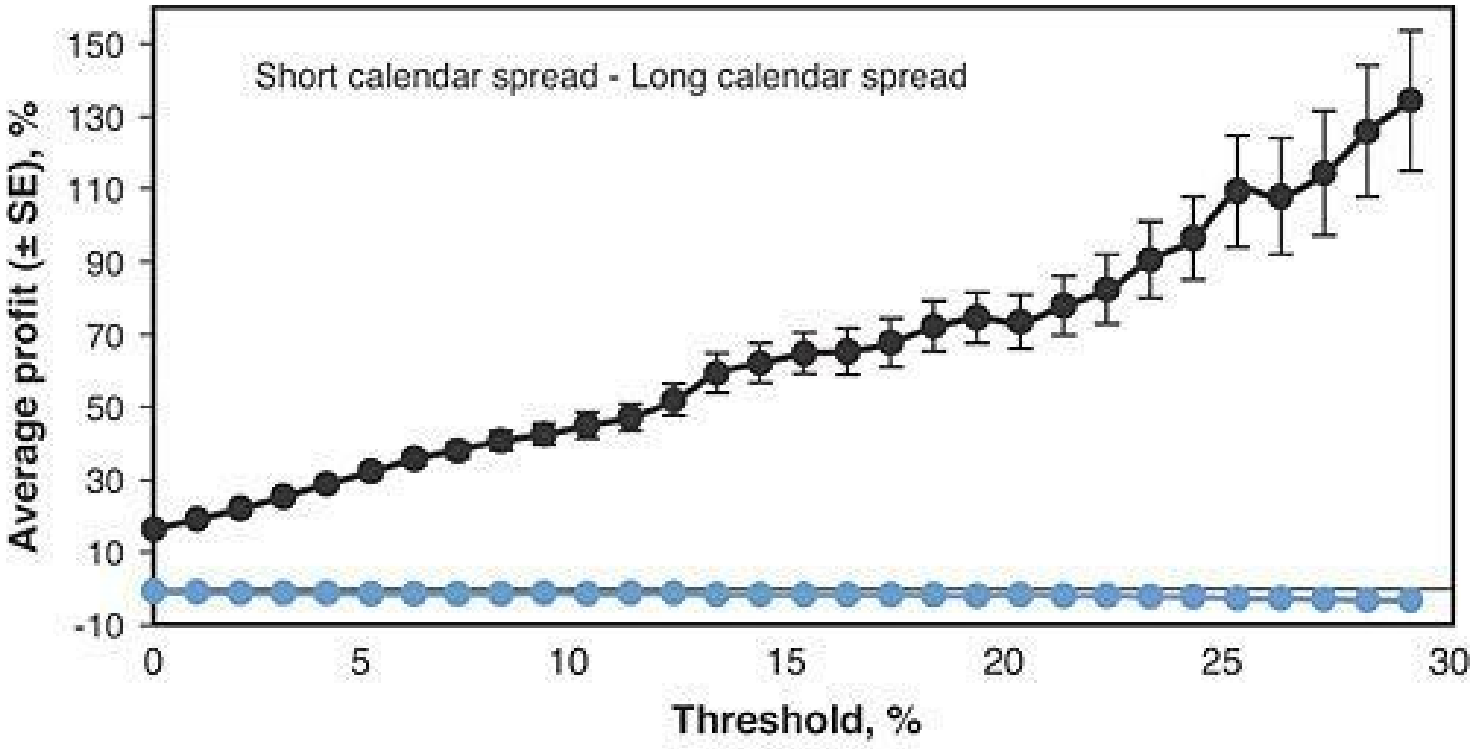
Qualitatively the same results are obtained when the short straddle strategy is compared with the long straddle strategy. (Except that up to a 13% *threshold* value, the differences in profits are not so pronounced [Figure 5.3.1].) When the *threshold* values are high, the profits of the selected strategy strongly exceed those of the rejected strategy, and this trend does not change as in the previous example (Figure 5.3.1).

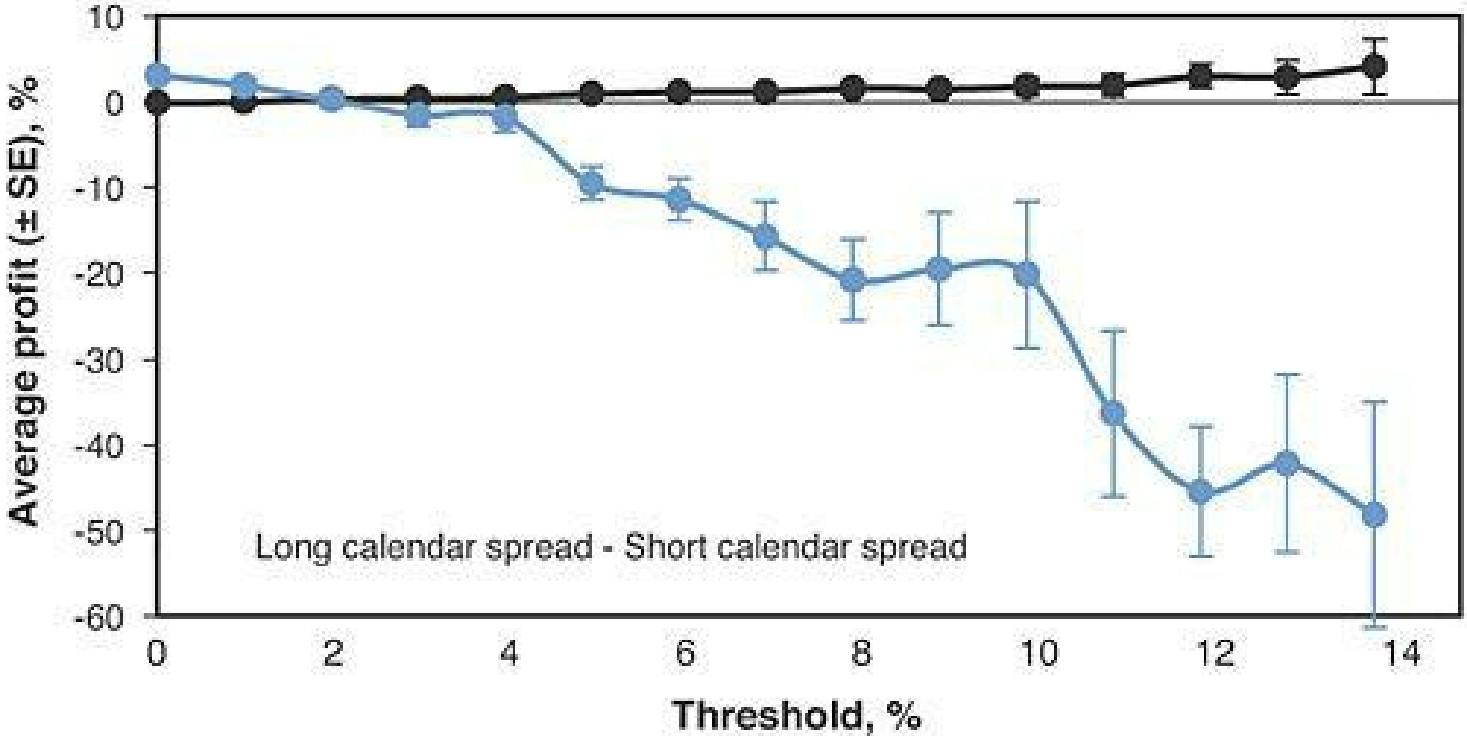
The next example is dedicated to another pair of option strategies—the long and the short calendar spreads. In this case the results are even more unambiguous; the profit of combinations for which the short calendar spread strategy is

estimated by the criterion as being preferable to the long calendar spread is always positive and grows linearly as the *threshold* value increases (Figure 5.3.2). Moreover, the maximum threshold value of 29% gives a high profit of 135%, which means that the *threshold* significantly influences profit potential. The rejected strategy shows constant unprofitability (Figure 5.3.2), which proves the high effectiveness of the criterion for this pair of strategies.

Figure 5.3.2. Relationship between average profit (± standard error) and

threshold value for two pairs of alternative strategies. Black symbols denote profit of the selected strategy; blue symbols denote profit of the rejected strategy.





Finally, consider the inverse situation

when the same criterion selects combinations for which the "long calendar spread" is expected to perform better than the "short calendar spread." In this case, similar to the previous one, the selected strategy shows stable profit growth (losses of the rejected strategy also grow rapidly) as the *threshold* value increases (Figure 5.3.2). However, this pattern appears not for all threshold values but only for those starting with 3%. Up to 3% the rejected strategy has slightly higher profit, which indicates that low *threshold* values are not applicable in this case. Data depletion starts at 15% *threshold* value.

It is worth noting that the "short calendar spread" strategy demonstrates strong

profit growth when it is selected as the best one (the top line in Figure 5.3.2 [a]) and strong losses growth when it is rejected (the bottom line in Figure 5.3.2[b]). At the same time the long calendar spread strategy shows the opposite trends; both profits (Figure 5.3.2[b]) and losses (Figure 5.3.2[a]) are bounded by narrow value ranges. The reason for this is that different strategies have different profit/loss potentials and different margin requirements. This imposes strict constraints on the use of absolute profits for selection of option strategies. (This issue is discussed in the next section.)

The research presented in this section proves that average profit (and ranking

analysis previously described) can effectively reflect the effectiveness of the criterion in selecting the best option strategies. Similar to the coefficient of criterion effectiveness (k), the average profit depends on the value of *threshold* parameter. This provides two additional utility functions: average profit and the ratio of average profit to its variability (the Sharpe ratio). Together with the other utility functions obtained as a result of ranking analysis, these two functions optimize the *threshold* parameter.

5.4 Synthetic Approach to Criterion Effectiveness

Analysis

Two methods of criterion effectiveness analysis have been discussed in this chapter. One method is commonly adopted to solve various evaluation problems. The other method was developed especially to solve a specific problem (selection of option strategies) while taking into account the drawbacks and uncertainties of the standard approach.

The first method, based on the average profit and its variability, is applicable to financial instruments possessing linear payoff functions. However, it has

numerous drawbacks when applied to evaluation of complex option combinations with nonlinear payoff functions. In particular, profit potential of option strategies can either be limited or unlimited. When comparing two strategies by average profit, the strategy with an unlimited potential may be unprofitable for most combinations yet generate huge profits for a few of them. The average profit is then positive and can even be rather high. At the same time the second strategy with a limited profit potential can generate modest profit for all or the majority of combinations; still its average profit could be lower than that of the first strategy. Does this mean that the first strategy is preferable to the

second one? It is hard to answer in the affirmative. Although the first strategy might have higher average profit, it comes from a few outliers. Application of the Sharpe ratio can help because it reflects not only the profit average, but also its variability. Nevertheless, an unambiguous decision is unreachable if the first strategy is preferable according to profit and the second one is better according to the Sharpe ratio. This is just one, but not the only, problem so far. Additional difficulties arise from the difference in loss potential among different option strategies. If we add the adverse complications arising from differences in margin requirements (resulting in differences in the basis used

to calculate profitability of option strategies), the essential drawbacks of the traditional evaluation method and necessity of a complementary evaluation system become apparent.

Ranking analysis can be considered as a complementary evaluation system. In contrast to the common method, it handles frequencies of certain events putting aside absolute values of individual outcomes. Using the frequency of positive outcomes (regardless of their magnitude) makes it possible to get rid of the influence of rare extreme outliers. Besides, unlike the common method, ranking analysis does not only estimate the absolute profit, but indicates also to what extent the profit

potential is realized. This is achieved by way of estimation of a special coefficient measuring the proximity of the realized gain to its maximum obtainable value. At the same time, ranking analysis possesses a number of features that make it impossible to gain a full understanding of criterion capabilities in selecting option strategies.

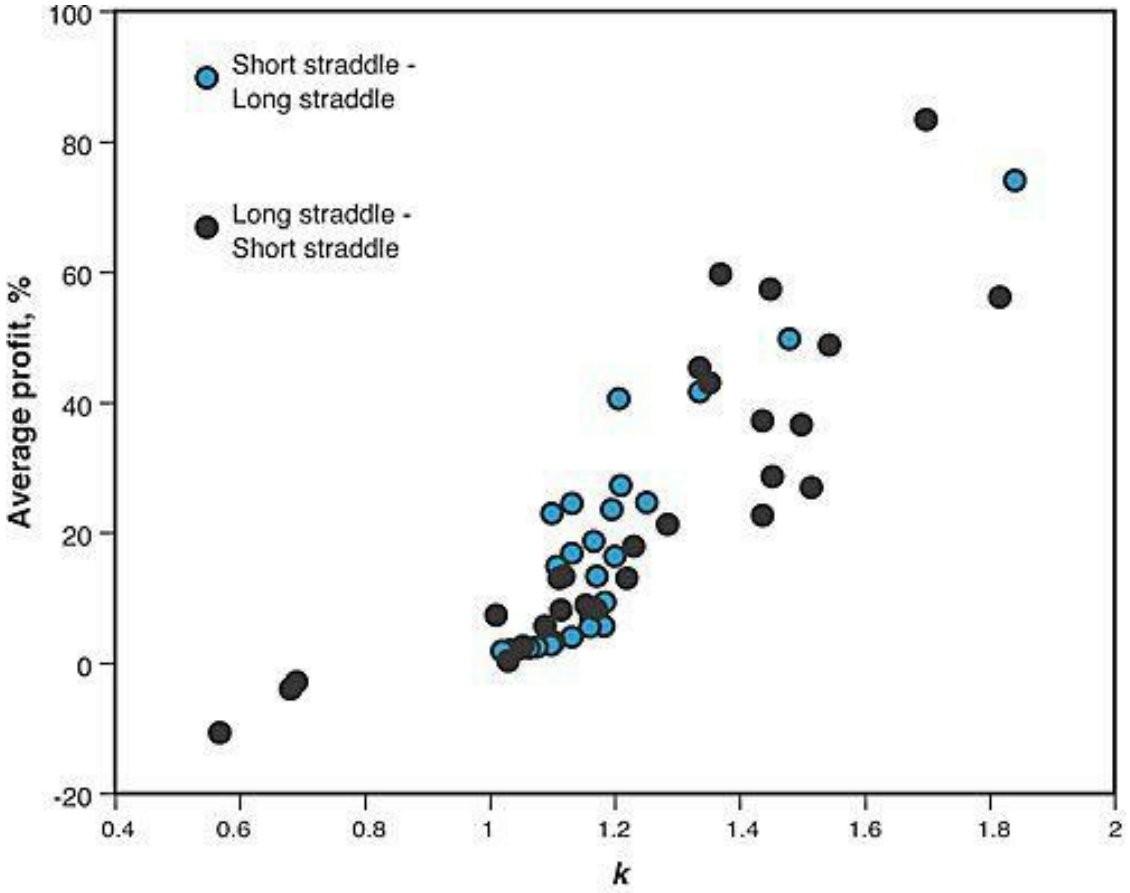
Therefore, the optimal solution would be to combine both methods. Because these methods are based on the processing of different items of information corresponding to the same objects (combinations, strategies, underlyings, their sets and subsets), they should be considered as complementary, and their

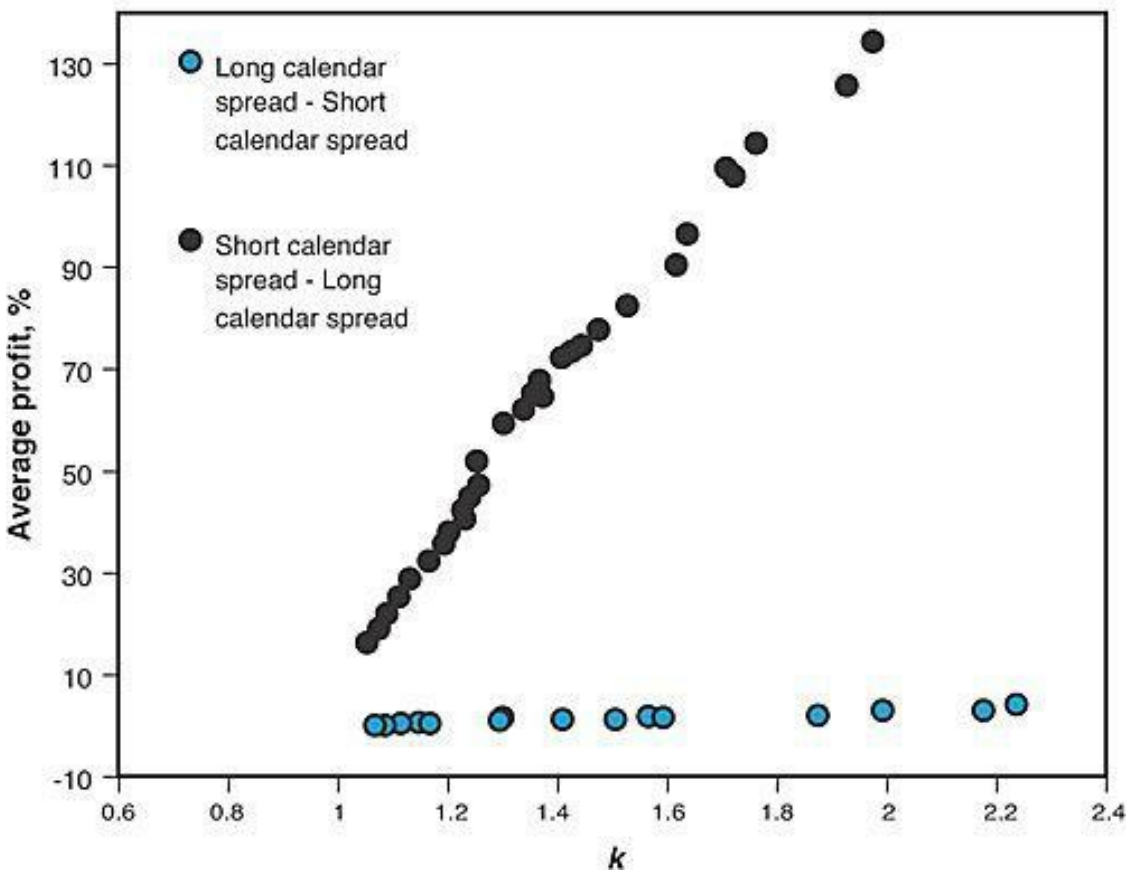
results might to a certain extent be similar. To understand the degree of this similarity, we analyze the interdependency of the two indicators (*k* and average profit).

Consider the correlation between the coefficient of criterion effectiveness and the average profit for two pairs of strategies examined in previous examples. Each point on the chart has an average profit and *k* as its coordinates and corresponds to a certain threshold value. As expected, comparison of the short and the long straddles reveals an interrelationship between average profit and *k* (Figure 5.4.1). The dispersion of points is wide in both cases: when comparing the first strategy to the second

one and vice versa. It means that these indicators can be considered as mutually complementary in reflecting the forecasting properties of a criterion. Therefore both indicators should be used when evaluating the criterion effectiveness.

Figure 5.4.1. Relationship between the average profit and the coefficient of criterion effectiveness (*k*) for four pairs of alternative strategies.

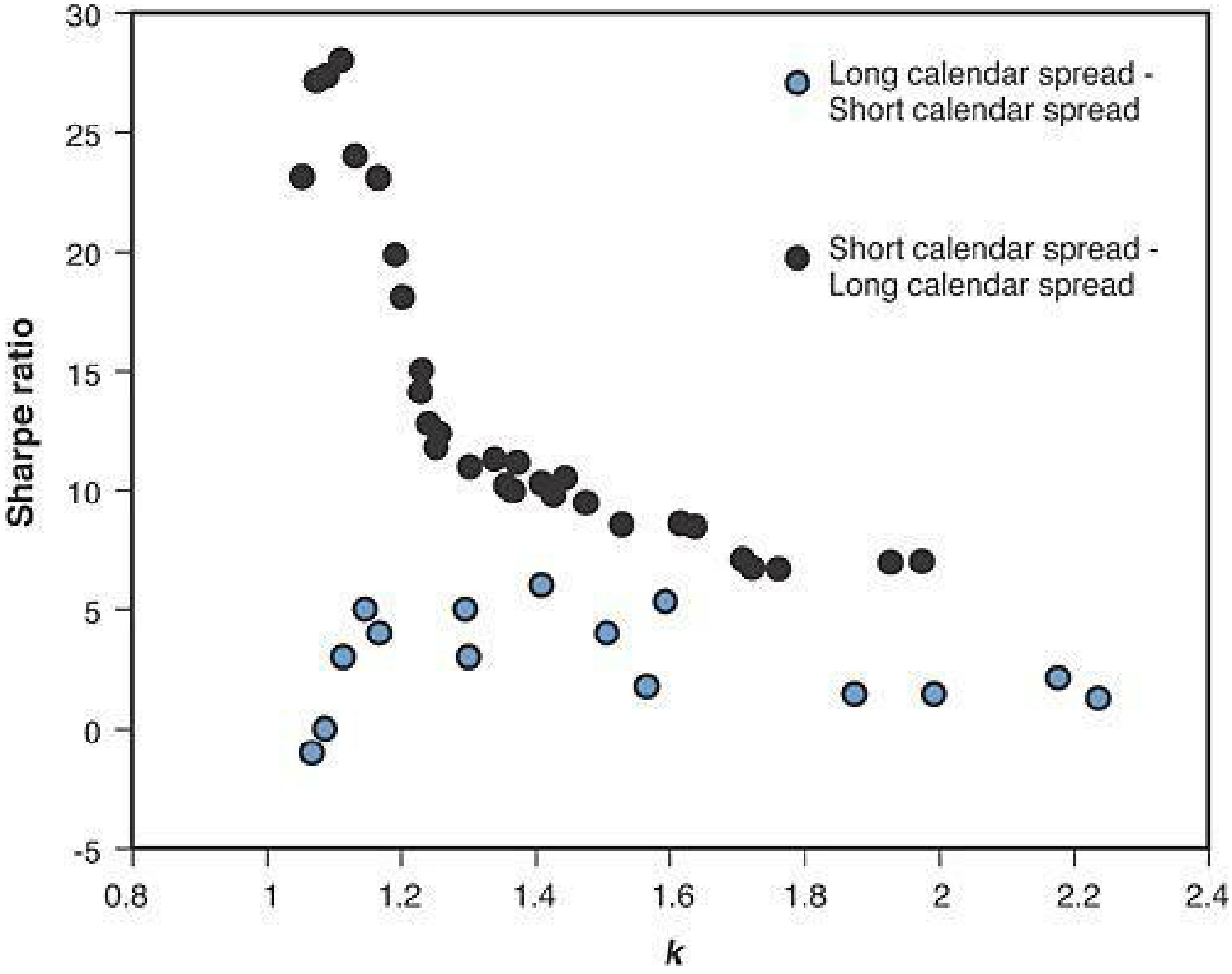

Short straddle - Long straddle
Long straddle - Short straddle
Average profit, %
100
80
60
40
20
0
-20
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
k


Long calendar spread - Short calendar spread
Short calendar spread - Long calendar spread
Average profit, %
130
110
90
70
50
30
10
-10
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
k

Comparison of calendar spreads demonstrates a much stronger relationship between the average profit and the coefficient of criterion effectiveness. This holds true in both cases: when comparing the short strategy with the long one and vice versa. Correlation coefficients are close to one, and almost all the points are situated along the straight lines (Figure 5.4.1). Such a strong correlation means that information contained in one of the indicators is duplicated in another one. At first glance, there is little sense in using both methods, and we can be content with only one of them. However, this conclusion could prove to be a

rather hasty one. Because profit is characterized not only by the average value, but also by variability (reflecting the risk and stability of the results), it would be worth looking at the relationship between *k* and the Sharpe ratio (which combines both the average and the variability of profit). Indeed, analyzing the relationship between two indicators brings about the contrary conclusion: There is an evident inverse relationship in one case and no correlation at all in the other case (Figure 5.4.2). It means that for this pair of strategies it would also be better to employ both methods of criterion effectiveness evaluation.

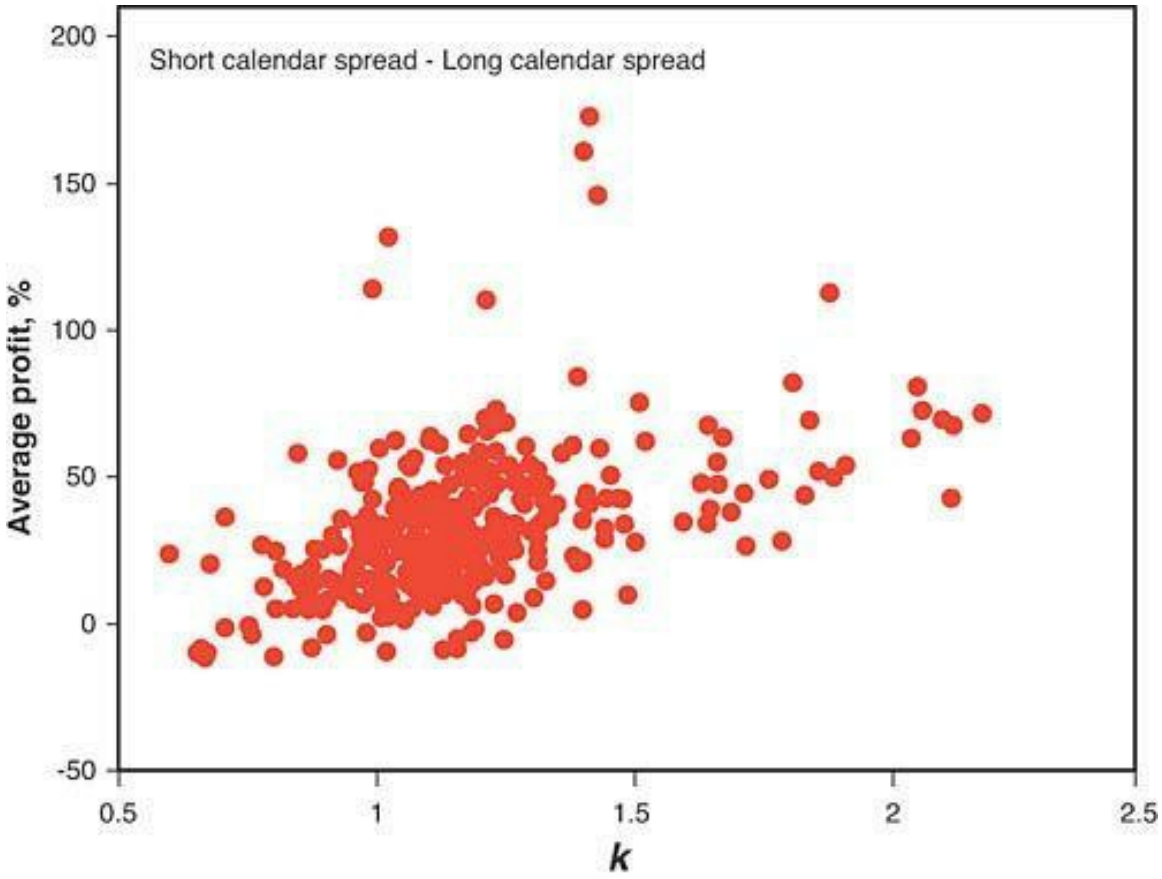
Figure 5.4.2. Relationship between the Sharpe ratio and the criterion effectiveness coefficient (*k*) for two pairs of alternative strategies.

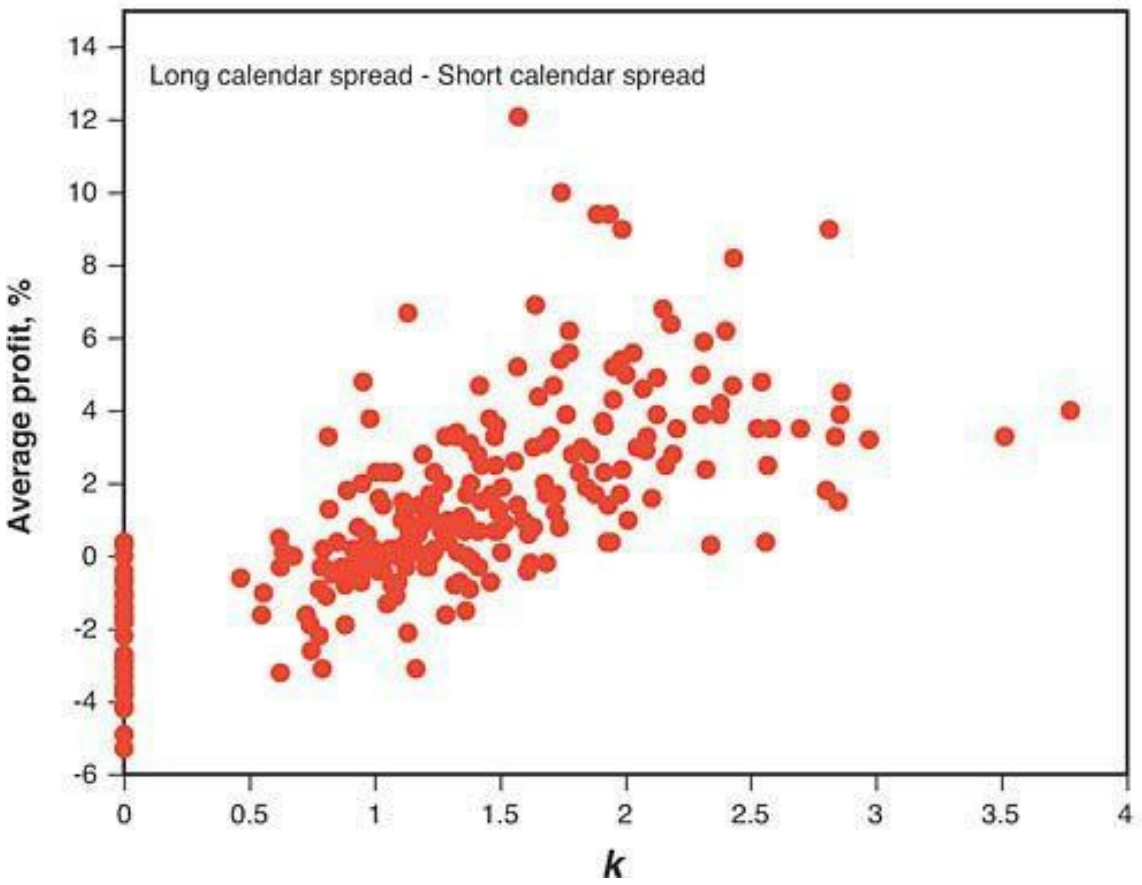


Additional insight into the degree and the shape of the interrelationship between two indicators of criterion effectiveness can be obtained by analyzing their correlation at some particular point on the *threshold* scale.

This can be illustrated by the comparison of the short and the long calendar spreads when the *threshold* value equals 5%. In this analysis every point corresponds to a certain date (in total, there are 347 points). Both comparisons (short with long spreads and long with short spreads) reveal direct relationship between two indicators. However, the correlations are far from perfect (Figure 5.4.3; $R^2 = 0.22$ in the first case and $R^2 = 0.48$ in the second case). High dispersion of points on Figure 5.4.3 provides additional confirmation that information contained in these two indicators overlaps only in part and is complementary rather than duplicating.

Figure 5.4.3. Relationship between the average profit and the criterion effectiveness coefficient (*k*) when the threshold value is 5%.


Short calendar spread - Long calendar spread
Average profit, %
200
150
100
50
0
-50
0.5
1
1.5
2
2.5
k


Long calendar spread - Short calendar spread
Average profit, %
14
12
10
8
6
4
2
0
-2
-4
-6
0
0.5
1
1.5
2
2.5
3
3.5
4
k

It is possible to produce a rigorous quantitative expression for the degree of overlapping of information contained in these two criterion effectiveness indicators. In statistics the determination coefficient (squared correlation coefficient) is interpreted as the percentage of the dependent variable variation explained by the independent variable. In our situation it is impossible to distinguish between the dependent and the independent variable because none of them influences each other, but rather they are affected by a number of additional factors that influence both variables simultaneously and cause their interdependence. Nevertheless it would

be correct to interpret the determination coefficient of the relationship between k and the average profit as a fraction of information about the criterion effectiveness contained in both indicators (that is, as the fraction of overlapping information). Hence, the fraction of new unreplicated information entering the evaluation system due to the use of an additional indicator can be expressed by the difference between 1 and the determination coefficient. The comparison of the short calendar spread with the long calendar spread gives us $R^2 = 0.22$. The inverse comparison gives $R^2 = 0.48$. This implies that the percentage of new information concerning the criterion forecasting

ability that can be obtained due to the application of two indicators is 78% in the first case and 52% in the second case.

To summarize, the analysis of criterion forecasting abilities in selection of option strategies requires application of both evaluation techniques. The synthetic, combined application of these two methods can be especially productive when they form the basis for construction of utility functions intended to estimate optimal parameter values. In the next section we develop a working model intended to find an optimal *threshold* value on the basis of simultaneous application of five utility functions.

5.5 The Model for Optimizing the Threshold Parameter

In this chapter we frequently reverted to the concept of *threshold* that strongly influences both the potential profit and criterion effectiveness. In this section we build a model intended for optimization of the *threshold* value on the basis of maximizing the system of several utility functions. The optimal threshold value may vary strongly between different utility functions. Furthermore, the maximum value of one

of the utility functions may coincide with the minimum value of another. Hence, our goal will be to develop an integral model allowing reduction of a number of utility functions to a single function that would have a sole easily distinguishable maximum. Such a model permits determining the optimal *threshold* value unambiguously.

Five utility functions develop the optimization model. Each of them is an empirically derived relationship between a certain indicator and the *threshold* values. The model includes utility functions based on the following indicators:

- The coefficient of criterion

effectiveness *(k)*

- k/k_{max} ratio (expresses the degree of proximity of actual k to its maximum obtainable value)
- Average profit
- Profit Sharpe ratio
- The number of combinations in subset **K** (expresses portfolio diversification)

The values of these functions are measured in different units and are scaled differently. Therefore, our first step is to reduce values of all five utility functions to a unified scale. This is done by subtracting the minimum value from each individual value and dividing the

result by the difference between the maximum and the minimum value. (This operation is executed for each utility function separately.) After this reduction the values of all five functions lie within the range from zero to one. The next step is to reduce the five utility functions to a single one. Two common methods will be used: additive and multiplicative convolution. When each parameter value corresponds to a constant number of utility function values (in our case, it is constant and equals 5), then additive convolution is equivalent to the arithmetic mean and multiplicative convolution—to the geometric mean (Chapter 7, "Basic Concepts of Multicriteria Selection as Applied to

Option Combinations," discusses different types of convolutions in detail.) We introduce an additional convolution type—developed specifically for the purpose of reducing many functions to a single one—the product of the maximum and the minimum values of the five utility functions corresponding to each threshold value. (It will be called the *minimax convolution*.) As shown here, this convolution produces the most unambiguous result.

We commence by building the optimization model intended to compare the long straddle and the short straddle strategies. To make it more illustrative, the graphs of all five normalized utility functions are shown in the common

coordinate system (Figure 5.5.1). It is evident that the shapes of different utility functions vary substantially and that their maximums and minimums do not coincide (that is, occur at different *threshold* values). Obviously, it is impossible to determine the optimal *threshold* value that maximizes all five utility functions by means of simple visual examination. This can be done only through application of different convolution techniques. We use three types of convolutions and compare their results. Additive and multiplicative convolutions are of irregular broken shape and do not produce the unambiguous result because they have several maximums (Figure 5.5.2). On the

contrary, the minimax convolution represents a unimodal function with the unique optimal *threshold* value of 4% (Figure 5.5.2).

Figure 5.5.1. Five utility functions built for optimization of the threshold parameter when the long straddle strategy is compared with the short straddle strategy.

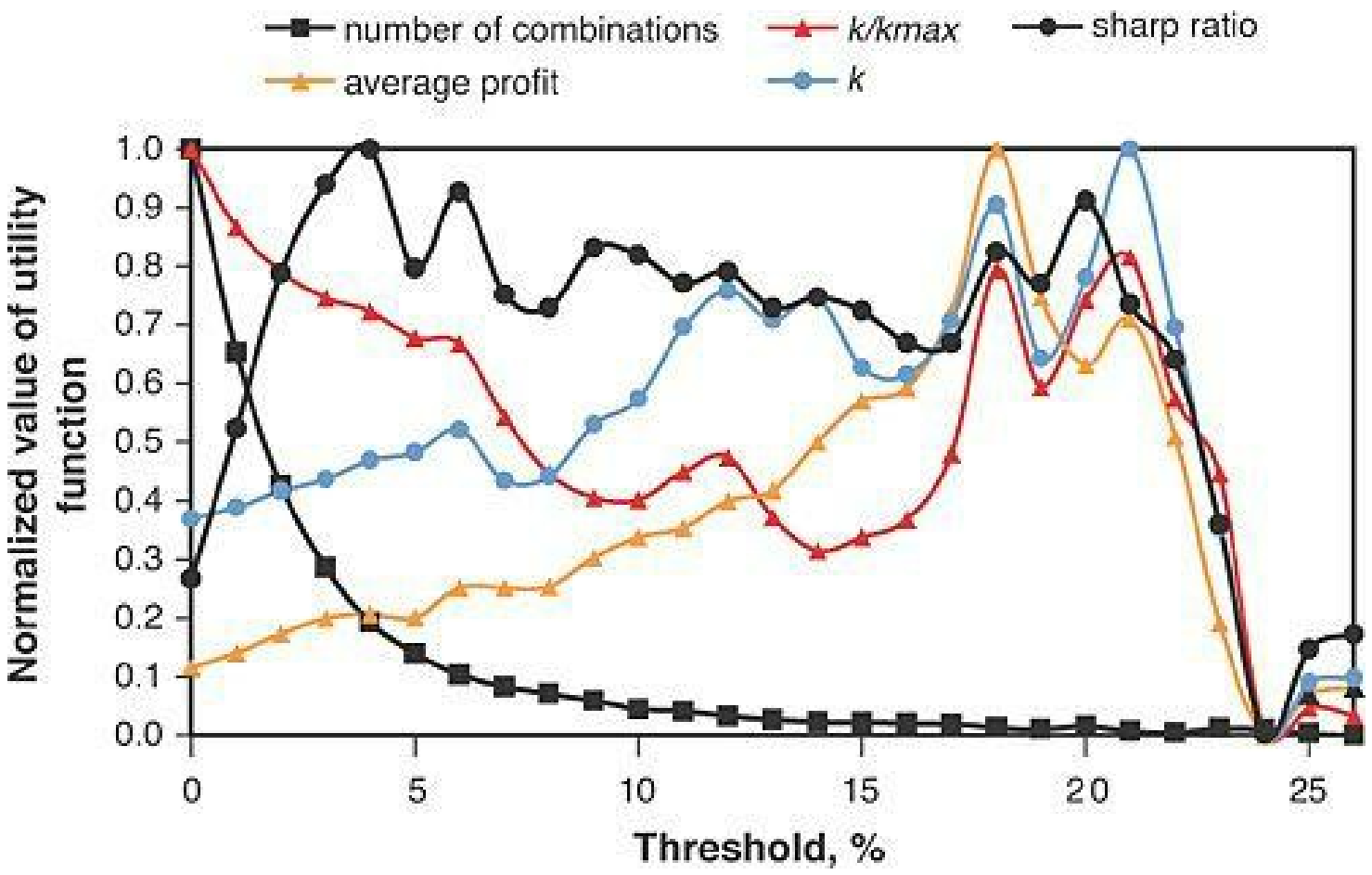
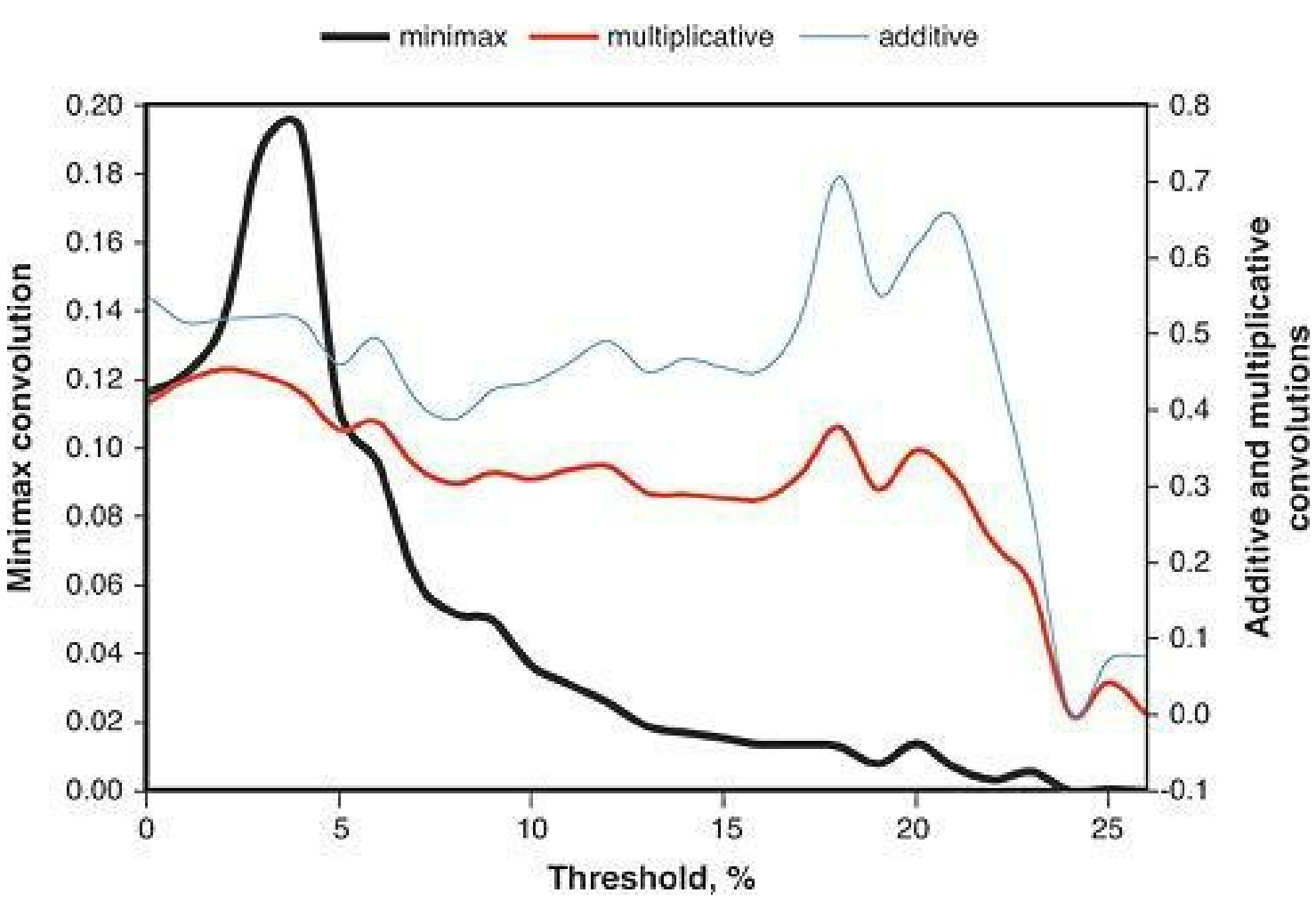


Figure 5.5.2. Three types of convolution of five utility functions shown in Figure 5.5.1.



When the short calendar spread strategy is compared with the long calendar spread strategy, the utility functions have completely different shapes. Three of five functions monotonically decrease, and the two others monotonically increase as the value of *threshold* increases (Figure 5.5.3). Again it is

impossible to determine visually the optimal *threshold* value corresponding to simultaneous maximization of all five utility functions. The most reliable solution is to apply one of the convolutions. In this case the additive convolution is absolutely useless because it is concave and has maximums at the two opposite ends of the range (Figure 5.5.4). Multiplicative convolution also gives little information because its function does not possess any clear maximum and is rather flat across a wide range of *threshold* values. In contrast, minimax convolution is again unimodal and gives the result that is easy to interpret (Figure 5.5.4). The optimal threshold value is now 6%.

Figure 5.5.3. Five utility functions built for optimization of the threshold parameter when the short calendar spread strategy is compared with the long calendar spread strategy.

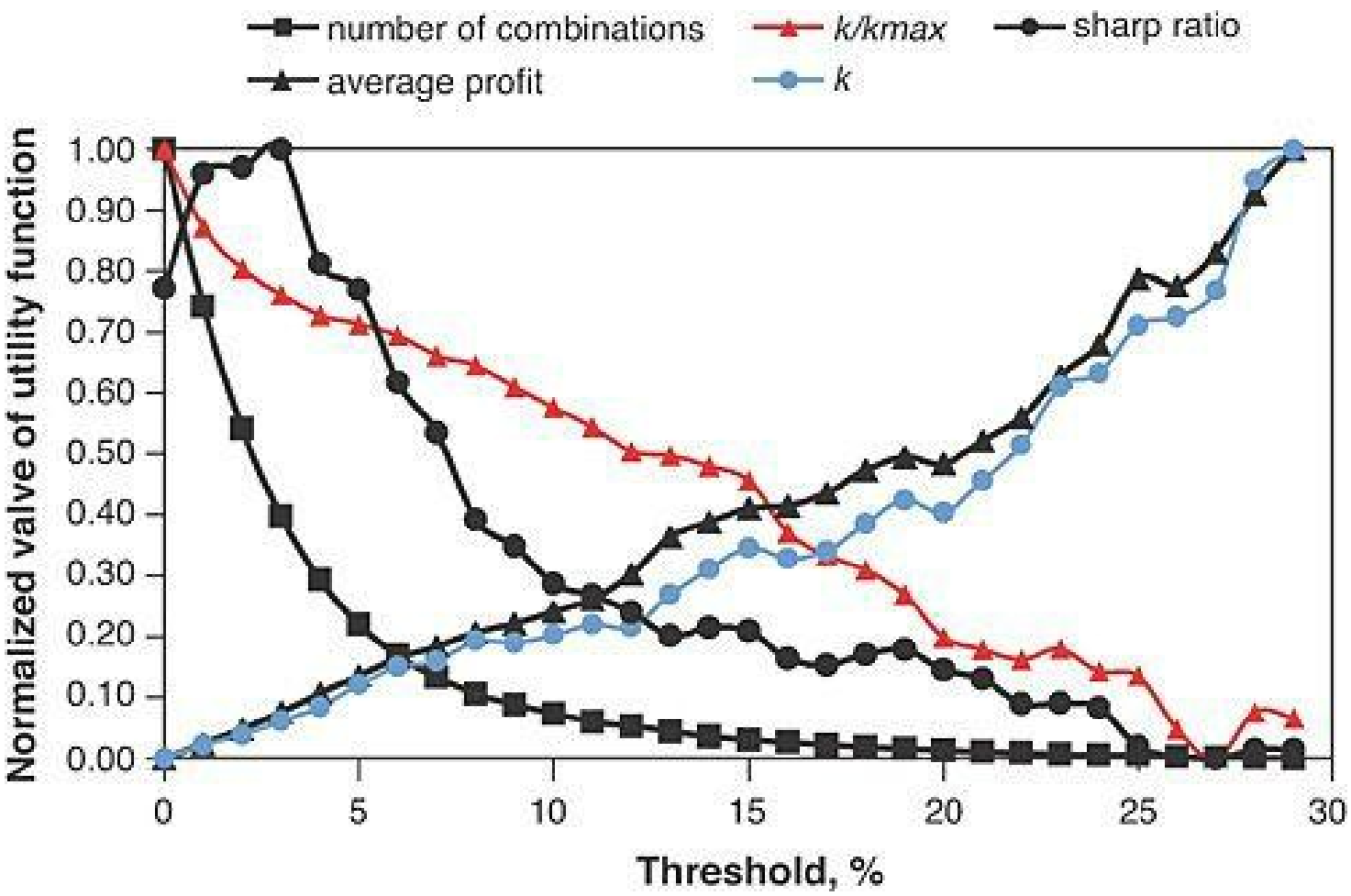
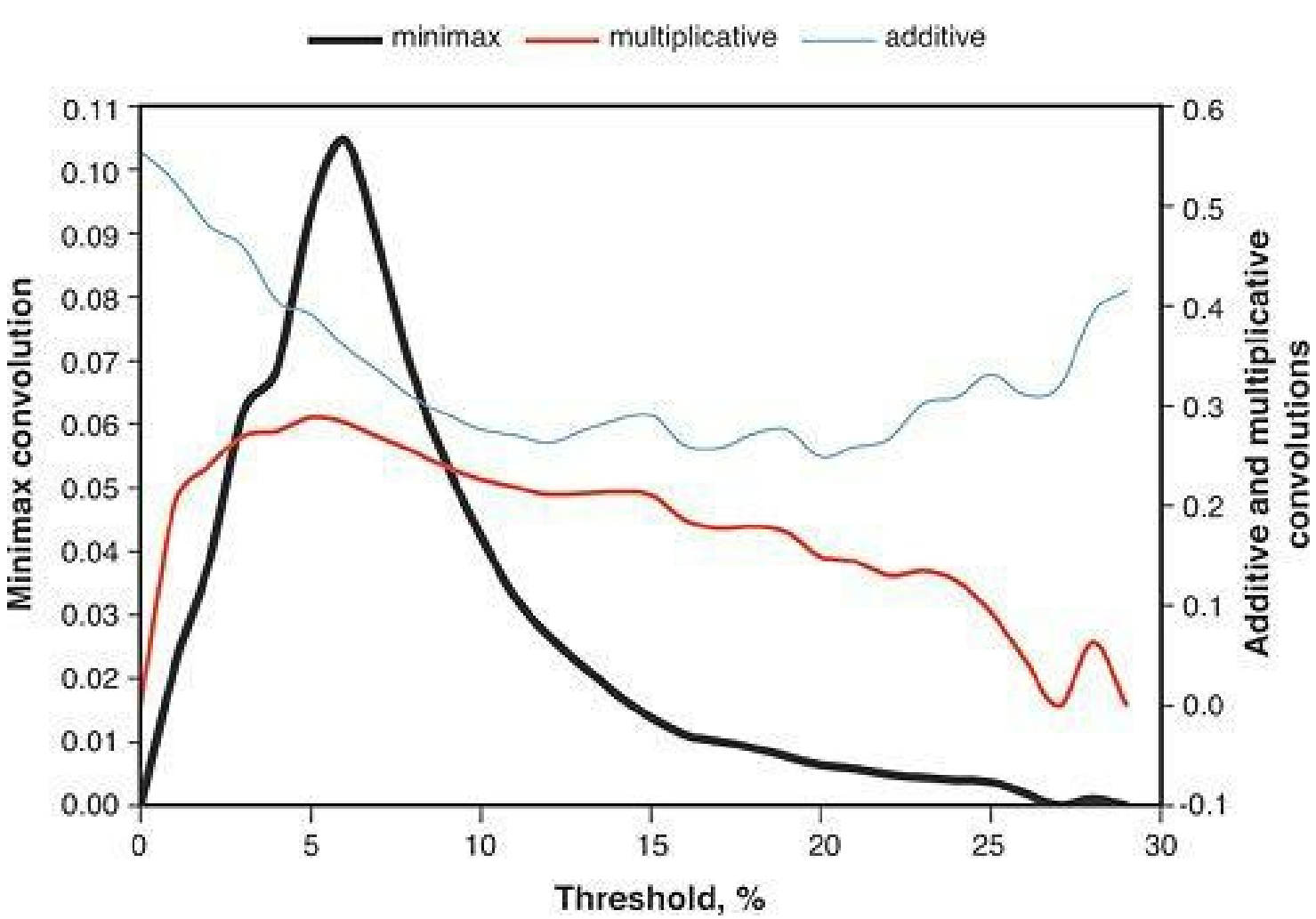


Figure 5.5.4. Three types of convolution of five utility functions shown in Figure 5.5.3.



The described model is easy to utilize and allows the simple determination of the optimal *threshold* value. It can be successfully applied to any other criteria and to the entire possible variety of option strategies. Moreover, the principles of this model can be adapted to other situations where optimization of

a certain parameter on the basis of several utility functions is required. The minimax convolution developed for this purpose is an efficient instrument facilitating the solution of multiparametric analysis when other methods do not lead to the appropriate results.

5.6 Summary

In this chapter we described two complementary methods intended to select superior option strategies. One method represents an adaptation of the generally accepted technique, whereas another one is our original development.

(The application of this method can be extended to various assessment problems; see an example in Izraylevich and Tsudikman, 2009c.) Because both techniques have their specific advantages and disadvantages, their synthetic simultaneous application is expected to produce the most sophisticated results. Introduction of a specific parameter, the *threshold* value, was shown to increase the predictive power of criteria.

Evaluation methods discussed in this chapter represent merely examples of the systematic approach as applied to the local problem of selecting option strategies using strictly formalized criteria. These examples were

simplified for the sake of clearness. Thus, in all the situations discussed we limited ourselves to only one criterion and to only two alternative strategies. Of course, the application of additional criteria or their combinations and an introduction of additional strategies would produce a more diversified and complex picture. Nevertheless, the methods and general principles of the systematic approach remain the same regardless of the complexity of the scrutinized system.

It is important to recognize that patterns and regularities detected under some market conditions may shift fundamentally if the basic parameters of the system change. Therefore, when

considering real option trading, all parameters must be thoroughly analyzed and optimized, and their potential effect on option strategy selection must be estimated and taken into account.

chapter 6. Selection of Underlying Assets

6.1 Introduction

In this chapter we consider the problem of selecting underlying assets given that an option strategy and a combination creation algorithm are defined a priori. Using the term *selection of underlying assets*, we refer to the selection of option combinations corresponding to

those underlyings. We assume the mechanism of combinations generation to be strictly predetermined. We also suppose that a sole option strategy is assigned to every underlying. This means that the most suitable option strategy and the best combination are chosen for each underlying asset (as discussed in Chapter 4, "Selection of Option Combinations," and Chapter 5, "Selection of Option Strategies"). Under these conditions every underlying asset is represented by a unique combination, which is created using a certain technology within a certain strategy. Therefore, selecting the best combinations from this set comes down to selecting the best underlying assets.

Following the same order as in the previous chapter, two main topics are addressed here. First, we analyze the ability of criteria to identify underlying assets possessing the best characteristics of future profit. This is necessary to ascertain the effectiveness of criteria in solving the problem of underlying selection. Assessment of criteria effectiveness is performed by evaluation of their abilities to order combinations (each of which represents a certain underlying) by their profit potential.

The second topic deals with the problem of profit maximization. We construct a model intended to optimize the parameter, which specifies the number

of underlying assets to be selected. The model is based on the development of utility functions. The latter represent empirically derived relationships between several indicators and parameter values. The indicators are numerical characteristics expressing potential profit and risk. The parameter is a number of underlying assets chosen using a certain criterion.

6.2 Analysis of Criteria Effectiveness in Selection of Underlying Assets

The analysis of criteria effectiveness

will be illustrated by the example of the short strangle/straddle strategy. The following algorithm of combinations creation is applied. For every underlying asset we construct a sole combination (either strangle or straddle) possessing the most symmetric payoff function relative to the current underlying price. (The symmetry is assessed by the method described in section 4.4.1.) Combinations are created using option contracts with the nearest expiration date; numbers of Calls and Puts are equal.

Reviewing the evaluation of criteria effectiveness (Chapter 3, “Evaluation of Criteria Effectiveness”), we have proved the necessity of using a 100-day

averaging period to achieve the acceptable level of reliability. Here we apply the same methods using September 10, 2007, as a starting point. Five hundred U.S. stocks (selected on the basis of liquidity of their options) were chosen as underlyings. One combination corresponding to the short strangle/straddle strategy is created for each of the 100 dates and for each of the 500 stocks. The set of combinations obtained in this manner can be presented as a table consisting of 500 lines (corresponding to the number of stocks) and 100 columns (corresponding to the number of dates). Each element of this table (in total 50,000 combinations) is characterized by two indexes: criterion

value corresponding to the valuation date and profit value corresponding to the expiration date. Both criterion and profit values are expressed as a percentage of margin requirements.

In every column of this table the combinations are ordered according to the respective values of a certain criterion. The average of the criterion (or profit) index in the *n*-th line of this table represents the average of the criterion (or profit) value for combinations situated at the *n*-th place in the ordering. Altogether we have 100 orders (equal to the number of dates), hence average values are calculated on a 100-element sample. In addition to averages, we can estimate profit

variability within each line (that is, for each *n*-th place in the ordering). The variability is expressed as a standard error (standard deviation divided by the square root of the number of elements in a sample, 100 in this case).

Execution of the aforementioned procedures provides us with averages and standard errors of criterion and profit values for combinations occupying the top positions in the orderings, the second positions, and so forth up to the 500th positions. To analyze the effectiveness of another criterion, it is necessary to re-sort combinations according to their values and repeat all these procedures.

The following method will be applied to evaluate criteria effectiveness in selecting underlying assets. Suppose that a certain amount of capital is invested in a single underlying asset (to be more precise, in a single combination built on the basis of one underlying). The profit of such investment is equivalent to the profit of the combination occupying the top place in the order. Alternatively the whole capital can be allocated equally among n underlyings positioned at the first n places in the order. In this case the percentage profit of the investment equals the average of percentage profits of n combinations.

Criteria effectiveness will be characterized by the functional

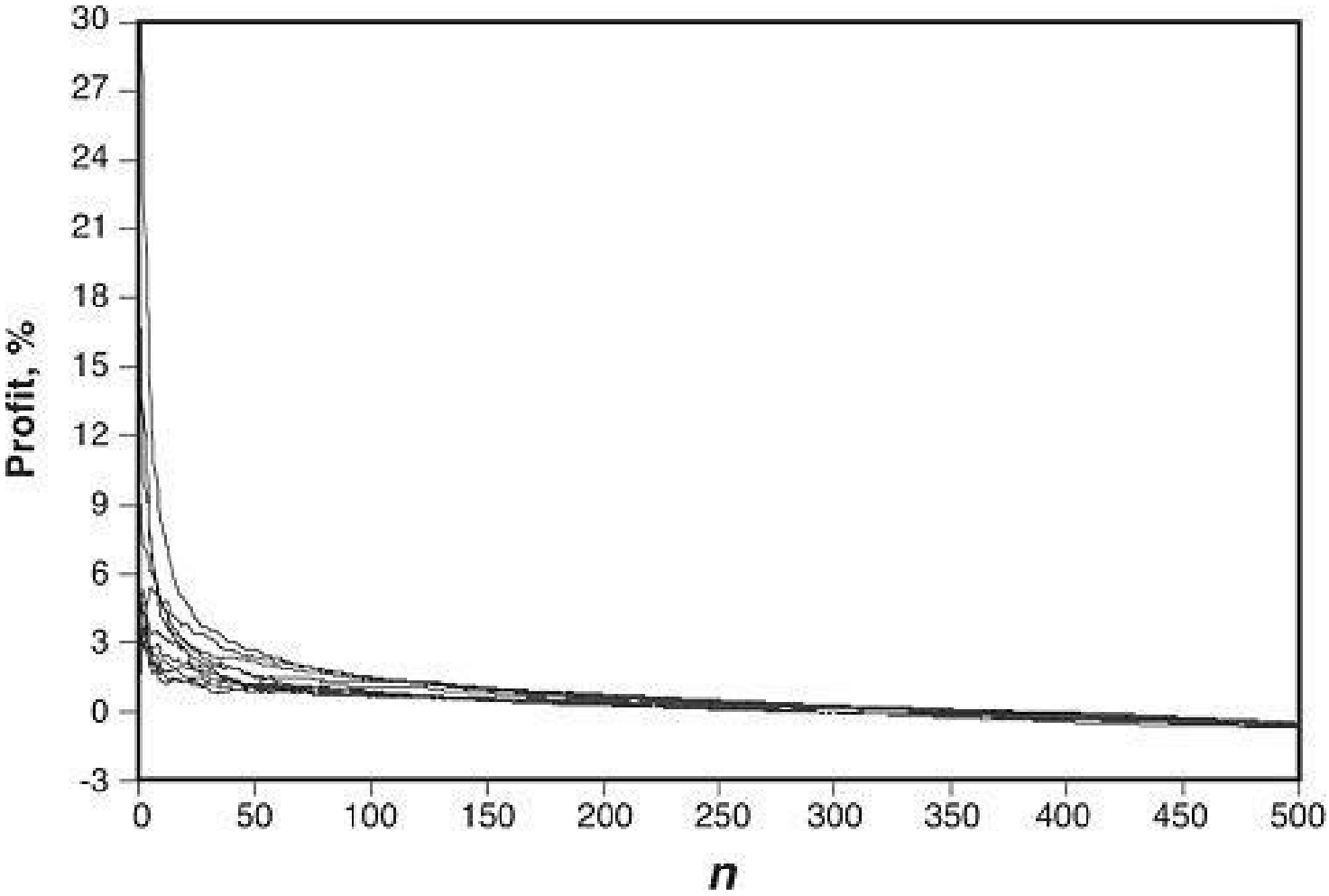
relationship between the profitability of the investment and the number of underlying assets (n) used to allocate funds. If a criterion is effective in selection of underlyings, then the ordering of combinations by criterion values should match their ordering by profitability. This means that combinations ranked higher in the criterion ordering should on average be more profitable than those ranked lower. If that's the case, the more underlying assets we use to allocate funds, the lower the profit of the investment.

Application of the described method reveals marked negative relationships between the profit of the investment and the number of underlying assets (Figure

6.2.1). This conclusion holds true for all the criteria without exception. The shape of these relationships is of particular interest; in most cases the profit falls exponentially when the number of underlying assets increases. This implies that the inclusion of every additional combination with a lower position in the ordering into the portfolio leads to a sharp profit decline. Allocation of funds among underlyings occupying the top 200 positions in the ordering causes profits to decline to zero, whereas the use of an even higher number of underlyings leads to losses (Figure 6.2.1).

Figure 6.2.1. Relationship between the profit of the investment and the number of underlying assets (*n*) used to allocate capital among combinations corresponding to these underlyings. Capital is distributed equally among the first n combinations ordered by a criterion. Combinations are built under a short strangle/straddle strategy.

Each line corresponds to one of 11 criteria.



Relationships presented in Figure 6.2.1 indicate that all the criteria are more or less successful in selecting underlying assets. However, due to the exponential

form of most relationships, the differences between individual criteria are hardly distinguishable. Besides, it is impossible to show the degree of profit variability in this figure. To illustrate the individual features of criteria and demonstrate the variability of the results, the data in Figure 6.2.1 needs to be presented in a different way.

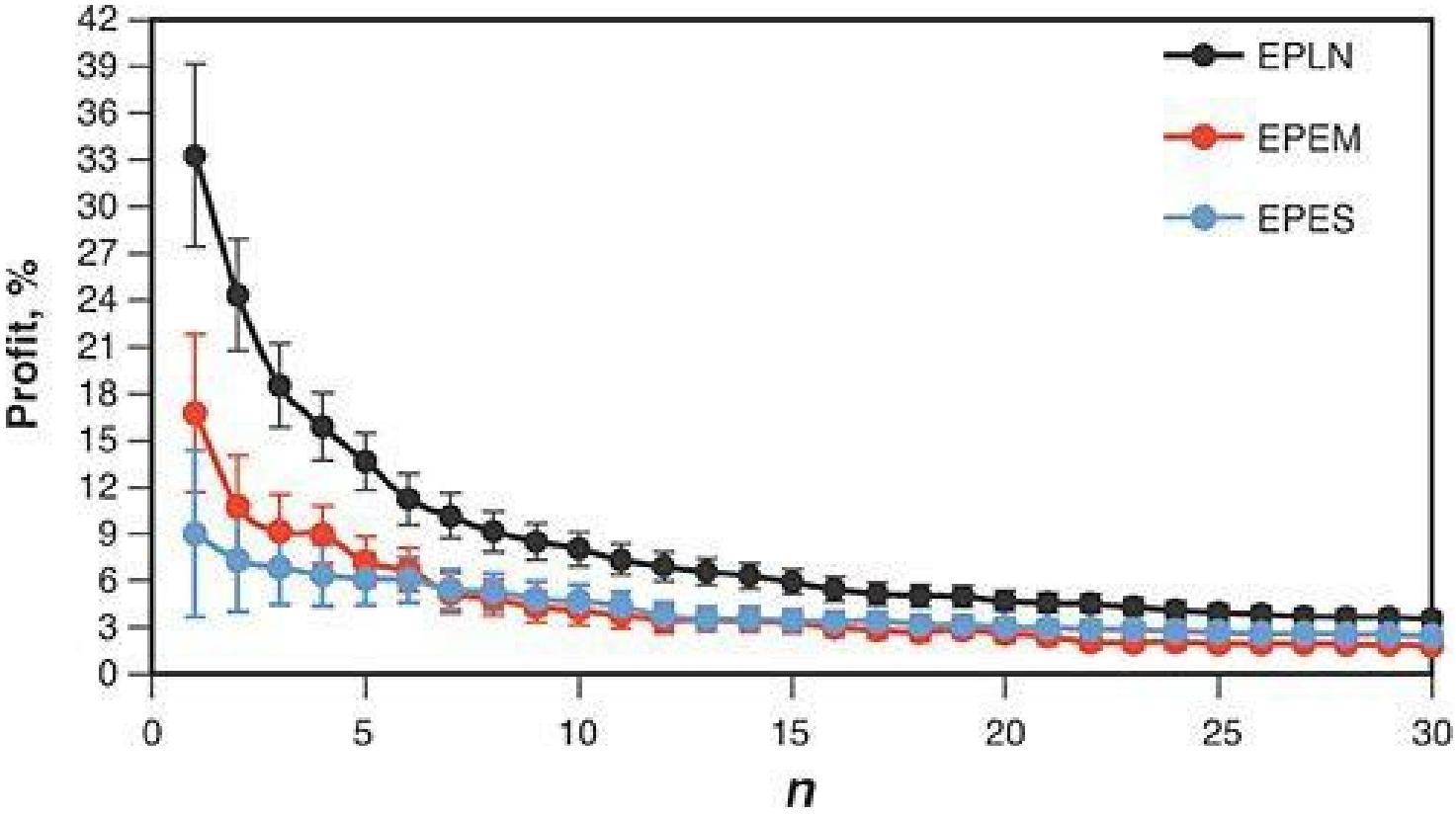
To make the figure more legible, we reduce the number of underlyings used to distribute capital from 500 to 30. We also add standard error values to average profits for every number of underlying assets. This gives an insight into the degree of variability of the results. We illustrate the relationships between profit and *n* by the example of

the three criteria forecasting the expected profit on the basis of different distributions: lognormal (EPLN), empirical (EPEM), and symmetrized empirical (EPES).

The EPLN criterion shows the highest rate of profit decline; the corresponding curve has a clearly exponential form (Figure 6.2.2). Ordering underlyings by EPEM criterion leads to smoother profit decline. Finally, in the case of EPES criterion, the profit falls rather linearly than exponentially (Figure 6.2.2). This means that ordering underlying assets by EPLN criterion matched the ordering by realized profit in the best way. EPES is the worst criterion and EPEM holds the middle place. Hence we can conclude

that in this particular case EPLN is the most effective of the three investigated criteria.

Figure 6.2.2. The same as Figure 6.2.1 but for a smaller range of underlying asset numbers. Average profits are depicted with circles; standard errors are shown as vertical bars. Abbreviations of the criteria are the same as in Figure 3.2.4.



Regardless of the applied criterion, standard errors are higher when the number of underlying assets used to distribute capital is small (Figure 6.2.2). This is indicative of the high profit variability within combinations occupying the first several positions in the ordering by criteria values. Although

the average profit of low-ranked combinations is lower, it is, nevertheless, more stable. Therefore, despite the maximum average profit of top-ranked underlying assets, it is necessary to include more than just a few of them into the portfolio. This would enhance the diversification of the portfolio and thereby reduce its volatility and risk. Optimizing the number of underlying assets in the following sections, we rely on the indicator that takes into account not only the average profit but its variability as well. (The Sharpe ratio serves as such indicator.)

Examples presented show the capability of the criteria to select the best

underlying assets (at least in some cases). However, the effectiveness of different criteria is certainly heterogeneous. Because our research so far was restricted to only one strategy and to a short time period (100 days), it cannot fully reflect the forecasting abilities of criteria. In the next section we expand time horizons and test the criteria effectiveness on different option strategies.

6.3 Multistrategy, Long-Term Evaluation of Criteria Effectiveness

6.3.1 Methods

The long strangle/straddle strategy, which is inverse to the short strangle/straddle strategy previously examined, is added to the investigation. The set of underlying assets are expanded to 2,500 stocks. The effectiveness of five different criteria is analyzed: expected profit on the basis of lognormal (EPLN) and empirical (EPEM) distributions, profit probability (PPLN) and the ratio of expected profit to losses (RPLL) on the basis of lognormal distributions, and the ratio of implied to historical volatility (*IV/HV*).

We need to apply such algorithms for the creation of combinations and strategies selection so that the effectiveness of these procedures do not influence the evaluation of criterion effectiveness.

The initial set of underlying assets containing 2500 stocks is divided into two subsets. One subset consists of the stocks that are most suitable for the short strangle/straddle strategy; the other one consists of the stocks suitable for the long strangle/straddle strategy. Stocks are assigned to these subsets using the *IV/HV* criterion in accordance with the procedure described in Chapter 5. Earlier it was explained (Chapter 4) why we use this particular criterion for strategy selection. It ensures effective

evaluation of not only individual combinations but also of underlying assets; as such, that allows selecting strategies without evaluating or selecting certain combinations. Such a feature enhances the reliability of the criteria effectiveness analysis.

Next, for every underlying asset we select one combination (either strangle or straddle) with the most symmetric payoff function. (All combinations contain equal amount of Calls and Puts and are composed of contracts with the nearest expiration date.) Using one of the five criteria we select 50 combinations out of every subset. At the same time we select 50 random combinations in each subset. Both selection procedures are

reproduced consecutively for all dates (separately for every criterion).

For every date we compute the total profit or loss of the 50 combinations selected by the criterion and of the 50 random combinations. Profits and losses are estimated assuming that each combination consists of one Call and one Put contract. The difference between the total profit of the 50 combinations selected using the criterion and the total profit of the 50 random combinations expresses the advantage provided by criterion application. Summarizing these differences while moving forward in time, we get a pictorial rendition of the profit accumulating due to criteria use.

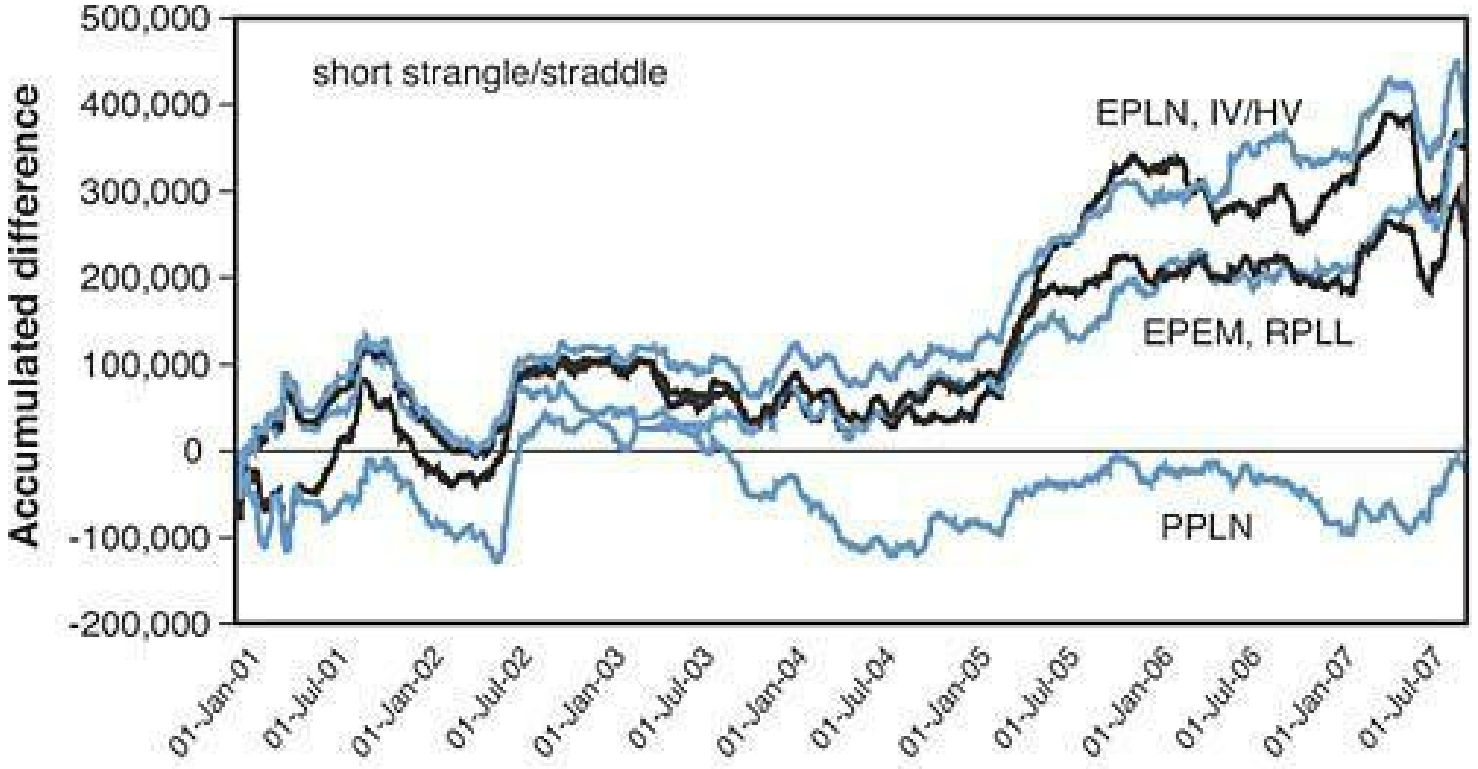
6.3.2 Results

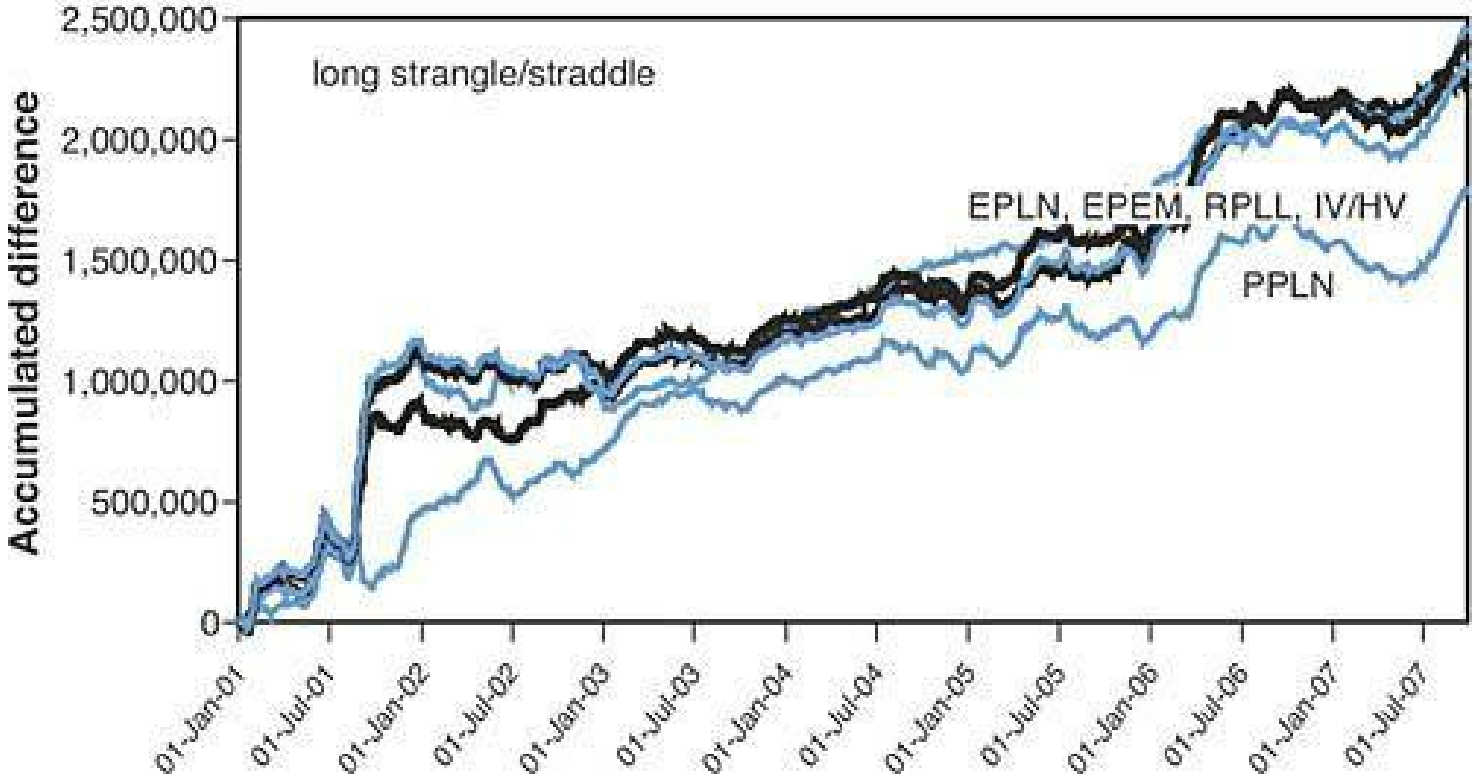
Figure 6.3.1 illustrates the dynamics of the accumulated difference between profits of combinations selected by criteria and of random combinations. Application of criteria to underlyings selection has a weaker effect within the short strategy than within the long one. Although in the first case the average accumulated amount came to about $300,000, in the second case it ended up $2,400,000 (Figure 6.3.1). The reason is that the long strategy has unlimited profit potential whereas the maximum achievable profit of the short strategy is

limited. Although absolute values of accumulated differences are not indicative (they depend on capital allocation and other specific methods adopted in this study), they are useful for comparative analysis of criteria and strategies.

Figure 6.3.1. Dynamics of accumulated difference between total profit of 50 combinations selected using criteria and total profit of 50 randomly selected combinations. Abbreviations

of criteria and strategies appear on the figure.





Four out of the five criteria showed

good forecasting abilities in selecting underlying assets within the short strategy. Accumulated difference charts for these criteria reveal steady (yet not quite smooth) growth (Figure 6.3.1). Although the PPLN criterion (forecasting the profit probability on the basis of lognormal distribution) did not show convincing effectiveness in solving the selection problem, the other four criteria were effective most of the time.

Applying the same criteria to selecting underlying assets within the long strategy produces even more convincing evidence of high effectiveness of the criteria. Accumulated differences grow permanently and relatively smoothly through the entire time period (Figure

6.3.1). The only exception is again the PPLN criterion that turned out to be less effective. In this case the accumulated difference grew less consistently and slower relative to the other four criteria.

6.4 The Optimization Model for the Number of Underlying Assets

The foregoing examination has proved that the criteria are indeed an effective tool for the selection of underlying assets with superior profit potential as compared to rejected alternatives. Using

this selective feature we can rank the whole set of underlying assets (to be more precise, combinations corresponding to them) according to their respective criteria values. On average every element of such ordering will be superior to the following ones.

However, even being provided with a hypothetically perfect ordering, how can we draw a line separating good combinations from ordinary and poor ones? In other words, how many top-ranked combinations should we consider as candidates for inclusion in the investment portfolio? To answer this question we use utility functions described in Chapter 5. The number of selected underlyings serves as the

argument of this function, whereas its value is the indicator expressing the expedience of selecting a particular number of underlying assets.

6.4.1 Utility Indicators

One of the utility indicators combines profit with its variability (Figure 6.2.2). This indicator is calculated as the ratio of the average profit to the standard error. This ratio (which is a version of the Sharpe ratio) was used in section 3.4.3 to evaluate criteria effectiveness; hence we are not going to elaborate further on it here. Another utility indicator is the maximum drawdown.

Because the functional relationships between these indicators and the number of underlying assets cannot be derived by analytical methods, we obtain them empirically.

The maximum drawdown usually reflects the risk of a certain investment strategy. Often used in optimization of trading systems, it shows the maximum losses incurred during a certain time period. In our interpretation the maximum drawdown is the loss of the combination that is the most unprofitable among others belonging to a certain rank in the series of orderings. This definition can be explained using the framework utilized in section 6.2 to evaluate criteria effectiveness. Suppose that all

data are grouped into a table, consisting of 500 lines (according to the number of underlying assets) and 100 columns (the number of dates). If within each column the combinations are ordered according to their criterion values, then the minimum profit value within the *n*-th line is a maximum drawdown corresponding to underlyings occupying the *n*-th positions in the 100 orderings. Thus, the maximum drawdown is a loss of the most unprofitable combination. (It will be expressed as a percentage of margin requirements.)

Consider the relationship between maximum drawdown and position of an underlying asset in the ordering by criteria values (*n*). We use the EPLN,

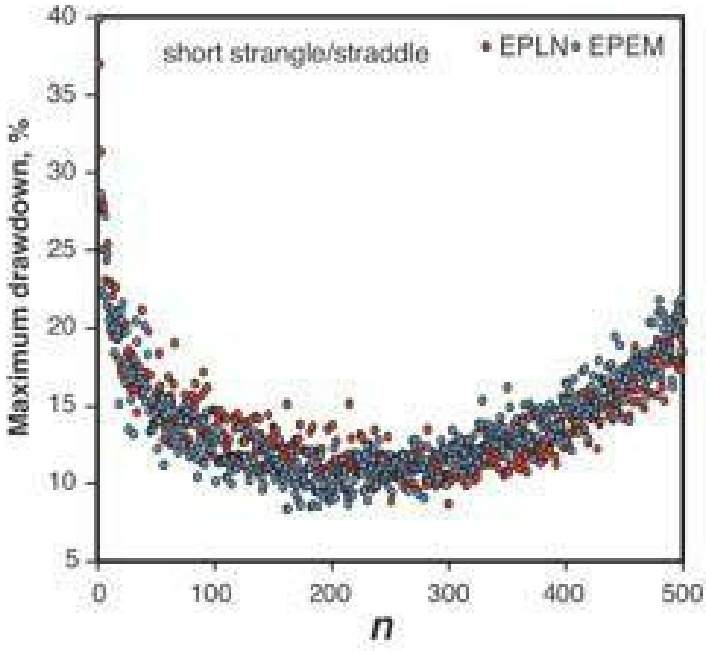
EPEM, PPLN, and RPLL criteria and both short and long strangle/straddle option strategies.

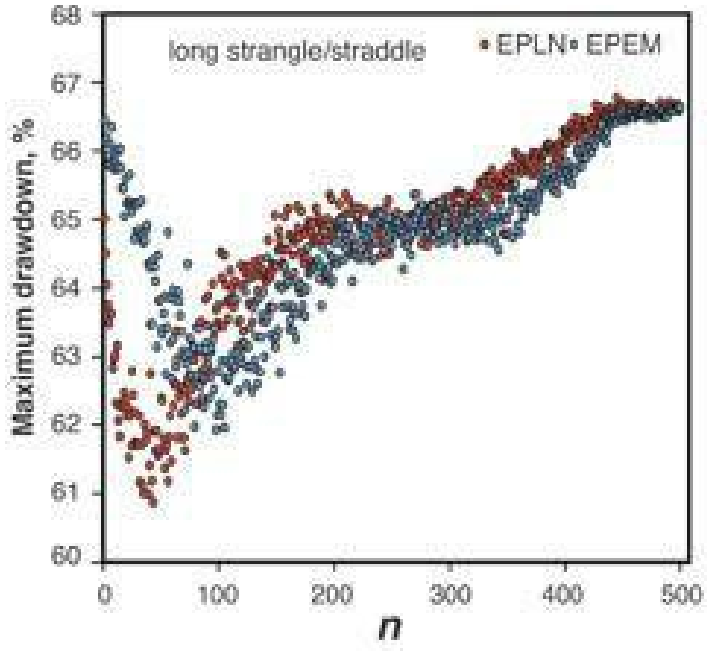
For the criteria forecasting profit expectations (EPLN and EPEM), the relationships between the maximum drawdown and *n* are nonlinear and rather concave (Figure 6.4.1). For the short strategy the maximum drawdown appears in the first and the last positions of the ordering (that is, the best and the worst combinations according to their criteria values). Underlying assets in the middle of the ordering (combinations with intermediate criteria values, around 200 to 300) have the minimum drawdown. In the case of the long strategy, the situation is quite similar

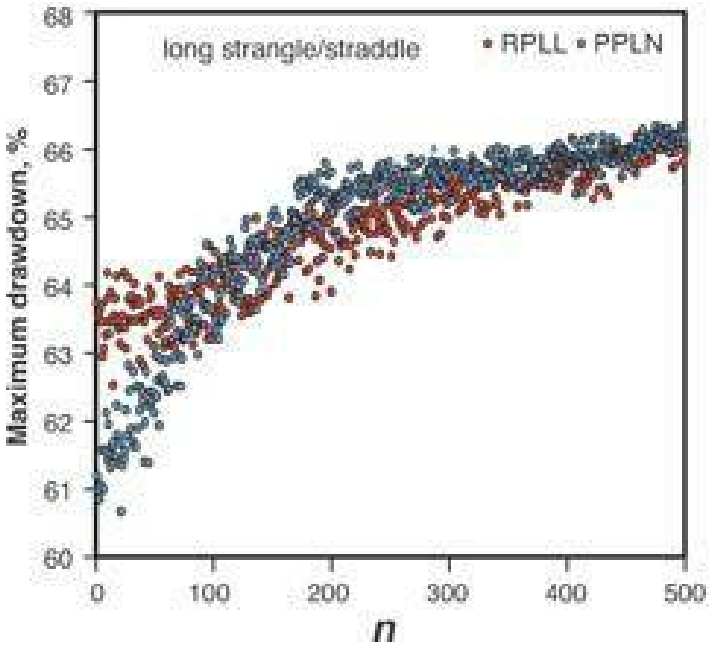
although the curve shape is slightly different. When sorting combinations by the EPLN criterion, the minimum drawdown is observed for combinations ranked around the 50th position in the ordering. For the EPEM criterion, combinations ranked around the 100th position are the best.

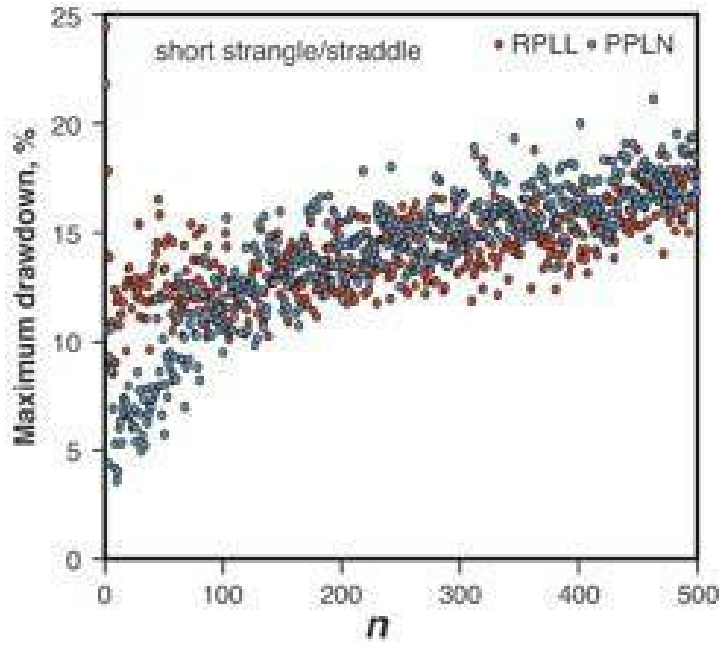
Figure 6.4.1. Relationship between maximum drawdown and position of an underlying asset in the ordering by criteria values (*n*). Abbreviations of

strategies and criteria appear in the figure.









For criteria forecasting the profit probability (PPLN) and the ratio of expected profit to losses (RPLL), the relationships have completely different shapes (Figure 6.4.1). The minimum

drawdown is observed for underlying assets found at the top of the ordering (combinations with the best criteria values) in the case of both the long and the short strategies. The maximum drawdown appears for underlyings positioned at the bottom of the ordering (combinations with the lowest criteria values). Note the difference between the PPLN and RPLL criteria. In the first case the maximum drawdown increases nonlinearly as the position of an underlying asset increases whereas in the second case the relationship is linear and less evident.

In the next section we proceed to the construction of utility functions intended to determine the optimal number of

underlying assets.

6.4.2 Utility Functions

There are two indicators expressing utility of investment in combinations occupying the n-th position in the ordering. Based on these indicators we need to construct functions that would express the utility of the capital allocation among the n combinations ranked on the first n postitions in the ordering. For example, all funds may be invested in the first combination in the ordering. On the other hand, they can be allocated among two combinations corresponding to the underlyings

occupying the first and the second positions. And so on.

The utility functions will be used to determine the optimal number of underlying assets. The optimality means that allocating capital among this number of underlyings maximizes the potential profit of such investment and minimizes its risk. Hence utility functions should reflect the relationship between profit (or risk) and the number of underlying assets used to distribute funds.

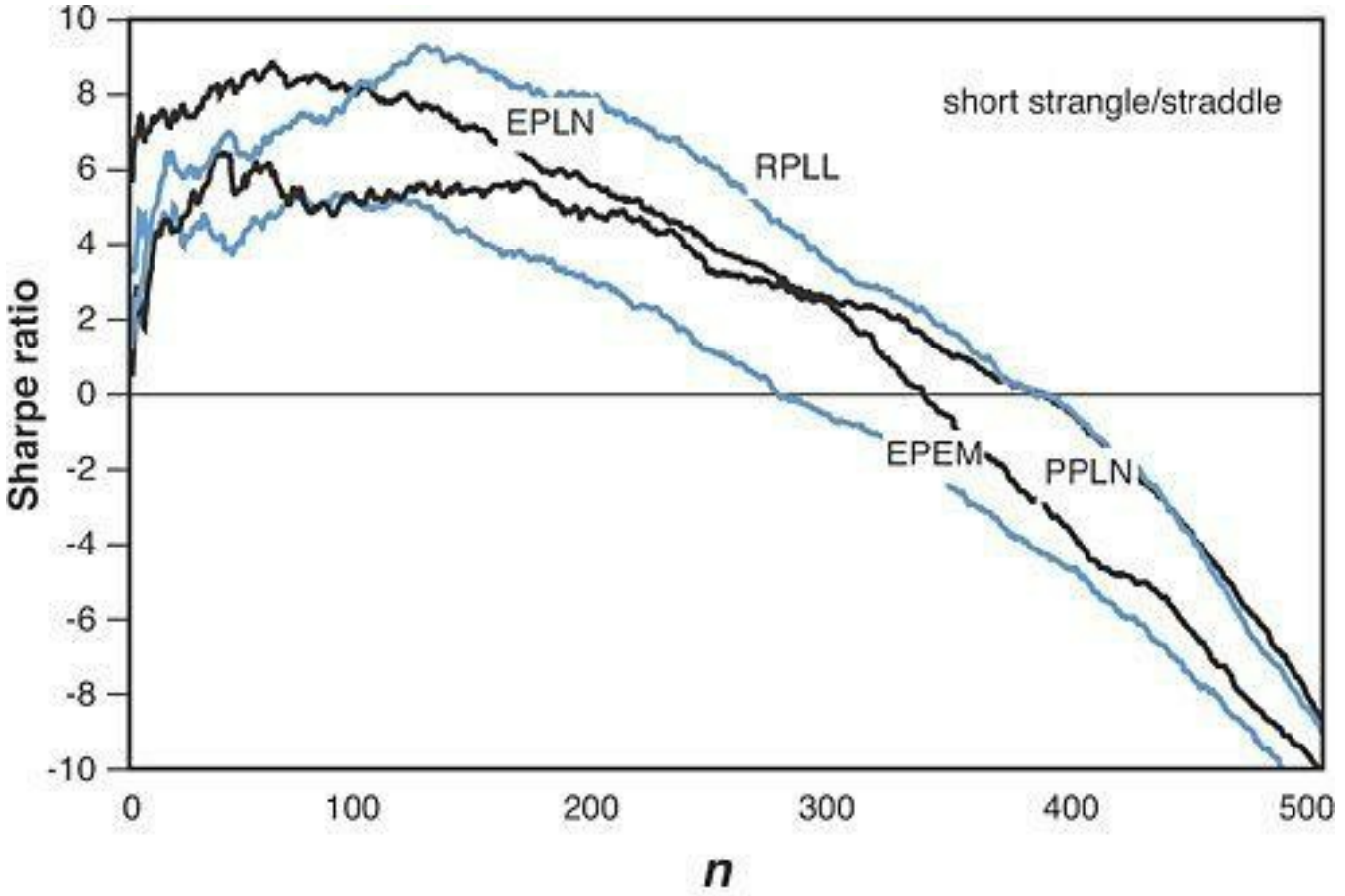
To illustrate the construction of utility functions, we use the data from section 6.2: 500 underlying assets and a 100-day time period. Each day is represented by a separate ordering of 500 combinations.

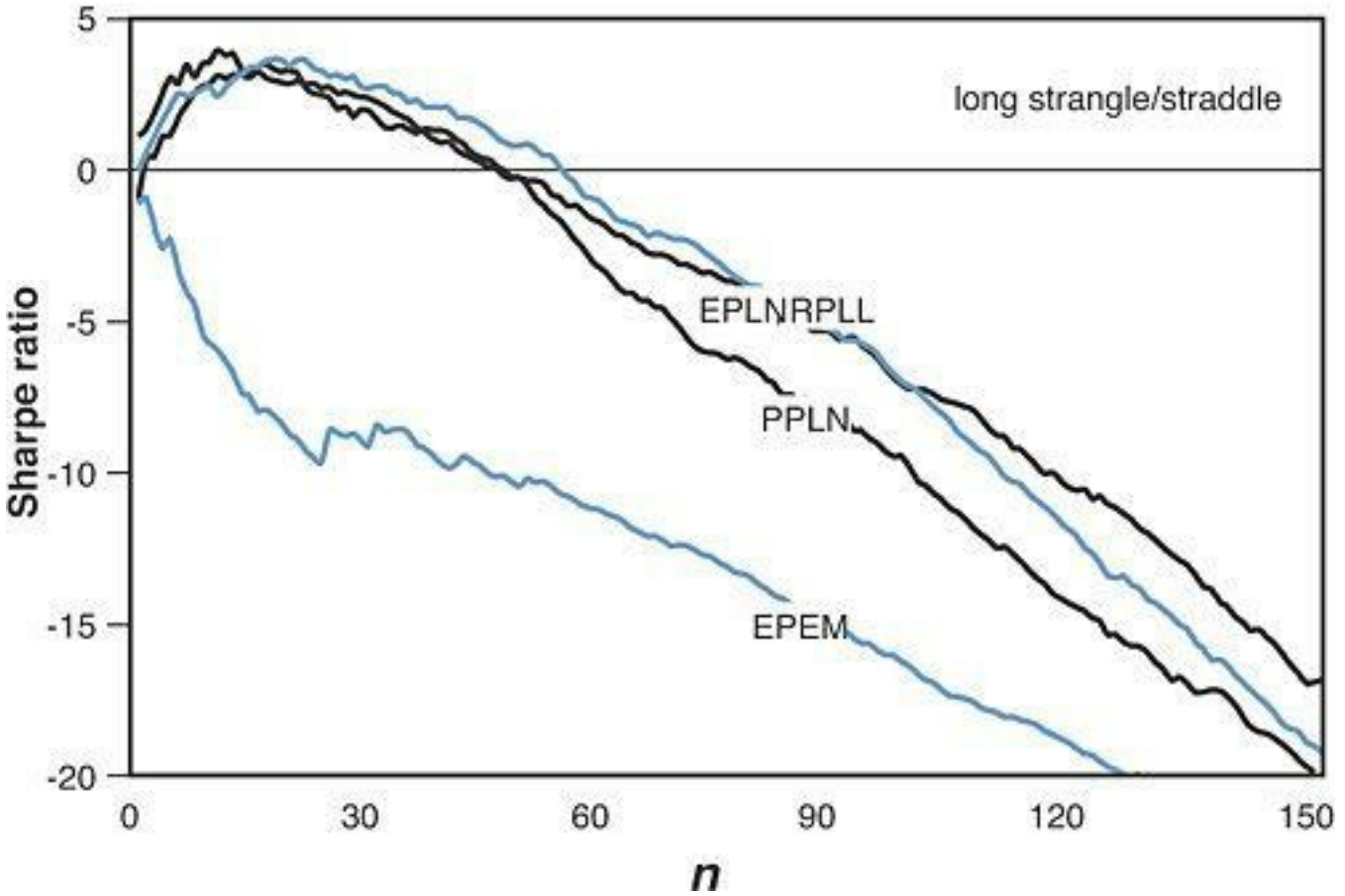
It means that we have 100 combinations occupying the first positions of these orderings, 100 combinations on the second positions, and so on up to 500 positions. For the sake of simplicity, we assume that the funds are allocated equally among all combinations and the investment amount is equivalent to the margin requirements.

The percentage return under equal allocation of funds among n underlyings equals the average of percentage returns of combinations occupying positions 1 to n in the ordering. Because we use 100 orderings (100-day time period), the return is calculated as the average of $100 \times n$ combinations' returns. The same data is used to estimate the standard

error and the Sharpe ratio. Figure 6.4.2 shows utility functions constructed on the basis of Sharpe ratios for two opposite strategies and four criteria.

Figure 6.4.2. Utility functions based on the relationship between the Sharpe ratios and the number of underlying assets (*n*) used for capital allocation. Abbreviations of criteria and strategies appear on the figure.


short strangle/straddle
EPLN
RPLL
EPEM
PPLN
Sharpe ratio
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
100
200
300
400
500
n


long strangle/straddle
EPLNRPLL
PPLN
EPEM
Sharpe ratio
5
0
-5
-10
-15
-20
0
30
60
90
120
150
n

In section 6.2 it was shown that, regardless of the criterion applied, maximum profit could be achieved by allocating funds among a few first underlying assets (Figures 6.2.1 and 6.2.2). However, utility functions based on the Sharpe ratio suggest another optimal solution. With only one exception, none of the utility functions reach their maximums on the first positions in the orderings (Figure 6.4.2). The highest utility of the short strategy is achieved when funds are allocated among the first 60 underlyings as ordered by the EPLN criterion. In the case of using the EPEM, PPLN, and RPLL criteria, the maximum utility is

achieved when funds are allocated among the first 90, 43, and 127 underlying assets respectively (Figure 6.4.2). For the long strategy the similar situation is observed in three cases out of four. The maximums of utility functions suggest that funds are optimally allocated among the first 20 to 30 underlyings (Figure 6.4.2).

The second utility function is based on the drawdown occurring when funds are allocated among *n* underlying assets. The value of this function is equivalent to the average of maximum drawdowns corresponding to *n* first positions of the ordering. For example, if the worst combination of the 100 top ones (that is, situated at the first places in the

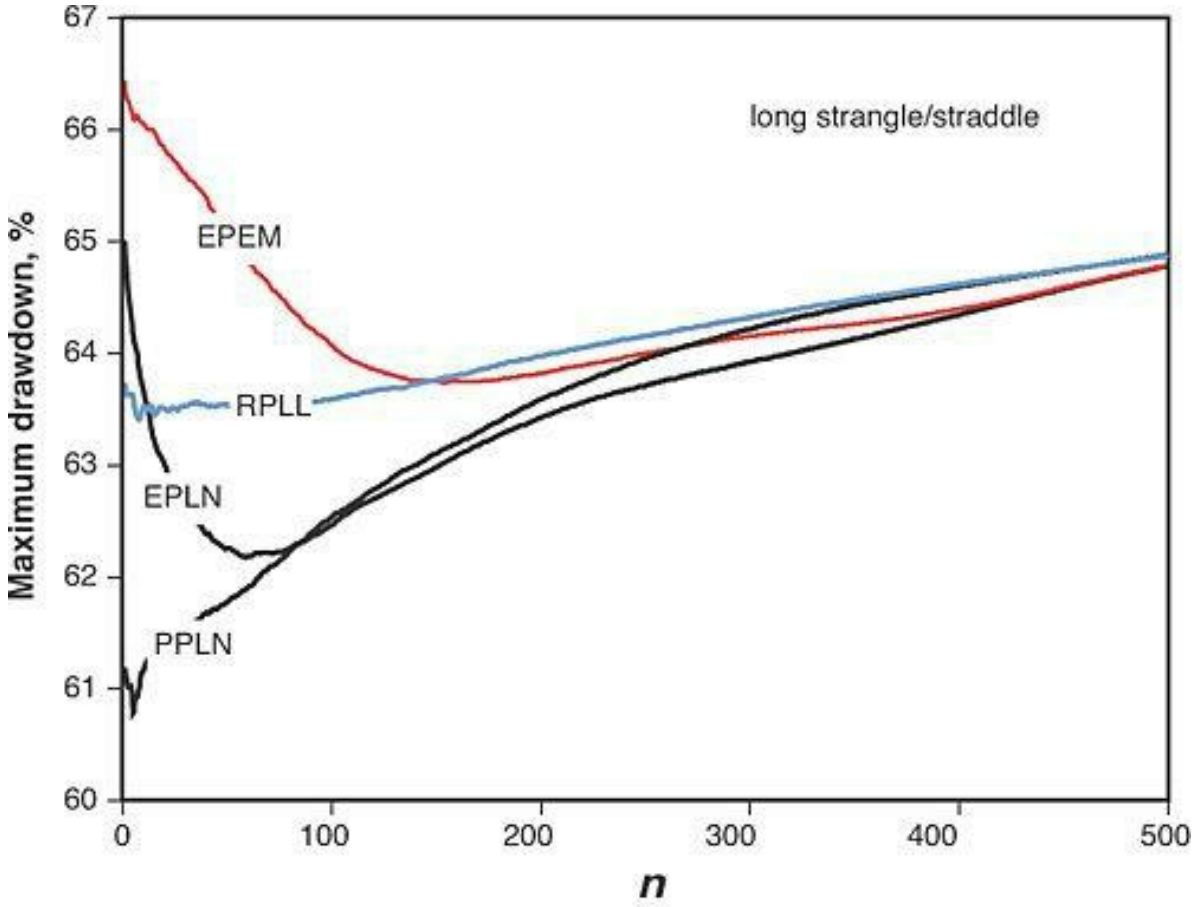
orderings) gives a 4% loss and the worst combination of the 100 ones occupying the second positions gives an 8% loss, then for n=2 the utility function value is 6%. Because the value of the utility function expresses a loss, the optimal n corresponds to the minimum of this function.

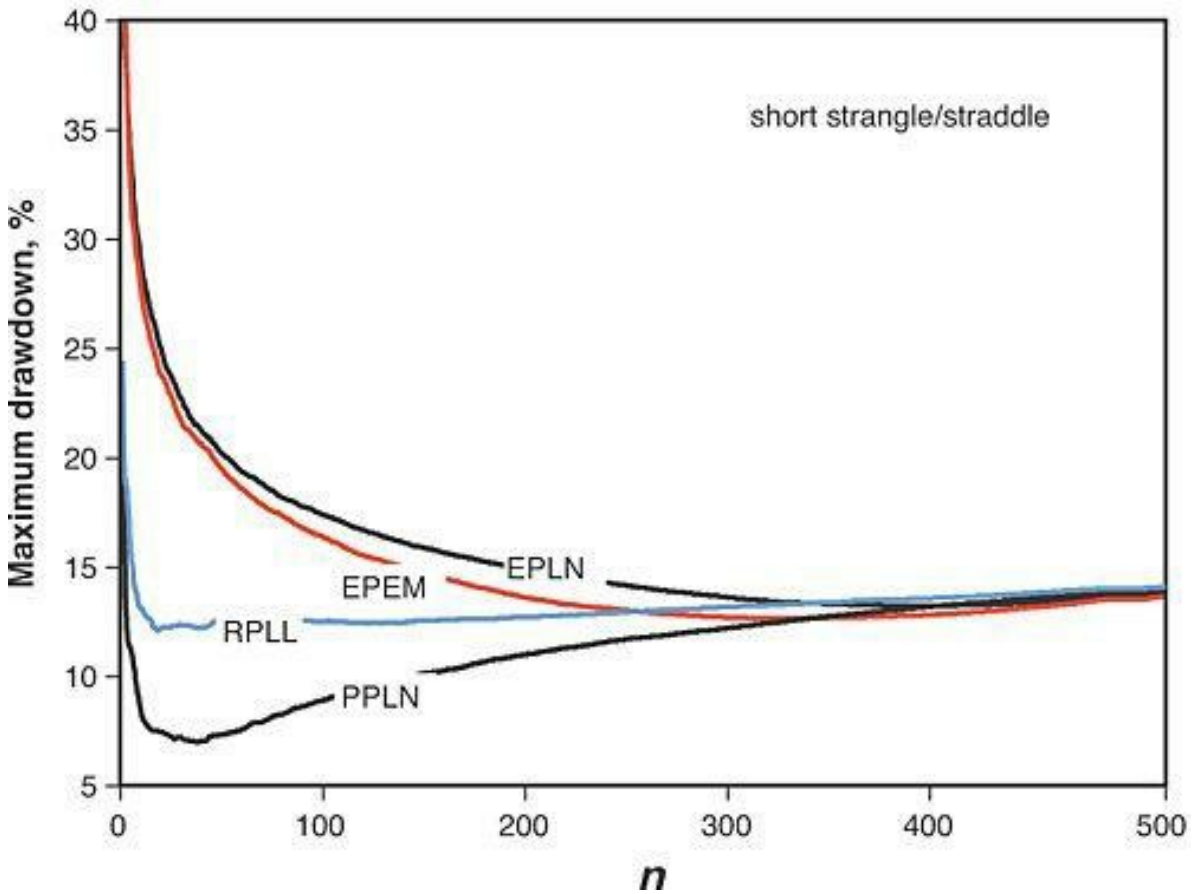
Figure 6.4.3 shows utility functions based on the maximum drawdowns obtained for two option strategies and four criteria. Note that optimal funds allocation based on the maximum drawdown does not coincide with optimal allocation based on the Sharpe ratio (compare Figures 6.4.3 and 6.4.2). In the case of the short strategy and using EPLN and EPEM criteria, utility

functions achieve their minimum under allocating funds among the significant number of underlying assets (392 and 325 accordingly, which exceed substantially the Sharpe ratio optima). On the other hand, for the PPLN and RPLL criteria, the optimal solution is to allocate funds among few underlying assets (39 and 18 accordingly, which is less than the Sharpe ratio optima). The same difference between criteria is noted in the case of the long strategy (Figure 6.4.3). Utility functions for EPLN and EPEM criteria suggest allocating funds among a significant number of underlying assets (59 and 155 accordingly), whereas for PPLN and EPLL optimal funds allocations are

achieved with fewer underlyings (5 and 8 accordingly).

Figure 6.4.3. Utility functions based on the relationship between the maximum drawdown and the number of underlying assets (*n*) used for capital allocation. Abbreviations of criteria and strategies appear on the figure.


long strangle/straddle
Maximum drawdown, %
67
66
65
64
63
62
61
60
EPEM
RPLL
EPLN
PPLN
0
100
200
300
400
500
n


short strangle/straddle
Maximum drawdown, %
40
35
30
25
20
15
10
5
EPLN
EPEM
RPLL
PPLN
0
100
200
300
400
500
n

The lack of coincidence between the optima of two different utility functions is extremely important and useful. It means that these functions reflect different nonoverlapping information. Hence, their simultaneous combined application would raise the reliability of the whole procedure intended to detect the optimal parameter value.

6.4.3 Convolution of Utility Functions and Deriving Optima for Different Strategies and Criteria

In Chapter 5 we discussed the methods of combining several utility functions

into a resultant one. The main requirement to the combining method is unambiguity of results. In the current context it means that the combined function should have the only maximum pointing to the optimal number of underlying assets (that is, it must be unimodal). In addition to the widely used methods of combining empirical functions (such as multiplicative and additive convolutions), we introduced another one—minimax convolution—giving more reliable and unambiguous results (at least in the case of combining utility functions for optimization purposes). When combining only two functions, minimax convolution is equivalent to a multiplicative one.

Values of utility functions based on the Sharpe ratio and on the maximum drawdown are measured in different units and are scaled differently. Therefore, we need to reduce empirically obtained values of both functions to a common scale. This is done by three consecutive operations:

1. Find the minimum and the maximum values of every function.
2. Subtract the minimum from each value.
3. Divide the result by the difference between the maximum and the minimum.

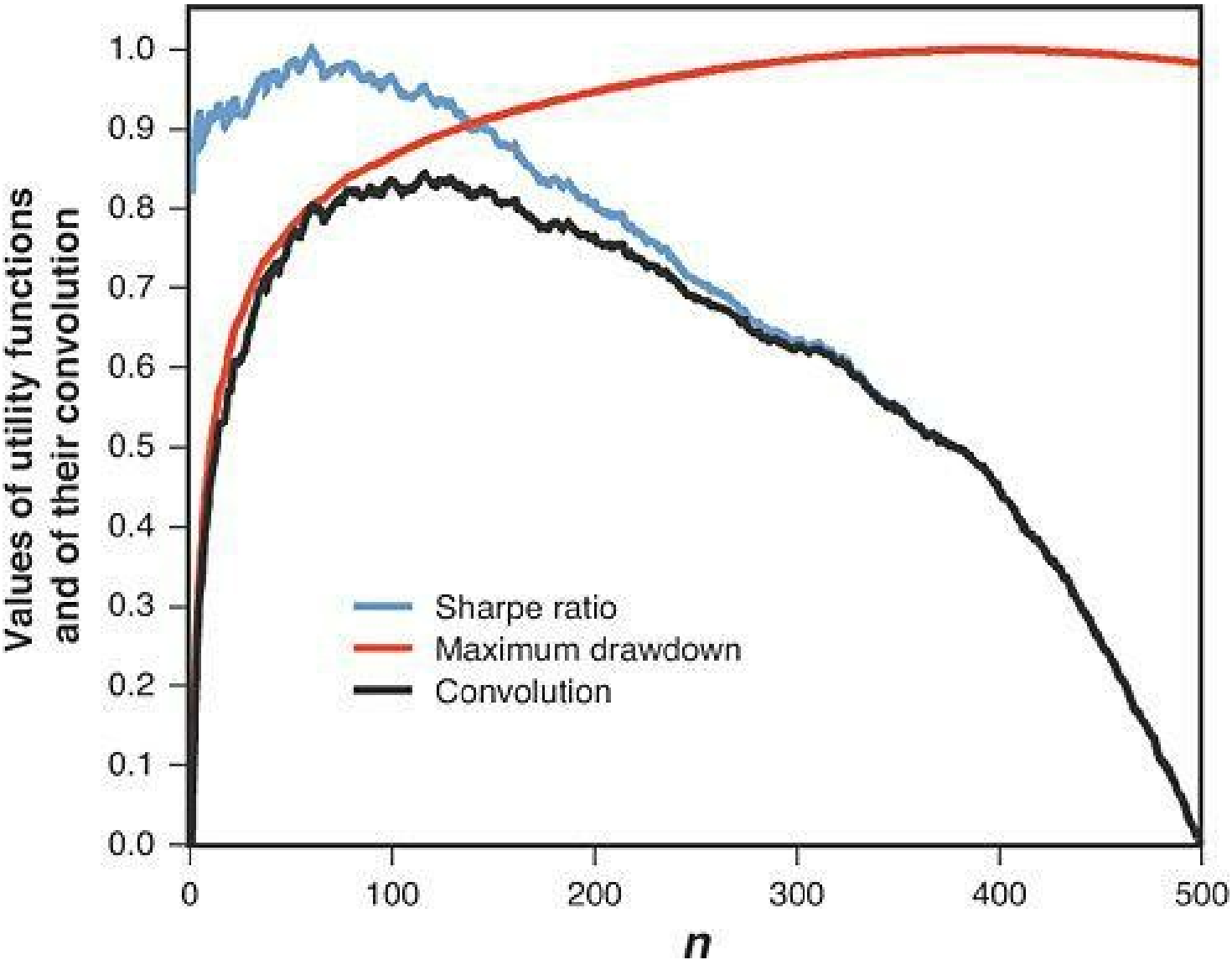
After such transformation the values of

both functions will lie within the 0 to 1 interval. Because optimization based on the maximum drawdown requires minimization of the utility function, its transformed values should be subtracted from 1 to be compatible with the function based on the Sharpe ratio.

As a result of the preceding operations, we obtain two utility functions possessing the common argument (n) and the same value range (from 0 to 1). To obtain the optimal n value, these functions are converted to a single one using multiplicative convolution. Figure 6.4.4 shows the example of two utility functions and their convolution for the short strangle/straddle strategy and the EPLN criterion. The convolution is

rather smooth and unimodal. It reaches the maximum at $n = 116$. This result is easy to interpret: The optimal investment solution would be to distribute capital among 110 to 120 underlying assets.

Figure 6.4.4. Utility functions based on the Sharpe ratio and the maximum drawdown and their convolution. Option strategy: short strangle/straddle; criterion —EPLN.



Using the same method we can obtain optimal parameter values for the other criteria and option strategies. The results for 11 criteria and 4 strategies are summarized in Table 6.4.1. It is evident that the parameter optimum is highly

versatile and varies considerably between different criteria (within a certain strategy) and between different strategies (within a certain criterion). Obvious regularities in distribution of optimal values are hard to define. Nevertheless, we can observe that when using criteria forecasting profit probabilities rather than expected profits, the optimal decision is to allocate funds among fewer underlying assets. We can also note that for calendar spreads it is optimal to use a smaller number of underlying assets than for strangles/straddles.

Table 6.4.1. Optimum

Values of the Parameter Specifying the Number of Underlying Assets (n) Used for Capital Allocation. Criteria Abbreviations Are the Same as in Figure 3.2.4.

	Option Strategy			
Criterion	Short Strangle/Straddle	Long Strangle/Straddle	Short Calendar Spread	Long Calendar Spread
EPLN	116	59	6	69
EPEM	120	134	1	32
EPES	138	128	2	118
PPLN	14	5	4	1
PPEM	5	3	8	55
PPES	1	20	4	3
RPLL	123	8	4	1
RPLE	3	1	1	14
RPLS	2	78	1	1
BEVR	19	9	44	1
IV/HV	66	1	36	1

6.5 Summary

The research presented in this chapter confirms that in most cases the criteria can effectively be applied for selection of underlying assets. The forecasting abilities of the criteria are stable in time and hold for both the long and the short option strategies. The revelation of promising trading opportunities is accomplished by way of sorting combinations according to their criteria values. The higher the position of the underlying asset in the resultant ordering, the higher its profit potential

is.

The effectiveness of criteria application depends on the procedure used to optimize the parameter specifying the number of underlying assets selected from the top of the ordering. We developed an optimization model based on the convolution of two empirical utility functions. This model produces unambiguous results that determine a narrow range of optimal parameter values.

Using this model we determined the optimal numbers of underlying assets for 11 criteria and 4 option strategies. However, you need to be careful with respect to the absolute values presented

in Table 6.4.1. These values relate to a narrow time interval (100 days). Furthermore, many parameters used in the computation of criteria (Chapter 2, "Review of the Main Criteria") and in the algorithms of selecting option combinations (Chapter 4) and strategies (Chapter 5), can fundamentally influence the figures.

Practical application of approaches and methods described in this chapter requires their adjustment to specific conditions. The set of criteria and their parameters, algorithms of creating combinations, time interval for optimization—all these factors need to be thoroughly adopted to the current market conditions.

Part III. Multicriteria Analysis

In previous parts of this book, we considered the individual properties of criteria that are constructed and applied independently from each other. Here we demonstrate that simultaneous application of several criteria allows exploiting different individual advantages inherent to each of them. The cumulative synergetic effects of such a

multifaceted approach are quite noticeable.

In Chapter 7, "Basic Concepts of Multicriteria Selection as Applied to Option Combinations," the basic concepts of multicriteria selection are adapted to options. We apply two techniques of the multicriteria analysis: the Pareto method and the convolution. Both approaches are explained in detail and accompanied with step-by-step calculation examples. We carry out a comparative analysis of the two alternative methods and outline their advantages and deficiencies.

The majority of criteria are interrelated. Their correlation is natural and follows

from intrinsic qualities of underlying valuation procedures. In Chapter 8, "The Impact of Criteria Correlation on Multicriteria Selection," we examine the extent and the shape of these relationships and describe their effects on the profitability of option combinations selected using the multicriteria analysis. We demonstrate that for both multicriteria methods the quality of selection is higher when the correlation of applied criteria is weaker. It is established that the development of criteria possessing high effectiveness characteristics and having at the same time a low degree of interdependence represents a key factor determining the success of multicriteria selection.

chapter 7. Basic Concepts of Multicriteria Selection as Applied to Options

7.1 Introduction

In Part II, “The Main Areas of Criteria Application,” we investigated different areas of criteria application. Each criterion was considered separately as

an independent valuation procedure. We ascertained that utilization of strictly formalized criteria positively influences investment potential. Although most criteria show statistically reliable predictive power, in most cases it is rather weak. Nevertheless, this small forecasting effect gives a considerable advantage to those who use it over those who ignore systematic criteria application.

The correlation between the profitability and the forecasts of any criterion is never perfect. It is usually unstable in time (when a criterion performs well at some time intervals and fails at others). Furthermore, the effectiveness of the same criterion can vary considerably

depending on its parameters, area of application and other specific factors.

Thus, we possess a broad range of criteria each of which is appropriate only under certain conditions. In other words, efficiency of every criterion is restricted to a definite set of parameters and circumstances. Therefore selection of option combinations by simultaneous application of several criteria seems to be more effective than selection based on monocriterion analysis. Concurrent utilization of many criteria allows exploiting different individual advantages inherent to each of them. The cumulative synergetic effect of such a multifaceted approach is expected to be quite noticeable.

In this chapter we consider multicriteria selection of trading opportunities arising in the option market. Various methods and techniques of multicriteria analysis are the subject of a whole section in mathematics. Without delving into details of this scientific theory, we address a practical issue: combining and simultaneous application of many criteria to determine profitable trading opportunities.

Similarly to the monocriterion case, the mission of multicriteria analysis is to order all available alternatives by the degree of their preference. We denote preference relation by the < symbol. A fully ranked set must satisfy several

basic conditions. Thus, $A < B$ and $B < C$ implies $A < C$ (transitivity condition). Apart from that, we must be able to compare any two elements with each other and get an unambiguous result (comparability condition). Quite often—when several alternatives are compared using a set of criteria rather than one separate criterion—these two conditions cannot be satisfied.

The following example illustrates the last statement. Suppose that upon comparison of three option combinations (*A, B, and C*) by three criteria, the following results were obtained: $A = (1; 2; 3)$, $B = (2; 3; 1)$, $C = (3; 1; 2)$—criteria values are given in parentheses. The alternative is considered to be

better if it exceeds another one according to most criteria. Obviously, B > A according to the first and the second criteria. Also, C > B according to the first and the third criteria. However, we cannot conclude that C > A because A is better than C according to two criteria (the second and the third) and thus A > C. We are doomed to walk along this vicious circle forever trying to find the best of three alternatives. Such a situation is caused by the absence of transitivity in this example.

Another idea, which intuitively seems to be appropriate, is to consider the alternative to be better if it is preferred to another one according to all criteria. It solves the problem of nontransitivity

and allows finding the best of all comparable alternatives. However, not all alternatives are comparable. In our example with *A*, *B*, *C* this condition is not met for any of these variants, and we have to declare them to be incomparable. In such situations only partial ordering of alternatives is achievable.

Thus, on one hand, we possess a whole set of effective evaluation criteria. On the other hand, complete ordering of trading variants via simultaneous application of many criteria is impossible due to nontransitivity and incomparability. These obstacles do not have a simple and unique solution. However, several methods allow

selecting the best alternatives on the basis of multicriteria analysis.

Partial refusal from transitivity and comparability leads to the Pareto optimum. Replacing several criteria with a new combined criterion is the essence of the *convolution* approach.

7.2 The Pareto Set

According to the previously discussed approach, one alternative can be preferred to another if it exceeds the latter according to all criteria (or to be more precise, if it is not worse according to all criteria and is better

according to at least one of them). As we have already seen, the ordering within the framework of such approach may be impossible because alternatives A = (1; 2; 3), B = (2; 3; 1) and C = (3; 1; 2) are incomparable. However, we can compare them with alternatives D = (1; 2; 2) and E = (3; 1; 1) and conclude that $A > D$ and $C > E$, so D and E are inferior to A and C. *A set of alternatives that are not exceeding each other (are not dominating each other), but at the same time dominate over the rest of alternatives, is called the Pareto set.* In our example *A, B, and C* belong to the "dominating" Pareto set (they are preferred to other variants according to the values of three criteria), whereas *D*

and *E* do not. Note that although *B* does not dominate over *E* (*B* is worse according to the first criterion, is better according to the second one, and is equal according the third one), *B* belongs to the Pareto set and *E* does not. The reason is that alternative *B* cannot be excluded from the dominating set because *A* and *C* do not dominate over it. Formally speaking, for every alternative X that does not belong to the Pareto set θ, there is at least one $Y \in \theta$ so that *Y* > *X*.

Thus, instead of selecting a single best variant (or any desirable number of best variants as in the case of complete ordering by one criterion), we obtain the set, each element of which can be

considered as “the best.” Note that we cannot control the number of elements forming the Pareto set; it can vary from case to case and does not depend on our wishes. It is possible to face a situation when only one single element dominates over the remaining alternatives (in this case the Pareto set consists of one element) or even the situation when no dominating alternatives exist at all. (In this case the Pareto set coincides with the initial set.) However, the latter case is extremely rare in practice. Problems arising when the Pareto set consists of the insufficient number of elements can be solved by applying the specific method that allows increasing the number of dominating elements (see

section 7.2.2).

7.2.1 The Algorithm of Forming the Pareto Set

The Pareto set can be formed by application of a relatively simple algorithm following from the definition given in the previous section. Because the Pareto set includes elements that do not dominate over each other but dominate over the alternatives not belonging to this set, the simplest algorithm consists in consecutive selection of elements and comparing the selected element with each of the rest. If we find an element dominating over the

selected one, we pass on to the next alternative because the selected element certainly does not belong to the Pareto set. If none of the elements dominates over the selected one, it is included in the Pareto set. Then we select the next element and go on until all the elements of the initial set are checked.

This easily programmable and intuitively understandable algorithm has one substantial drawback. Because every element of the sample has to be compared with almost all other elements, the total number of comparisons is considerable. (It is proportional to the square of the sample size.) The complexity of the comparison procedure also depends on the number

of criteria used. According to our experience, the time required to process (using the personal computer) a sample consisting of about 1,000 elements evaluated by two criteria is quite acceptable. However, it rises sharply as these parameters increase.

The more effective but more complicated algorithm is based on the preliminary ordering of elements according to criteria values. Imagine that an initial set is evaluated with two criteria. At the first stage all elements are ordered by one of the criteria in the descending order. The first element has the highest value of this criterion and hence belongs to the Pareto set; none of the other elements can dominate over it.

Moving further along the list of the elements in the descending order, we compare them (according to the value of the second criterion) with the found Pareto set member, and all elements that turn out to be dominated by it are excluded from consideration. Thus after the first run through the sample, we have one element that certainly belongs to the Pareto set and a number of candidates for inclusion in this set. At this stage we can name another element belonging to the Pareto set. It is the first of the candidates: Because the elements are ordered by the criterion values, it is not dominated either by the rest of the elements nor by the first Pareto set element. We include it in the Pareto set

and compare the rest of the elements with it excluding all dominated ones. This is the second run. We proceed with such iterations until the initial ordering is through. By this time the Pareto set is finally formed. This algorithm requires less operations (It is proportional to $N\ log(N)$, where N is the number of elements in the ordering.)

Consider an example illustrating the previously described algorithm. Suppose that an initial set consists of 30 option combinations constructed according to the short straddle/strangle strategy. To form such an initial set, 30 stocks were randomly selected, and a single combination (either strangle or straddle) possessing the most symmetric

payoff function was constructed. (The symmetry was assessed by the method described in section 4.4.1.) All combinations were evaluated by two criteria: expected profit on the basis of lognormal distribution (EPLN) and *IV/HV* ratio. Only options with the nearest expiration date (March 16, 2007) and strikes closest to the current stock price were used. (The evaluation was conducted on March 1, 2007.) The results are presented in Table 7.2.1.

Table 7.2.1. Forming the Pareto Set for 30 Option Combinations. Values of

Two Criteria—Expected Profit on the Basis of Lognormal Distribution (EPLN) and IV/HV Ratio—Were Estimated on March 1, 2007. Combinations Are Sorted by the EPLN Criterion. Asterisks Denote Combinations That Became the Candidates for Inclusion in the Pareto Set After Comparison of All 29 Combinations with VG.

Stocks Included in the Pareto Set Are Shown in Gray.

Underlying stock	EPLN, %	*IV/HV*	Underlying Stock	EPLN, %	*IV/HV*
VG	28	1.7715	RMBS	9	1.6371
*IDIX	19	2.3308	ILMN	9	1.6235
ELN	18	1.7706	TRE	9	1.2231
POZN	16	1.5804	CWTR	9	1.4274
*ONXX	16	2.2605	PDLI	9	1.5428
*SUP	14	2.0942	IMMU	8	1.1897
KKD	14	1.4568	*MIR	8	2.4556
*GPS	10	1.9432	VPHM	8	1.3709
CHS	10	1.6729	FL	8	1.6726
TKLC	10	1.3761	NTES	8	1.5154
*KG	10	2.1543	LIFC	7	1.5247
*DNDN	10	1.8636	GYMB	7	1.6171
TOMO	9	1.3566	DELL	7	1.5708
IOC	9	1.2756	CTB	7	1.3106
SUNW	9	1.3232	HUN	7	1.5458

Ordering by the first criterion (EPLN) determines the first element of the Pareto

set—VG (Table 7.2.1). All other alternatives are compared to it according to the second criterion, and elements that are dominated by VG are excluded. As a result we get only seven combinations that can pretend to be included in the Pareto set: IDIX, ONXX, SUP, GPS, KG, DNDN, MIR (marked with asterisks in Table 7.2.1). The second element included in the Pareto set is IDIX. When the six remaining elements are compared with IDIX, five of them turn out to be dominated by it. To the contrary, MIR is not dominated by IDIX, thus becoming the third and the last Pareto set member.

The algorithm applied in this described example required merely 36 operations

(instead of 870 operations that would have been needed if we compared all 30 elements among themselves). If the initial set size is larger (hundreds of thousands could be quite a realistic estimate), the advantage of this algorithm will become even more evident.

7.2.2 Widening the Pareto Set and the "Layer" Notion

The practical application of the Pareto method is often hampered by the small quantity of elements constituting the Pareto optimal set. For example, when 1,000 combinations are evaluated by two criteria, in most cases the Pareto set

includes not more than 5 of them. At the same time evaluating such a great amount of variants, we expect to select at least about 50 best alternatives.

When widening the set of potential trading opportunities, it is inevitable to use suboptimal elements in addition to the optimal ones. Suboptimal elements are those that are optimal for the set derived from the initial one by excluding Pareto-optimal elements from it. In the previous example (Table 7.2.1) we found that 3 stocks out of 30 form the Pareto set. If we exclude these 3 stocks and repeat the whole procedure for the remaining 27 elements, the suboptimal Pareto set will arise.

Furthermore, suboptimal points can also be excluded from the initial set to define next suboptimal elements among the remaining ones. Such iterations can be repeated until the total number of optimal and suboptimal elements is sufficient. However, we should remember that each reiteration widens the set of the selected combinations with new elements, which are worse than the preceding ones. To distinguish between optimal, suboptimal, and subsequent suboptimal elements, we introduce the notion of the *layer*. The elements selected during the first iteration will be denoted as belonging to layer 1. The elements selected after the exclusion of layer 1 belong to layer 2; then comes

layer 3, and so on.

In our example (Table 7.2.1), combinations relating to stocks VG, IDIX, and MIR constitute layer 1. They must be excluded from the initial set for the purpose of building the second layer. After sorting the remaining 27 stocks by EPLN criterion, ELN stock turns out to be at the first place. (It means that ELN belongs to layer 2 of the Pareto set.) After excluding all elements dominated by ELN, only ONXX, SUP, GPS, KG, and DNDN remain. Because ONXX dominates over SUP, GPS, KG, and DNDN, layer 2 consists only of two stocks: ELN and ONXX. The same procedure is repeated for finding layer 3; we exclude ELN and ONXX from 27

stocks and find the Pareto set among the remaining 25 stocks. Layer 3 consists of three elements: POZN, SUP, and KG.

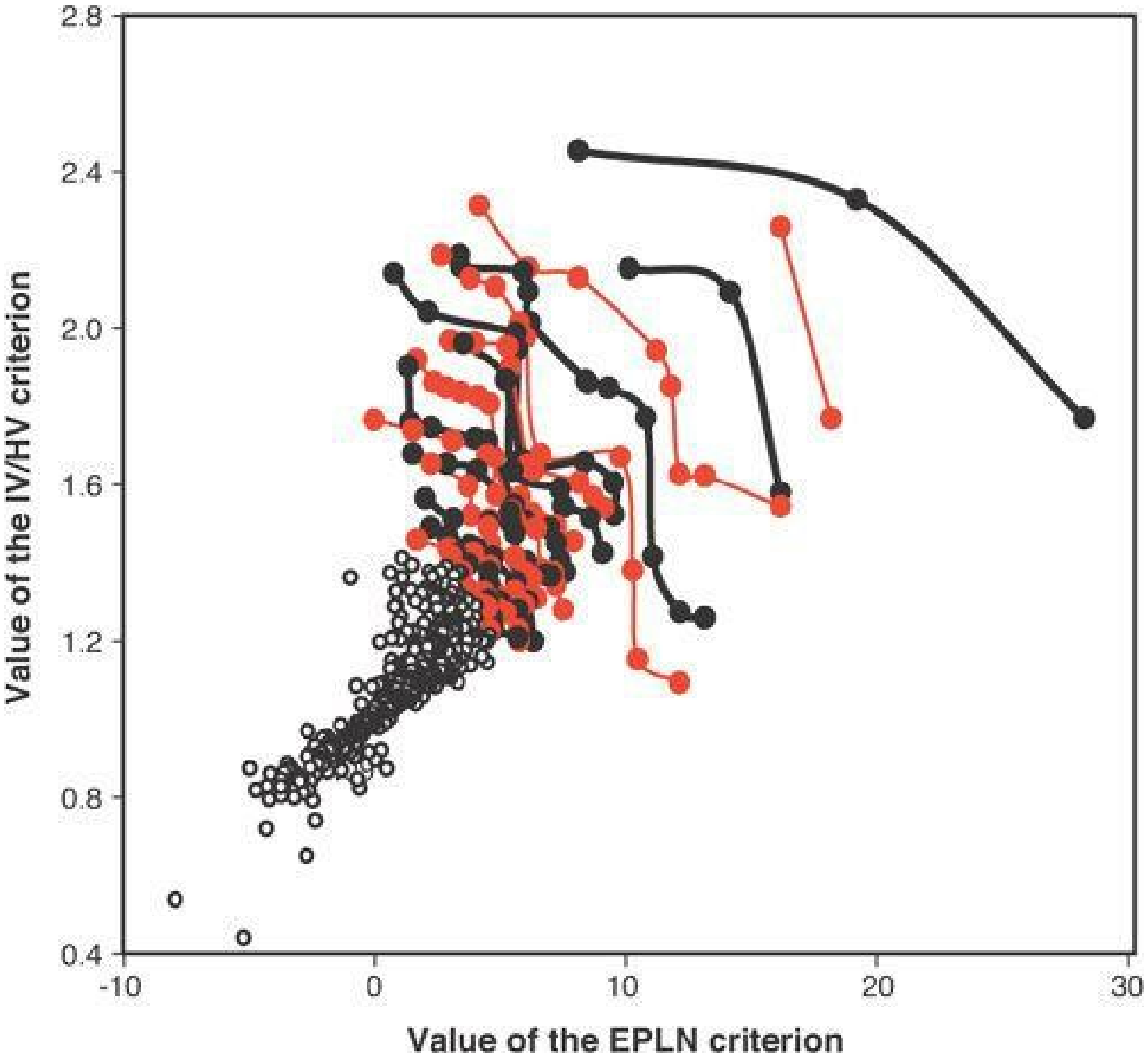
Table 7.2.1 shows that although combinations belonging to the second and the third layers are dominated by elements of the first layer, they have rather high criteria values. Therefore it would be worthwhile to add them to the Pareto optimal set (layer 1) if the number of its elements is insufficient.

Thus the selection of the best alternatives using one layer gives us 3 potentially best combinations, using two layers provides 5 (3 + 2) superior variants, and using three layers gives 8 (3 + 2 + 3) promising trading

opportunities. The Pareto set and its layer structure can be vividly demonstrated by plotting a chart with the values of the first criterion on one axis and the values of the second criterion on another axis. Each point of this chart represents an option combination, and its coordinates correspond to two criteria values. Figure 7.2.1 shows 20 Pareto layers (and all the remaining points of the initial set) for the pair of criteria used in Table 7.2.1. It can be easily seen that the first layers are situated in the top-right corner of the figure, in the area where both criteria have relatively high values. Subsequent layers are shifted lower and to the left, whereas points not belonging to the first 20 layers (shown

as contour circles) are in the area of low criteria values.

Figure 7.2.1. Layers of the Pareto set determined for 600 option combinations using two criteria—expected profit on the basis of lognormal distribution (EPLN) and IV/HV ratio. Points belonging to the same layer are connected by the solid line.



The concept of layers makes it possible to widen the set of selected combinations to any desirable limits, although we still cannot specify their

precise number (due to the fact that the exact number of elements included in each layer is uncontrollable). However, including each additional layer implies opening positions with worse criteria values than in the preceding layers. It means that if the criteria used are capable of ordering combinations by their trading potential (otherwise their application is useless!), the increase in the number of layers will inevitably lead to the decrease in the average profitability of selected combinations. To illustrate this important property we use 13 pairs of criteria. (All of them are described in Chapter 2, "Review of the Main Criteria.") The strategy and the method of option combinations creation

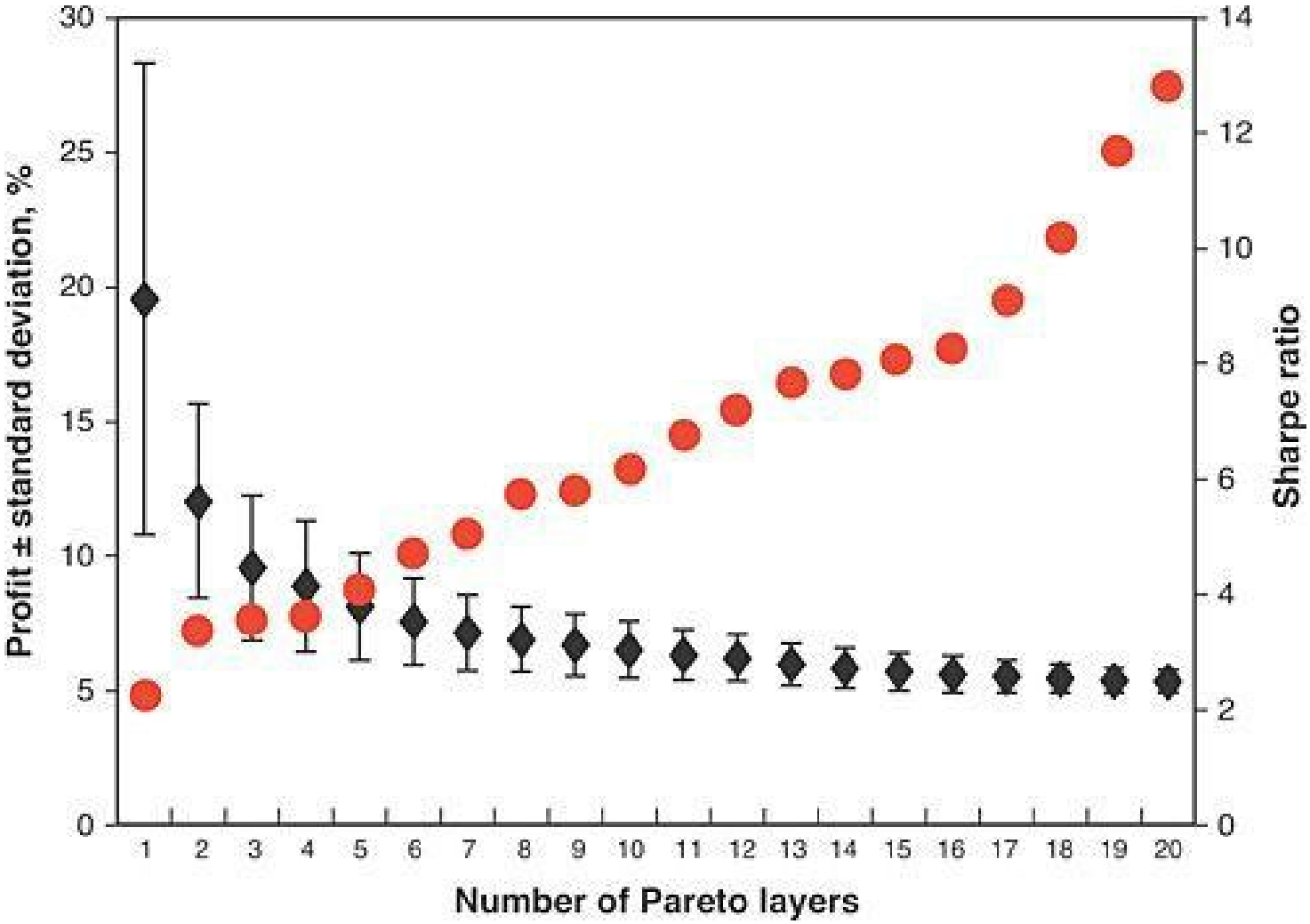
is the same as in the example of the Pareto set determination (see Table 7.2.1). However in the current example we widen the initial set to 600 combinations. (Six hundred stocks are utilized as underlying assets with one combination being created for each stock.) The profits and losses of combinations (expressed as a percentage of margin requirements) are estimated at the expiration date. For each of 13 pairs of criteria we determine 20 Pareto layers. Average profit/loss (and standard deviations) is calculated for all combinations included in the first *n* layers. (*n* runs from 1 to 20.) Figure 7.2.2 shows the result. The number of layers is plotted on the horizontal axis

and the average profit/loss values (± standard deviations) are on the left vertical axis. Obviously the average profit decreases as the number of layers used in the selection process increases. However, the negative effect of layers extension should not be overestimated. Note that the standard deviation value (which is the measure of profit variability) decreases as the number of layers increases. The ratio of the average to standard deviation (a version of the Sharpe ratio that expresses the profit and its variability simultaneously) is plotted on the right axis in Figure 7.2.2. In our example the Sharpe ratio steadily increases as the number of layers increases (Figure 7.2.2). It means

that due to the increase in the number of layers, portfolio diversification goes up, and consequently its variability (and the risk) goes down. Thus resolving the issue of the optimal number of layers to be used in the selection procedure requires a compromise between striving for profit maximization and portfolio risk minimization.

Figure 7.2.2. Relationship between profit (rhombi), Sharpe ratio (circles), and the number of Pareto set layers used to select superior option

combinations.



The previous example demonstrates that the increase in the number of layers leads to the decrease in the average profit due to the augmentation in the number of selected combinations. In

view of this finding, it would be useful to determine how the increase in the number of layers influences the growth rate of the number of selected combinations. To establish the relationship between these two variables, we have to find out the number of combinations included in each Pareto layer. Figure 7.2.3 shows that for all 13 criteria pairs the total number of combinations sharply increases as every subsequent layer is added to the set of selected combinations. If only 10 layers are used, the number of selected variants ranges from 50 to 100, whereas utilization of 20 layers raises this number sharply (from 110 up to 400 variants). It is noteworthy that for some

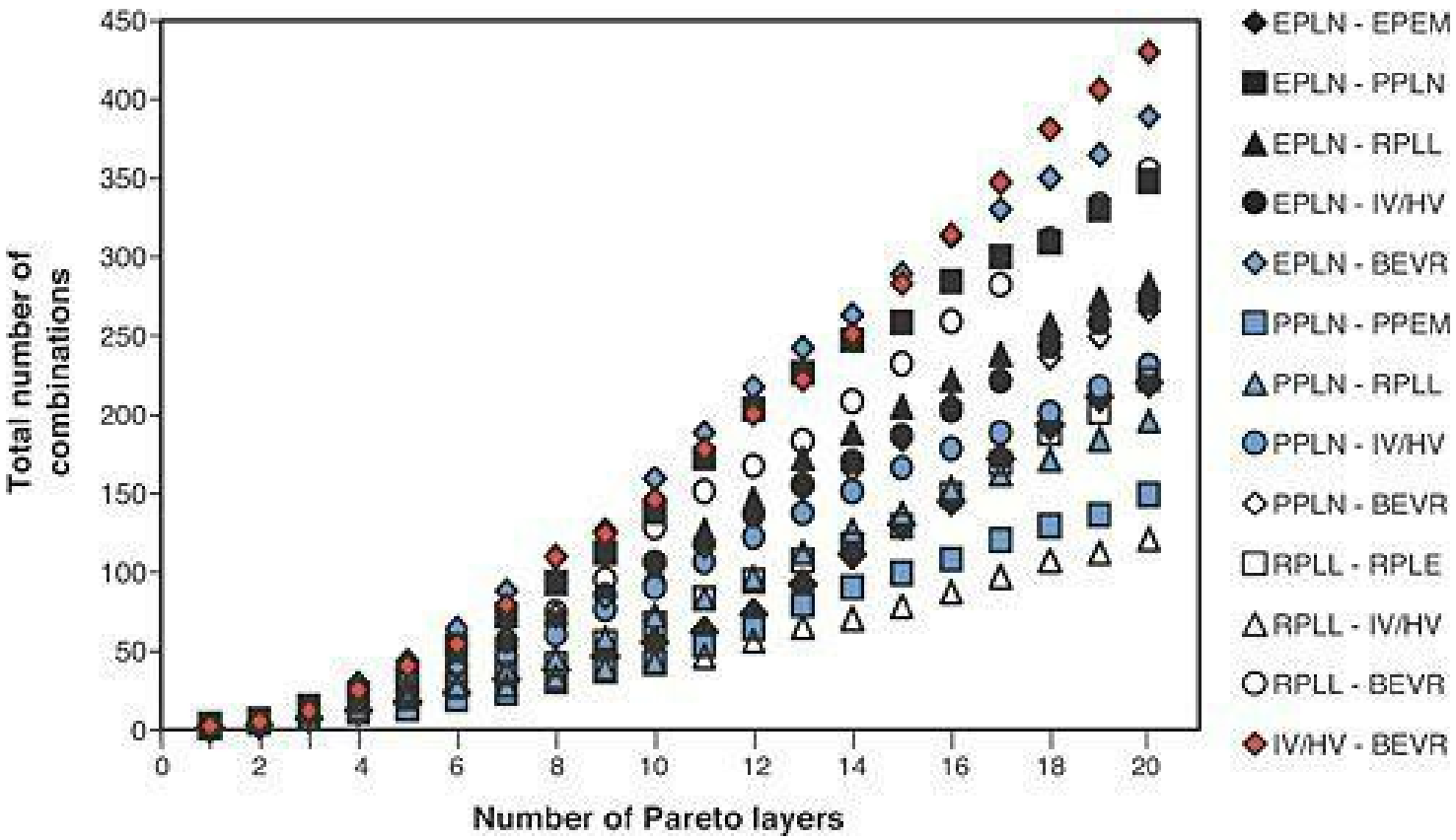
pairs of criteria the increase in the number of layers induces a relatively moderate growth in the number of selected combinations, whereas for other criteria pairs addition of layers leads to a strong rise in the number of selected combinations. What is the reason for such tremendous differences? The point is that most criteria are correlated to some extent. This implies that if a certain combination has a high value of some criterion, the value of another criterion will also be high (if these criteria are correlated). Below we demonstrate that the stronger the correlation between the two criteria, the lesser number of combinations is included in the Pareto set. Therefore the

great difference in the number of combinations selected by different pairs of criteria (Figure 7.2.3) is due to their different degree of correlation.

Figure 7.2.3. Relationship between the total number of selected combinations and the number of Pareto set layers used in the procedure of selection. The relationship is presented for 13 pairs of criteria with the following abbreviations:

EPLN, EPEM, and EPES are expected profits on the basis of lognormal, empirical, and symmetrized empirical distributions; PPLN, PPEM, and PPES are profit probabilities on the basis of the same three distributions; RPLL, RPLE, and RPLS are ratios of expected profit to loss for these distributions, IV/HV is the ratio of implied

volatility to historical volatility, BEVR is the break-even range normalized by the underlying asset price.



We paid so much attention to this topic

because the number of combinations selected for trading is an important practical issue. On one hand, the fewer variants selected, the higher criteria values and expected average profitability. On the other hand, risk management requires reasonable portfolio diversification thereby stimulating to select more combinations. Looking for a compromise between these two factors, we have to optimize the number of Pareto layers considering the relationship between the growth rate of the number of combinations and the correlation between criteria.

7.3 Convolution

An alternative way to perform the multicriteria analysis is to reduce a number of criteria to a single one. The resulting compound criterion is called a convolution. This procedure takes into account the whole criteria vector and frequently performs considerably better than other valuation techniques. Below we consider the basic methods of criteria convolving.

The sum of criteria represents an *additive convolution*. Multiplying criteria values by weight coefficients can modify the degree of their influence on the resultant convolution; the higher the criterion weight, the stronger its

effect on the selection results.

The product of criteria represents a *multiplicative convolution*. Criteria values can be raised to a certain power before multiplying them. The higher the importance of the criterion, the higher the power should be (similar to using weights in additive convolution). Multiplicative convolution is appropriate only for criteria possessing exclusively positive values; otherwise multiplying two negative figures will convert two bad values into one positive convolution value. However, if only one criterion possesses negative values, this effect will not arise and application of the multiplicative convolution is possible. If one of the criteria is equal to

zero, multiplicative convolution is also equal to zero, which does not happen in the case of additive convolution. Generally, criteria with low values have greater impact in multiplicative convolution than in the additive one.

Additive convolution is more appropriate for criteria that are close in their nature and are similarly scaled. Particularly, these conditions are met by some of the criteria classified as forecasting (see classification in Chapter 1, "General Presentation and Review of Criteria Properties"). For example, combining "expected profit on the basis of lognormal distribution" and "expected profit on the basis of empirical distribution," it is natural to sum up

these criteria. On the other hand, multiplicative convolution seems to represent a more appropriate solution for combining the criterion expressing expected profit with the criterion predicting profit probability (on the basis of any distribution). In this case we exploit a useful product property; if the estimated profit probability is close to zero, the combined criterion will also converge to zero. However, there is an additional point in applying the product; if the expected profit is negative, then multiplying it by lower probability leads to the value that is closer to zero and hence the higher one. Nevertheless, it does not create any difficulties if combinations with negative expected

profits are excluded from consideration.

Apart from additive and multiplicative convolutions there is a *selective convolution* when the highest (or the lowest) value of all criteria is assigned as a convolution value. In Chapter 5, "Selection of Option Strategies," we introduced the method of *minimax convolution* for utility functions. Similar principles can also be applied to convolving several different criteria.

Calculating the convolution we should remember that criteria can be measured in different units and have different scales. There are several methods to reduce them to a unified form. We can apply the normalization method,

subtracting the average from criteria values and dividing the result by the standard deviation. Alternatively, we can subtract minimum values and divide the result by the difference between maximum and minimum values. In this case the values of all criteria will lie within the range from zero to one, which facilitates their convolution considerably. The former is more appropriate for the additive convolution; the latter is more suitable for the multiplicative one.

Another approach to the criteria convolving is to find a distance between a given element and an ideal one. This can be performed by reducing the criteria values to the (0, 1) interval and

assuming that the ideal variant has all criteria values equal to one (that is, all criteria reach their maximum achievable values). For each evaluated element j of the initial set, we calculate the convolution value R according to the following formula:

$$R_j = \sqrt{\sum_{i=1}^{n} (1 - X_{i,j})^2}$$

where $i = 1, 2 \ldots n$ is the number of criteria used; $X_{i,j}$ is the value of criterion i for element j. The best element is the one with the lowest distance from the ideal (that is, the one with minimum R value).

For the research described next we used additive convolution. Although this method is rather simple and quite rough, it is still the most appropriate one when conducting heterogeneous statistical studies requiring results comparability. In real trading, however, it is preferable to use more advanced convolution methods, similar to those previously described or others not mentioned here.

7.4 Comparative Analysis of Multicriteria and Monocriterion Selection Effectiveness

From the scientific point of view, multicriteria selection seems to be a much more advanced and sophisticated method relative to the monocriterion approach. At the same time, it is widely known that application of rough and approximate valuation techniques to complex dynamic systems (including financial markets) often gives better results than methods based on subtle adjustment. Hence, special research is required for answering the paramount question, namely, whether the multicriteria selection has any advantages over the monocriterion method.

To investigate this important issue in

details, we utilize 11 criteria and all their possible pairs (totally 55 pairs, Table 7.2.1). The same strategy (short straddle/strangle) and the same method of creating combinations as in the previous examples will be applied here. Criteria values will be calculated as of March 1, 2007. Profit and loss (expressed as percentage of margin requirements) of 600 combinations will be estimated for the closest expiration date (March 16, 2007).

For each criteria pair we determine the 20-layers Pareto set. For each layer we calculate average profit p_j^i of combinations belonging to it. (i is the number of the Pareto layer; j is the

number of the pair of criteria.) For all 55 criteria pairs we also build additive convolutions and calculate average profits of combinations selected according to them. To make selection results obtained by the application of convolution comparable with Pareto results, for each of 55 criteria pairs we calculate average profit s_j^i using the number of combinations equal to their number in each Pareto layer. For example, if for some pair of criteria there are 5 combinations in the first layer, 12 in the second, 24 in the third (and so on up to the 20^{th} layer), we calculate the average profit of combinations selected by convolution

using 5, 12, 24 (and so on) best combinations as well. As the result we obtain 1,100 average profits p_j^i (20 layers × 55 criteria pairs) corresponding to the Pareto method and 1,100 average profits s_j^i relating to the convolution. We also need to calculate profits m_j^i corresponding to monocriterion selection. To guarantee the comparability of m_j^i with profits corresponding to multicriteria selection (p_j^i and s_j^i), we use the following method. For one of two criteria constituting a certain pair, we calculate average profit for *n* best combinations. (*n* is equal to the number of

combinations included in the corresponding Pareto layer: 5, 12, 24, and so on in the previous example.) The same procedure is repeated for the second criterion. Then for every *n* the averages corresponding to the first and the second criteria are further averaged. The 1100 m_j^i values obtained this way represent average profits of monocriterion selection.

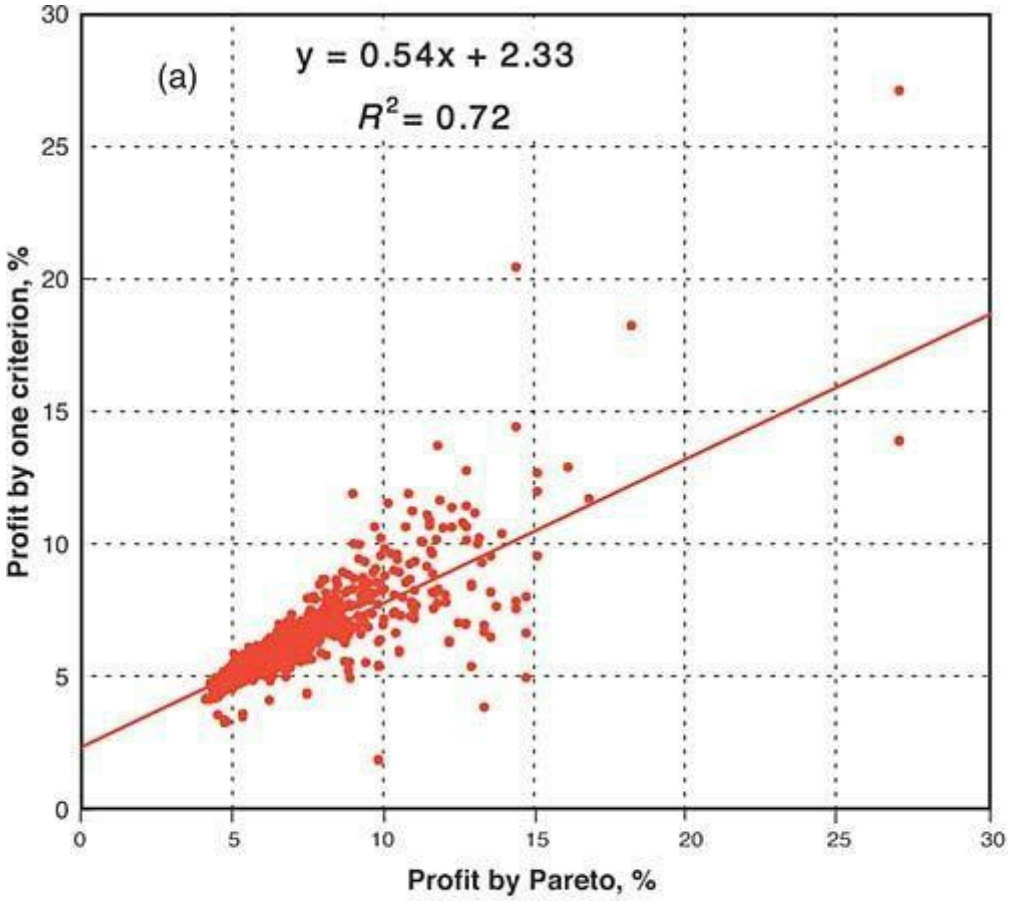
To determine whether the profitability of the Pareto selection method is higher than the profitability of monocriterion selection, the following regression model will be analyzed. Average profits obtained by application of the Pareto method (p_j^i) are plotted on the horizontal

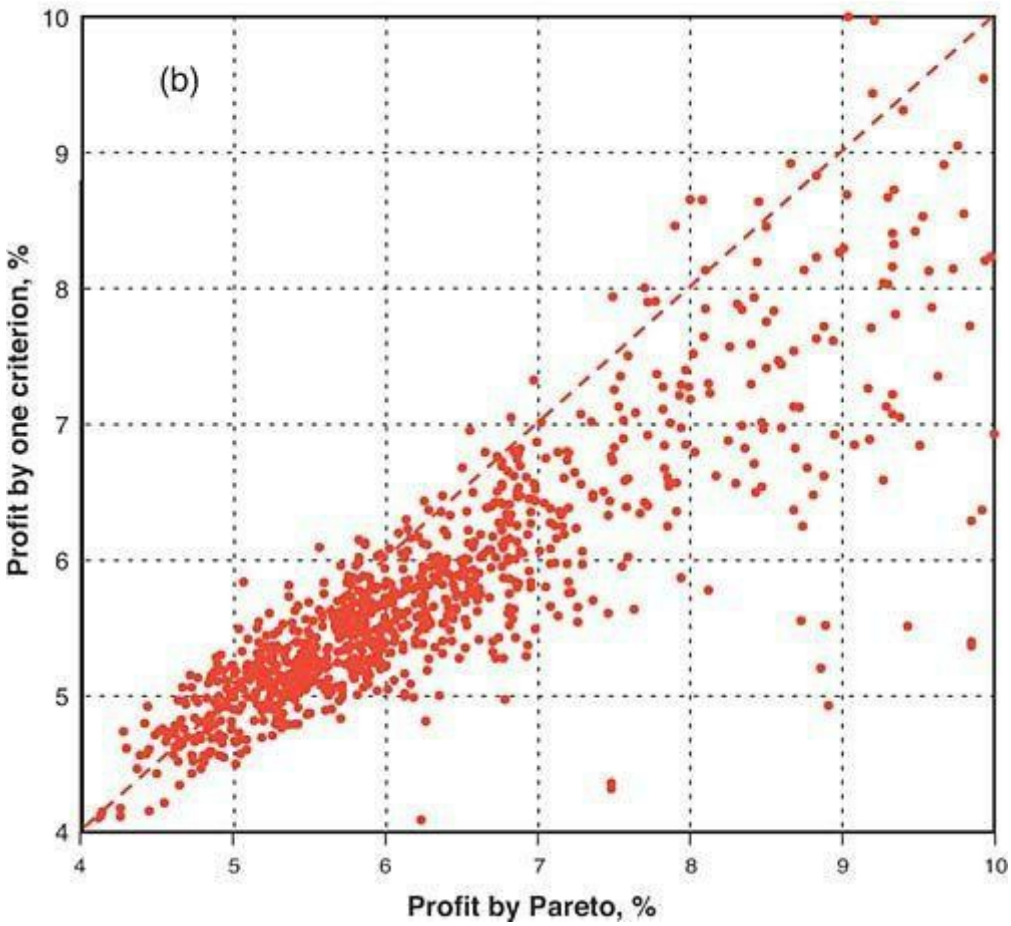
axis, whereas profits obtained by monocriterion selection (m_j^i) are plotted on the vertical axis. Totally 1,100 points (p_j^i, m_j^i) representing 55 criteria pairs and 20 layers are plotted on the two-dimensional chart. If the Pareto selection profit is similar to the monocriterion one, the points lie across the regression line with slope coefficient of 1 ($y = x$, which means that profits obtained by both selection methods are equal). The slope coefficient >1 implies the advantage of monocriterion selection whereas the slope coefficient <1 implies that the Pareto selection profit is higher. Figure 7.4.1(a) shows this regression. Two variables turned out to be highly

correlated ($r^2 = 0.72$) with the slope coefficient less than one ($b = 0.54$), and its difference from one is statistically significant at the high level ($p < 0.00001$). These results imply that multicriteria selection based on the Pareto method is better than monocriterion selection.

Figure 7.4.1. (a) Relationship between the average profit of combinations selected by one criterion and the average profit of

combinations selected by two criteria using the Pareto method. (b) The same for narrower range of average profits. The inclined dashed line divides the area where the profit of combinations selected by one criterion is lower (dots below the line) than the profit of combinations selected by the Pareto method (dots above the line).


(a)
y = 0.54x + 2.33
R^2= 0.72
Profit by one criterion, %
Profit by Pareto, %
0
5
10
15
20
25
30


(b)
Profit by one criterion, %
Profit by Pareto, %
4
5
6
7
8
9
10

The small area of Figure 7.4.1(a) contains the majority of dots. If this data is examined at the narrower profit range, we can visually observe that most dots are situated below the line with a slope coefficient of 1 (dotted line in Figure 7.4.1[b]). Totally, in 904 cases out of 1,100, multicriteria selection of option combinations using the Pareto method resulted in higher profit than monocriterion selection. (In 18 cases profits were equal.)

Our next task is to verify whether the second multicriteria selection method (the convolution) also provides higher average profits than the monocriterion selection. This issue will be studied in

the same manner as previously described—by analyzing the two-dimensional chart consisting of 1,100 dots with (s_j^i, m_j^i) coordinates. Average profits of combinations selected by convolution will be plotted on the horizontal axis, whereas monocriterion profits will be plotted on the vertical axis.

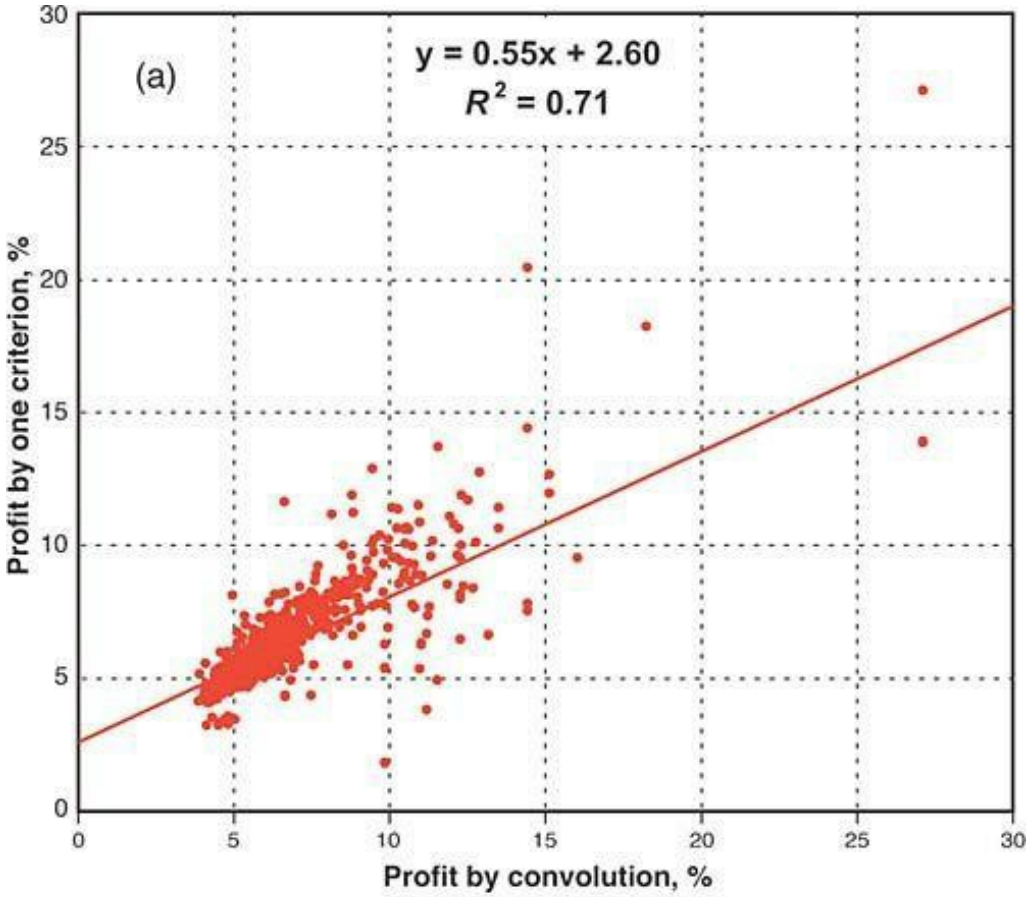
The results shown in Figure 7.4.2(a) demonstrate that both profits are strongly correlated ($r^2 = 0.71$), the slope coefficient of the regression line is lower than one ($b = 0.55$), and its difference from one is statistically significant at a high confidence level ($p < 0.001$). As it was in the Pareto case,

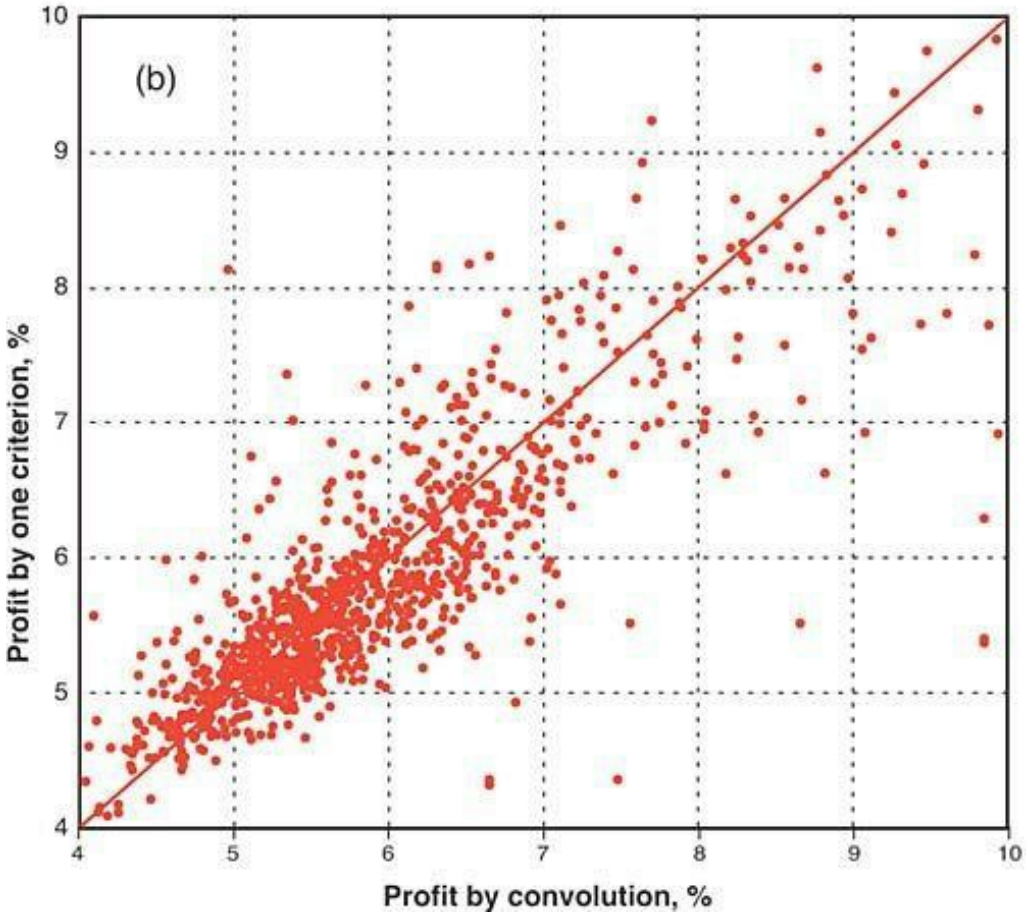
most dots are situated below the line with the slope coefficient 1, which can be easily distinguished at a narrower profit range where the majority of points lies (dashed line in Figure 7.4.2[b]). Totally, in 629 cases out of 1,100, selection by application of the convolution technique resulted in higher profits than monocriterion selection. (In 28 cases profits were equal.)

Figure 7.4.2. (a) Relationship between the average profit of combinations selected by one criterion and the

average profit of combinations selected by two criteria using the convolution method. (b) The same for narrower range of average profit. The inclined dashed line divides the area where the profit of combinations selected by one criterion is lower (dots below the line) than the profit of combinations selected by the convolution

method (dots above the line).


(a)
y = 0.55x + 2.60
R^2 = 0.71
Profit by one criterion, %
Profit by convolution, %
0
5
10
15
20
25
30


(b)
Profit by one criterion, %
Profit by convolution, %
4
5
6
7
8
9
10

The advantage of the Pareto method of multicriteria analysis over the monocriterion selection is more pronounced than the advantage of the multicriteria analysis realized via the convolution method. Comparison of Figures 7.4.1(b) and 7.4.2(b) reveals that although in the first case the majority of dots are situated below the line of parity between two selection methods; in the second case such allocation is less evident. Probably this phenomenon can be explained by the higher profitability of the Pareto method as compared with the selection by convolution. (This issue is discussed in detail next.)

The results presented in Figures 7.4.1

and 7.4.2 are based on March 1, 2007 data. We repeated the same analysis for other dates within the seven-year interval and obtained qualitatively similar results; multicriteria methods steadily produced higher average profits than monocriterion selection.

Thus application of multicriteria analysis generates higher profits than monocriterion selection. However, one important note has to be made at this point. Our conclusion holds true only in its general form, when the superiority of one of the two multicriteria methods has to be determined unambiguously. Indeed, the majority of dots lie below the line with the slope coefficient $b = 1$; however, not all of them do. Although

such distribution of data is not random but statistically significant, 178 dots under the Pareto selection and 443 dots under the convolution selection (which is 16% and 40% of cases respectively) are still situated above this line. Hence, it would be more precise to conclude that *in general* multicriteria analysis gives better results than monocriterion selection; at the same time, some criteria can solve the selection problem more effectively if applied separately than in combination with other criteria. It is quite difficult to determine the criterion that would generate steadily better results alone than in multicriteria association. Our experience shows that during some periods of time certain

criteria demonstrate better results in separate applications, whereas at other periods other criteria demonstrate the same effect. Finding factors influencing this dynamic is the subject of special research and falls beyond the boundaries of this book. For practical purposes we recommend application of multicriteria analysis because in most cases it shows steadily better results.

7.5 Comparative Analysis of Two Multicriteria Selection Methods: Pareto Versus Convolution

Each of the two basic methods of multicriteria selection—Pareto and convolution—has its own advantages and drawbacks. Which of these approaches to the multicriteria selection is preferable?

The method used in the previous section for comparative analysis of multicriteria and monocriterion selections can be applied here again. The same 11 criteria are utilized to form 55 pairs. For each pair the best combinations are selected by the Pareto method (20 layers) and by additive convolution. Within each criteria pair the number of combinations selected by convolution corresponds to

the number of combinations included in the Pareto layers. For each of the 55 criteria pairs and 20 layers, average realized profits are calculated at the expiration date. This provides 1,100 observations for each of the two multicriteria selection methods.

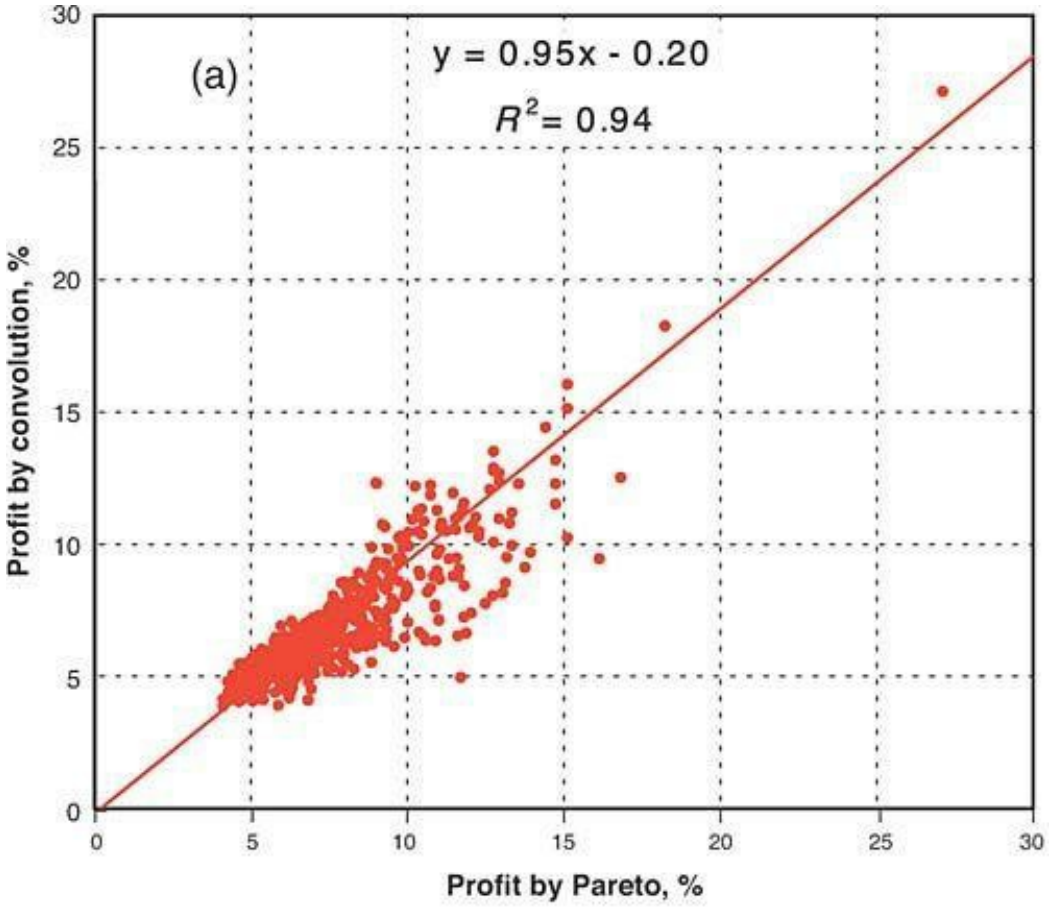
To verify whether profits under two various methods of multicriteria selection are different, we build the regression where average profit under the Pareto selection is plotted on one axis and profit under the selection using convolution on the other axis. The deviation of the slope coefficient of the regression line from 1 expresses the extent to which one method is overperforming another.

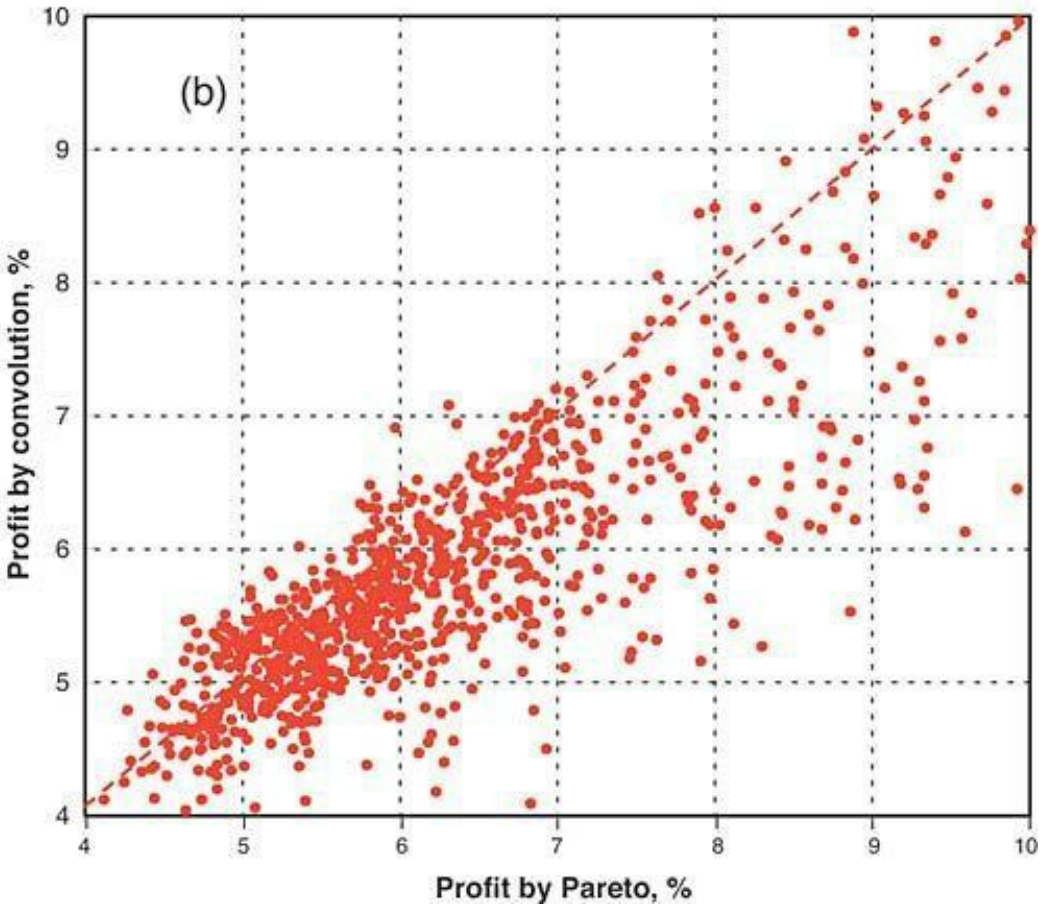
The results are shown in Figure 7.5.1(a). The average profits of combinations selected by the Pareto method are plotted on the horizontal axis, and the profits corresponding to the selection via convolution are on the vertical axis. The slope coefficient of the regression line is slightly lower than 1 ($b = 0.95$). Although its deviation from 1 is small, it is statistically significant ($p < 0.01$), which implies that this result is not random. Close examination of Figure 7.5.1(a) at narrower profit ranges permits us to find out that the majority of dots are situated below the line with slope coefficient 1 (Figure 7.5.1(b)). Totally the Pareto selection was more profitable than selection via convolution

in 774 cases out of 1,100, and in 96 cases the results were equal.

Figure 7.5.1. (a) Relationship between the average profit of combinations selected by convolution and the average profit of combinations selected by the Pareto method. (b) The same for narrower range of average profit. The inclined dashed line divides the area where

the profit of combinations selected by convolution is lower (dots below the line) than the profit of combinations selected by Pareto method (dots above the line).


(a)
y = 0.95x - 0.20
R^2= 0.94
Profit by convolution, %
Profit by Pareto, %
0
5
10
15
20
25
30


(b)
Profit by convolution, %
Profit by Pareto, %
4
5
6
7
8
9
10

Summarizing this study we conclude that the Pareto method of multicriteria analysis prevails over the convolution. However, although in most cases application of the Pareto method gives better results than selection by convolved criteria; in 230 cases (21% of all cases) the situation was quite opposite—the performance of convolution was better than that of the Pareto. It means that although in general the Pareto method is better than convolution, it is preferable to determine criteria and their combinations for which the selection of combinations by convolution may be more appropriate.

Quite often the results of the selection by

convolution coincided with the Pareto selection results (96 cases that make almost 9%). In many cases despite being different, the results of these two methods were very close (dots in immediate proximity to the dotted line in Figure 7.5.1[b]). This can be explained by the fact that many combinations are included in optimal sets of both methods. Such coincidences are natural and are caused by the similarity of the basic principles inherent to all multicriteria methods—striving for the selection of combinations with the highest possible values of all criteria.

Whatever the reason for this phenomenon, it would be useful to verify whether the extent of coincidence

between optimal sets selected by Pareto and by convolution influences the difference between the profitability of positions formed by these two methods. In other words, is it possible to explain the advantage of the Pareto method over the convolution (and, in fewer cases, inverse advantage of convolution over the Pareto method) by the extent of coincidence of two sets? To answer these questions we have arranged a series of investigations.

At the first stage we have to establish the relationship between the abundance of combinations included in both optimal sets and the total number of selected combinations. Figure 7.5.2 shows the relationship between the percentage of

coincidences (expressed as the number of combinations included in both optimal sets normalized by the total number of combinations constituting these sets) and the number of Pareto layers. One can clearly recognize the growth of the coincidence as the number of layers increases. When the number of layers exceeds 15, the selection results of both multicriteria methods coincide approximately by 90%. This relationship is caused by the fact that as the number of layers increases, the total number of combinations constituting the optimal set also increases (see Figure 7.2.3). Furthermore, the growth of the total number of combinations is accompanied by the increase in the percentage of

combinations included in both optimal sets (Figure 7.5.3).

Figure 7.5.2. Relationship between the percentage of combinations included in both optimal sets (reflecting the coincidence between two selection methods) and the number of layers used for 13 pairs of criteria.

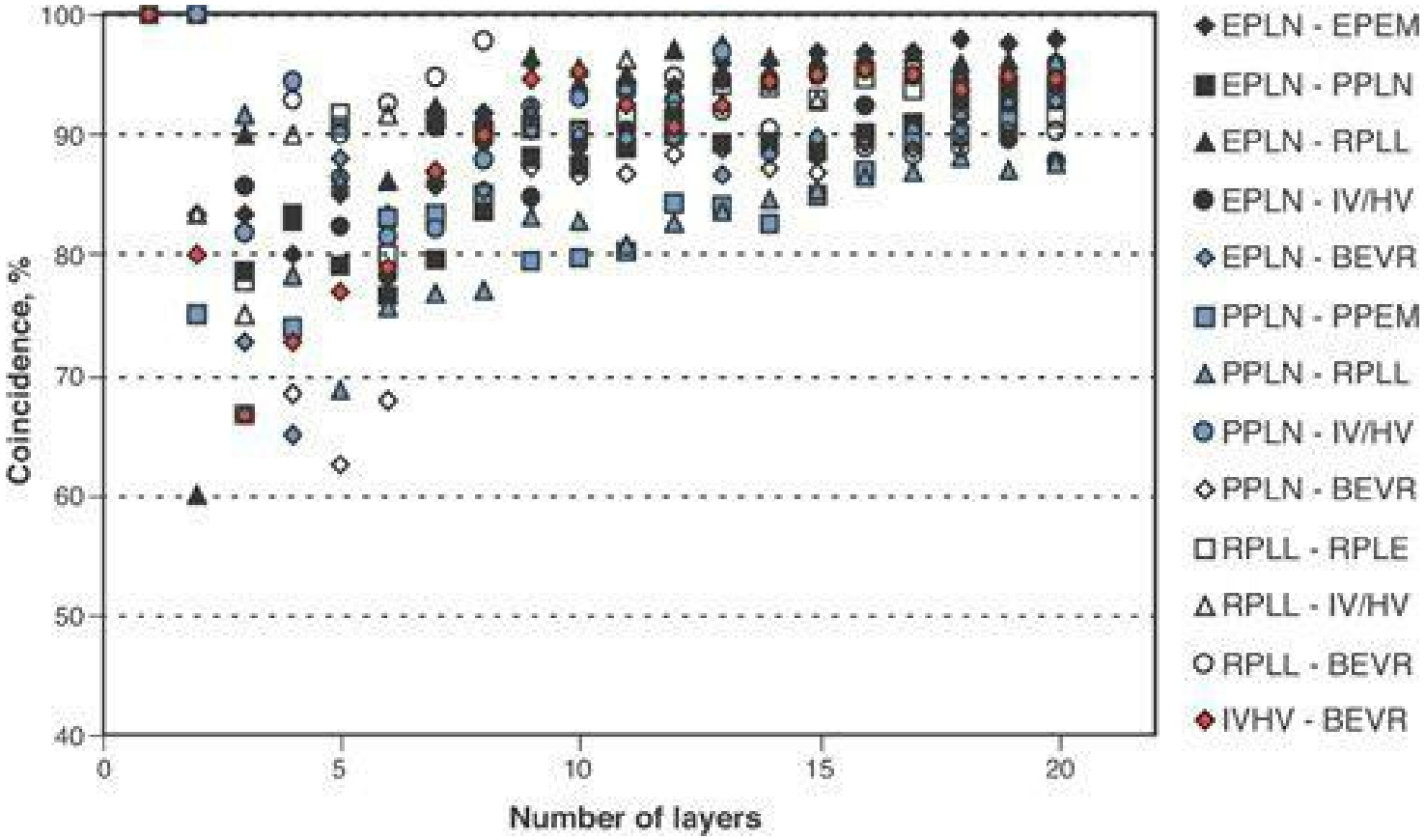
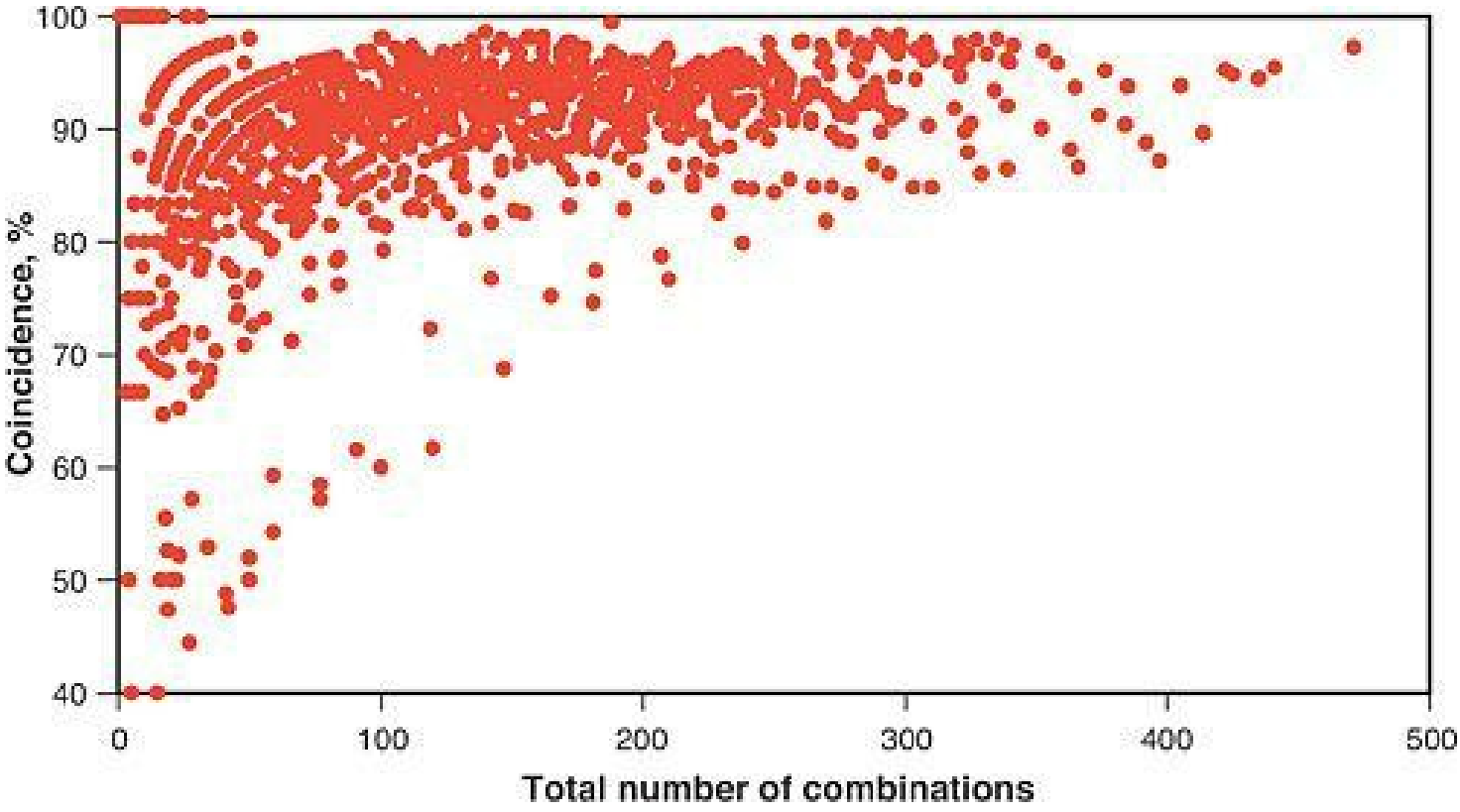


Figure 7.5.3. The relationship between the percentage of combinations included in both optimal sets and the total number of combinations selected by 55

pairs of criteria.



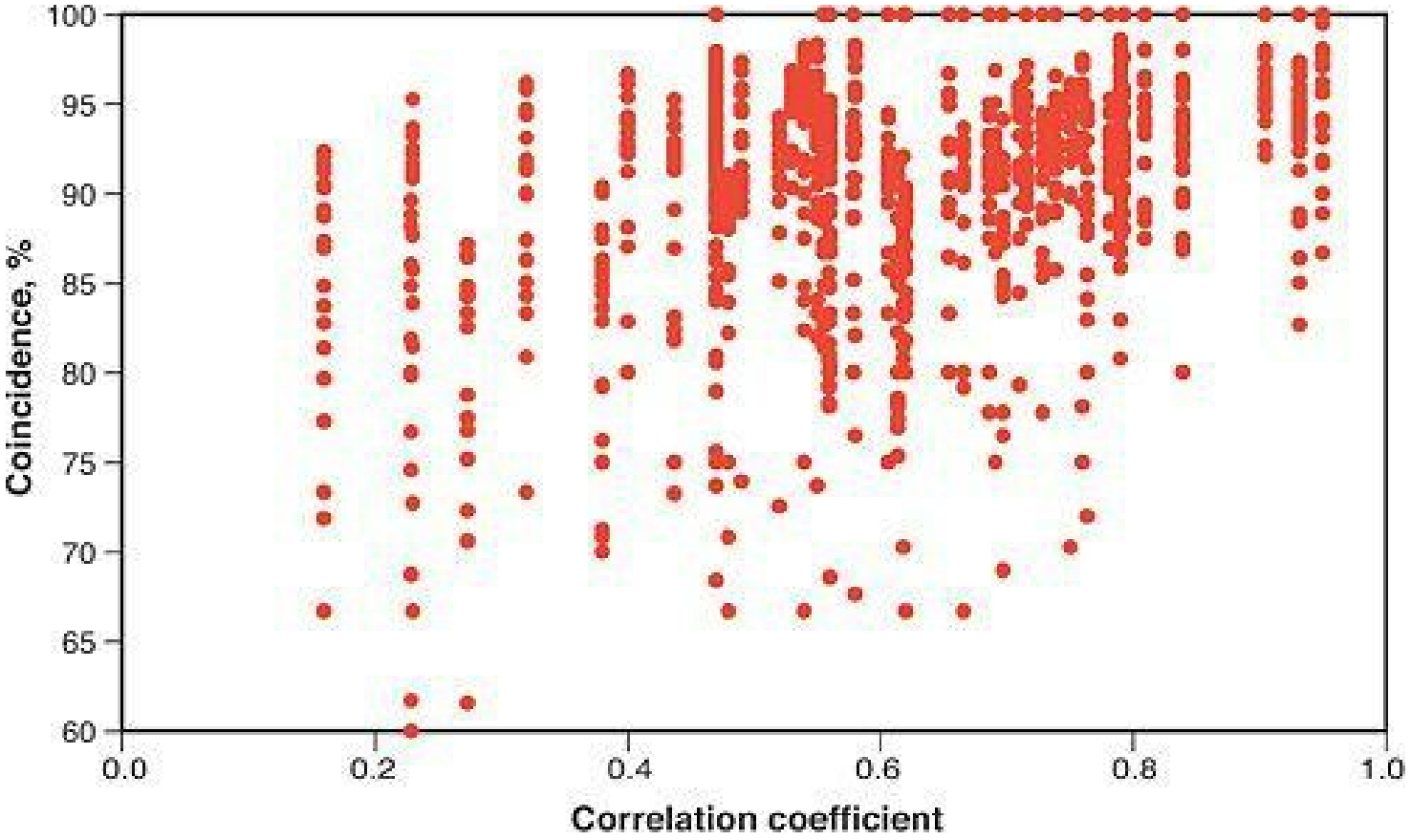
Thus, the greater the number of combinations required for positions opening, the lesser the difference between optimal sets formed via convolution and the Pareto method. Therefore, the question “Which of the selection methods should be used in

each specific case?" makes sense only when a relatively small number of combinations is to be selected from quite an extensive initial set. In all other cases the Pareto method can be used without any hesitation because, on the one hand, it selects the same combinations as the convolution by 90% and, on the other hand, in most cases it shows better performance than the convolution does (Figure 7.5.1).

The second stage of our investigation demonstrates that the total number of selected combinations is not the only factor affecting the extent of the coincidence between optimal sets obtained via the convolution and via the Pareto method. Figure 7.5.4 shows that

the correlation between criteria also influences the number of coincidences. (Different aspects of criteria correlation are discussed in detail in the next chapter.) The higher the correlation coefficient of two criteria, the higher the percentage of combinations included in both optimal sets. Hence the area where we have to answer the question “When is the selection via convolution better than via the Pareto method?” gets even smaller. In fact, we have to bother ourselves with this question only when selecting a small number of combinations according to the pair of low-correlated criteria. In all other cases the Pareto method is preferable.

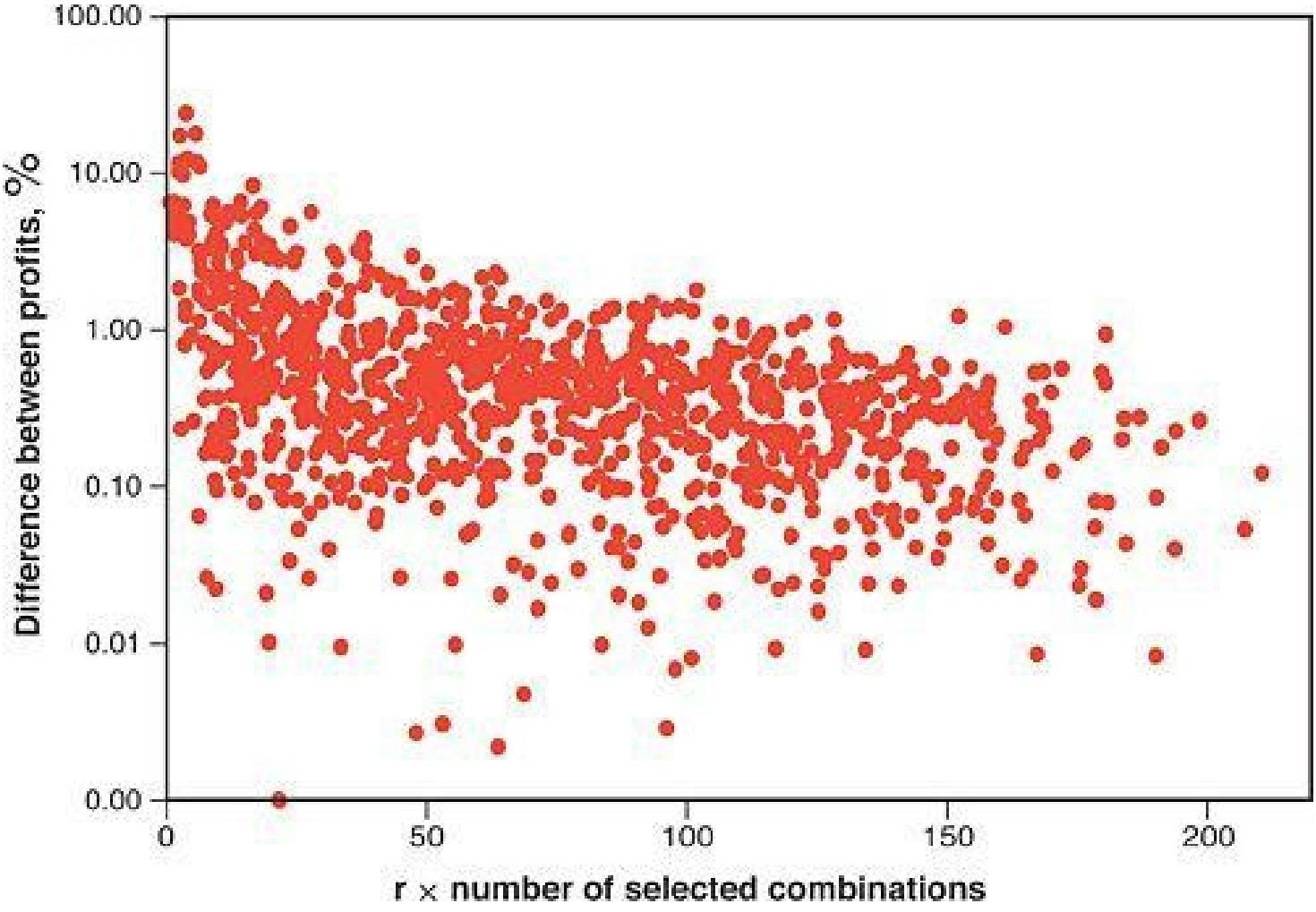
Figure 7.5.4. Relationship between the percentage of combinations included in both optimal sets (Pareto and convolution) and the correlation coefficient of criteria. The correlation coefficients were calculated for all 55 criteria pairs and for all 20 layers.



This statement can be illustrated by the relationship between two compound variables. The first variable should reflect the degree of convergence between profits realized via the Pareto method and via the convolution, whereas the second variable should combine the total number of selected combinations with correlations between criteria. The

relationship will be analyzed by constructing the regression with the absolute value of the difference between profits realized by two selection methods being plotted on the vertical axis and the product of correlation coefficient by the number of selected combinations on the horizontal axis. The regression shown in Figure 7.5.5 proves the correctness of our statement; only under low correlation and small numbers of combinations is the difference between two selection methods significant. When these indicators are high, both methods provide approximately the same results.

Figure 7.5.5. Relationship between the absolute value of the difference between profits realized by selection via the Pareto method and via the convolution (logarithmic scale) and the product of the correlation coefficient between criteria by the number of selected combinations.



7.6 Summary

The study described in this chapter proves that multicriteria selection of option combinations has definite advantages over selection based on

separate criteria. At the same time, the application of monocriterion analysis can sometimes be preferable to multicriteria selection. In practice these situations are infrequent; however, if the investor can reveal them, it will undoubtedly increase average trading profitability.

Our considerations about merits of multicriteria analysis are based on the investigation of 11 criteria with full review of all their possible combinations. This brings us to the most general conclusions concerning the advantages of multicriteria approach. However, in Chapter 3, "Evaluation of Criteria Effectiveness," it was shown that the effectiveness of different criteria

can vary substantially depending on many factors including the option strategy. This implies that simultaneous application of all available criteria is not the most optimal solution; only those criteria that possess the best effectiveness characteristics should be included in the procedures of multicriteria analysis. Combining these criteria also should not be random but based on the analysis of their compatibility, concurrency and correlation. (The latter will be discussed in the next chapter.) A skillful combination of different criteria makes the advantages of multicriteria selection even more pronounced.

In most cases the Pareto method turned

out to outperform the convolution. However, inverse situations—when average profitability of combinations selected via the convolution is higher than of those selected by the Pareto method—also occur. Besides, the effectiveness of both selection techniques is sometimes equal. Two methods generate almost the same profit when criteria are highly correlated or the number of combinations included in the optimal set is high. When criteria have low correlation or the number of selected combinations is small, additional analysis is needed to establish the superiority of any particular technique. To determine which of the methods is preferable in the given

circumstances, you should arrange detailed research of the particular trading system, including its parameters and other factors influencing it directly or indirectly.

chapter 8. The Impact of Criteria Correlation on Multicriteria Selection

8.1 Introduction

The majority of criteria are to some extent correlated. The reason is that while inventing various valuating techniques based on different principles

and approaches, we always strive for one general goal: to reveal situations when our estimate of underlying asset prices variability does not correspond to market values of their options. Approaching this problem from different aspects and using various mathematical constructions, we aim to invent various indicators reflecting (either directly or indirectly) the profitability of future trading. Therefore, the existence of an interrelationship between different criteria is quite natural and follows from their intrinsic qualities. The extent and the shape of this relationship influence the profitability of option portfolios formed using the multicriteria selection methods. Hence the impact of criteria

correlation must be taken into account when choosing criteria and their combinations.

8.2 Evaluation of Criteria Interrelationship

To evaluate the interrelationships between different criteria, we use 11 criteria and calculate correlations for each of their 55 possible pairs. The methods of evaluation and combinations creation are the same as in the examples given in Chapter 7, “Basic Concepts of Multicriteria Selection as Applied to Option Combinations”; the option

strategy is short strangle/straddle; the number of underlying assets is 600. Table 8.2.1 presents the values of 55 correlation coefficients. As we expected, there are no independent criteria, all of them are to some extent interrelated. Profit probabilities based on various distributions turned out to be the most correlated criteria. Expected profits on the basis of lognormal and empirical distributions are not highly correlated. Criteria based on the estimates of profit-to-loss ratios show relatively strong correlation, although they are weaker than in the case of criteria forecasting probabilities. The break-even range is the least-correlated criterion.

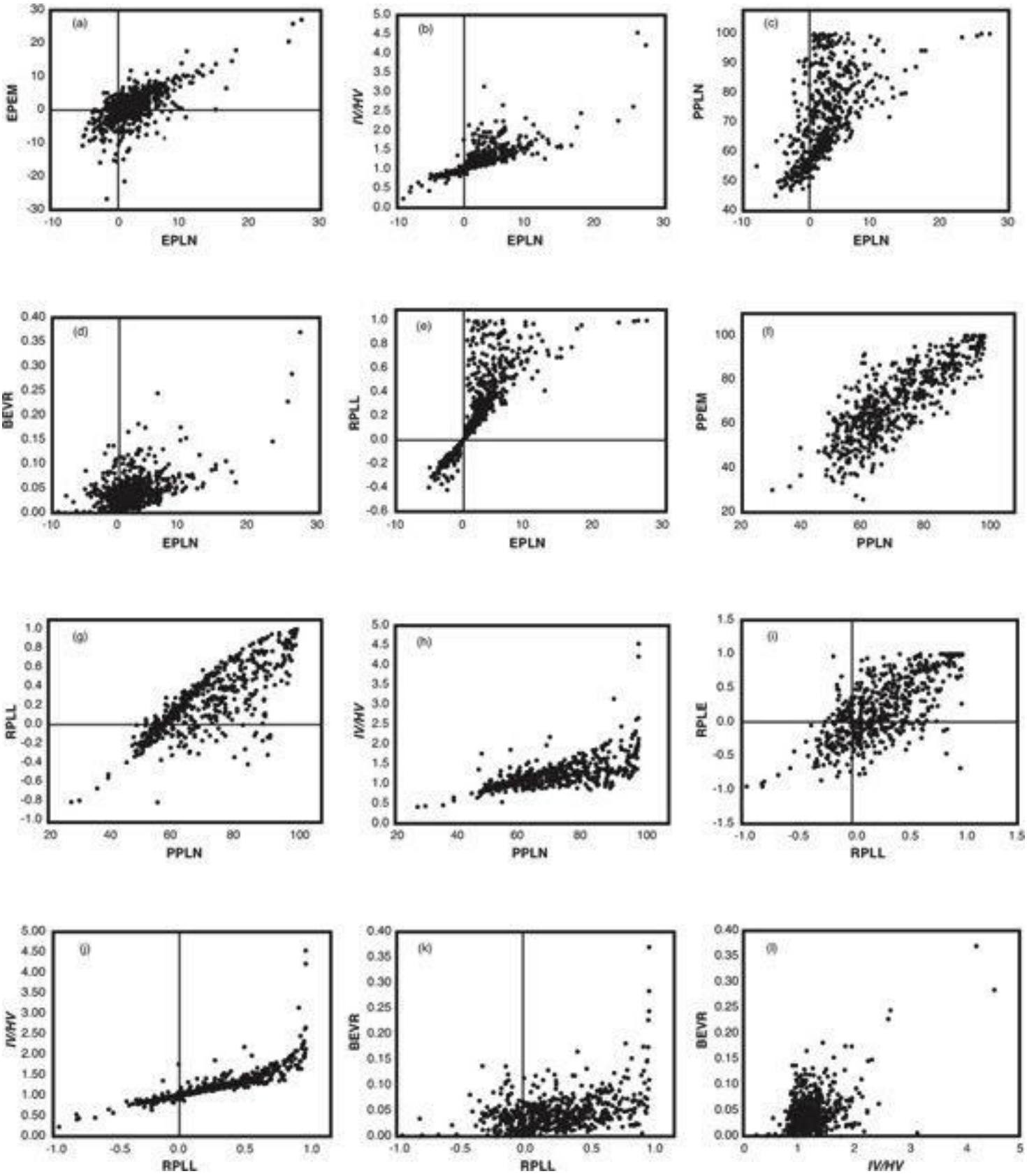
Table 8.2.1. Correlation Coefficients for 11 Criteria Calculated as of March 1, 2007 for 600 Stocks Using the Short Strangle/Straddle Strategy. Criteria Abbreviations Are the Same as in Figure 7.2.3

	EPLN	EPEM	EPES	PPLN	PPEM	PPES	RPLL	RPLE	RPLS	*IV/HV*	BEVR
EPLN	1,00	0,47	0,56	0,58	0,47	0,49	0,79	0,52	0,56	0,79	0,48
EPEM		1,00	0,95	0,32	0,54	0,47	0,47	0,76	0,71	0,47	0,16
EPES			1,00	0,40	0,53	0,55	0,55	0,75	0,79	0,54	0,23
PPLN				1,00	0,84	0,90	0,81	0,55	0,62	0,65	0,63
PPEM					1,00	0,93	0,68	0,79	0,74	0,58	0,50
PPES						1,00	0,73	0,71	0,78	0,60	0,55
RPLL							1,00	0,70	0,76	0,84	0,39
RPLE								1,00	0,93	0,62	0,22
RPLS									1,00	0,66	0,27
IV/HV										1,00	0,46
BEVR											1,00

The correlation coefficient is not the unique indicator characterizing the relationships within different pairs of criteria. The shapes of these relationships are also specific and possess many visual peculiarities. Some examples are shown in Figure 8.2.1.

Figure 8.2.1. Relationships

between different criteria constructed as of March 1, 2007 for 600 stocks using the short strangle/straddle strategy. Criteria abbreviations are the same as on Figure 7.2.3.



First, consider the EPLN-EPEM pair:

The correlation coefficient reveals a relatively weak relationship between these two criteria (r = 0.47, Table 8.2.1); however, Figure 8.2.1(9) shows that when criteria values are high (the top-right part of the figure), their correlation is much stronger than when criteria values are low (the bottom-left part of the figure). Because the area of high values is of particular interest for solving the selection problem, it would be reasonable to conclude that within the interval of high criteria values, the correlation is considerably higher than its value presented in Table 8.2.1. This means that when the interrelationship between two criteria is determined using the standard procedure of correlation

assessment, it can be considerably underestimated.

The opposite situation is observed for the EPLN-*IV/HV* pair. In this case the strong correlation under low criteria values is transformed into weak correlation under their high values (Figure 8.2.1[b]). It means that when determining potentially "good" combinations, these criteria often differ in their estimates; however, they agree with each other while discovering "poor" alternatives. The same tendency is observed in some other cases (Figures 8.2.1[c] and 8.2.1[g]), implying that the interrelationships between criteria within these pairs are considerably overestimated. Therefore, in the

practical selection of option combinations, these pairs of criteria should be considered as weakly correlated.

In some cases the correlation is homogeneous and does not change along the whole range of criteria values (Figures 8.2.1[f] and 8.2.1[i]). Finally, there are also situations when the correlation is strong under medium criteria values and gets weaker as we approach the boundaries of their ranges (Figure 8.2.1[e]).

Summarizing the data presented in Figure 8.2.1, we can conclude that all the criteria are correlated and that the extent of their interrelationship is

variable. Moreover, even within the same pair of criteria, the correlation can vary strongly depending on the range of criteria values under consideration. Our next task is to determine how the correlation between the criteria influences the profitability of option combinations selected by their values.

8.3 Criteria Correlation and Profitability of Pareto Selection

The following method is applied to determine whether the correlation between criteria influences the

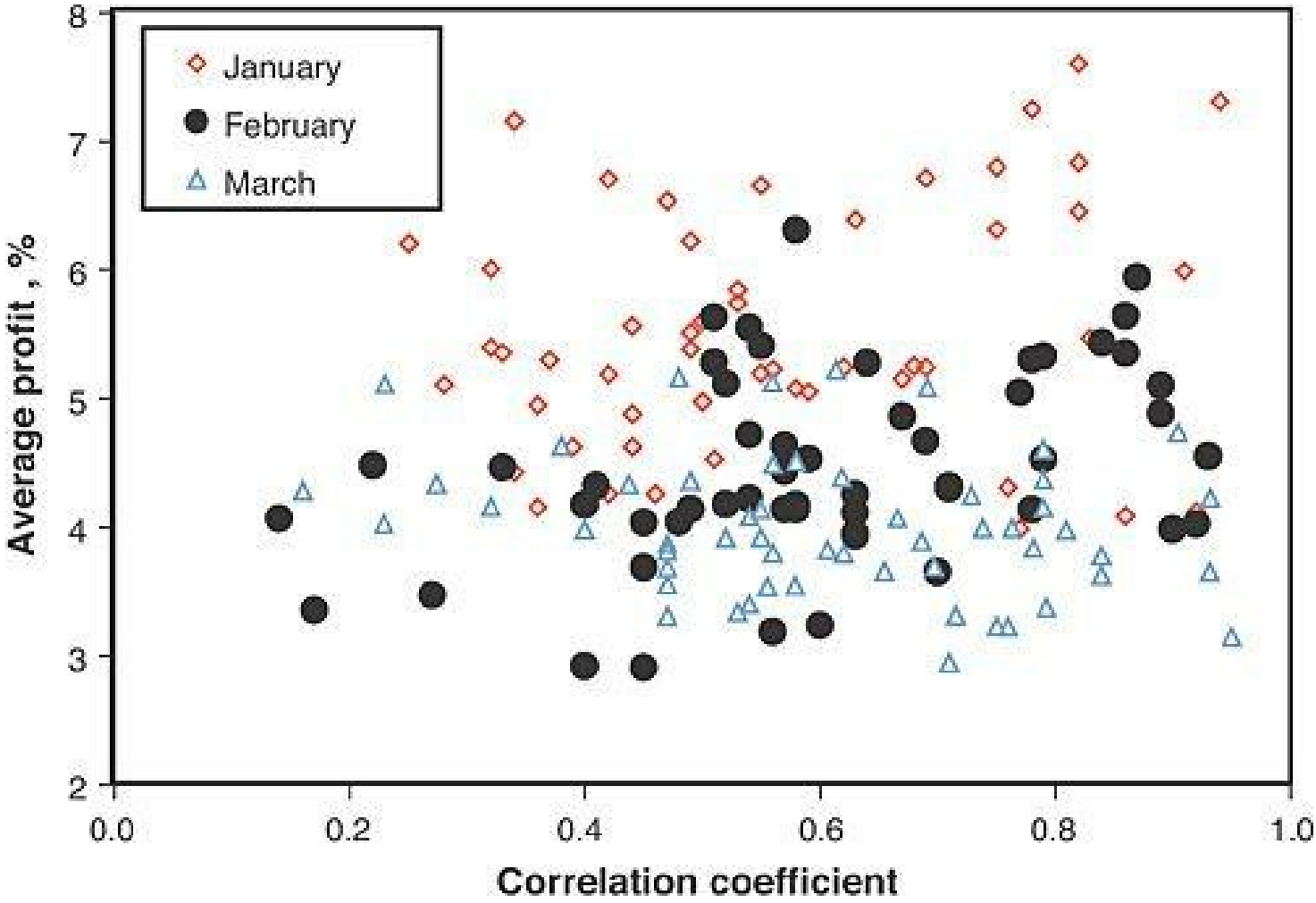
profitability of combinations selected on their basis. The relationship between the average profits of combinations selected using the Pareto method (the optimal set consists of 20 layers) and correlation coefficients of 55 pairs of criteria (see Table 8.2.1) is used as the basis of this study. To increase the reliability of results, we do not limit ourselves to one date but analyze three dates corresponding to the first three months of 2007. (Positions are open on the first working day of each month: January 3, February 1, and March 1.) We calculate combinations profits (expressed as a percentage of margin requirements) for the expiration date that are.

The data corresponding to January and

March did not reveal clear relationships between variables under examination (Figure 8.3.1). However, in February statistically significant positive correlation between profit and correlation was detected (significance level $p < 0.01$). Figure 8.3.1 demonstrates that in February high correlations between the criteria coincide with high average profits of combinations. This result contradicts our intuition because high correlation implies "similarity" of the criteria, which in turn means that using additional criteria does not considerably improve our ability to select the best variants. In the extreme case the correlation coefficient approaching 1 leads to full

identity of both criteria and consequently multicriteria selection reduces to a monocriterion one.

Figure 8.3.1. Relationship between the average profit of combinations selected using the Pareto method (20 layers) and the correlation coefficient between criteria. (Contour signs denote data for which the relationship was statistically insignificant.)



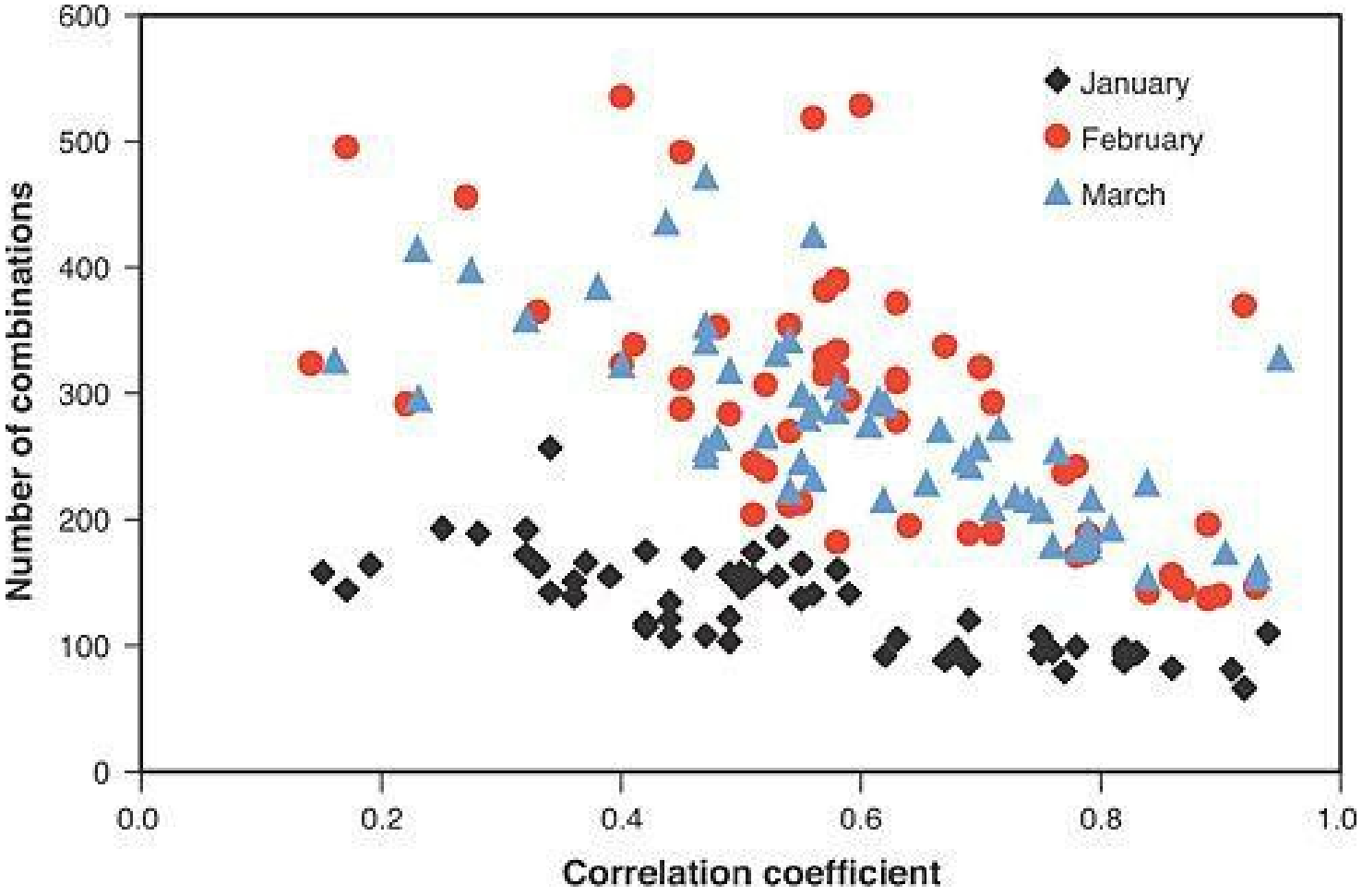
This contradiction forces us to delve deeper into the nature of the revealed positive relationship. If we do not manage to explain this phenomenon logically, the best solution is to refuse the multicriteria selection and work on the basis of a single criterion (or use

highly correlated criteria).

Fortunately, this paradox can be easily explained by the indirect influence of correlation on the average profit through the number of elements included in the Pareto set. Figure 8.3.2 vividly demonstrates that the higher the correlation between criteria, the lower the number of combinations included in the Pareto set. This finding is not surprising: The wider the spread of points (that is, the lower the correlation) in two-dimensional space analogous to the one shown in Figure 7.2.1, the more of them are cut off by each Pareto layer. (In the extreme case when correlation is absolute, the number of Pareto-optimal points does not exceed the number of

layers.)

Figure 8.3.2. Relationship between the number of combinations in the 20 Pareto set layers and the correlation coefficient between criteria.

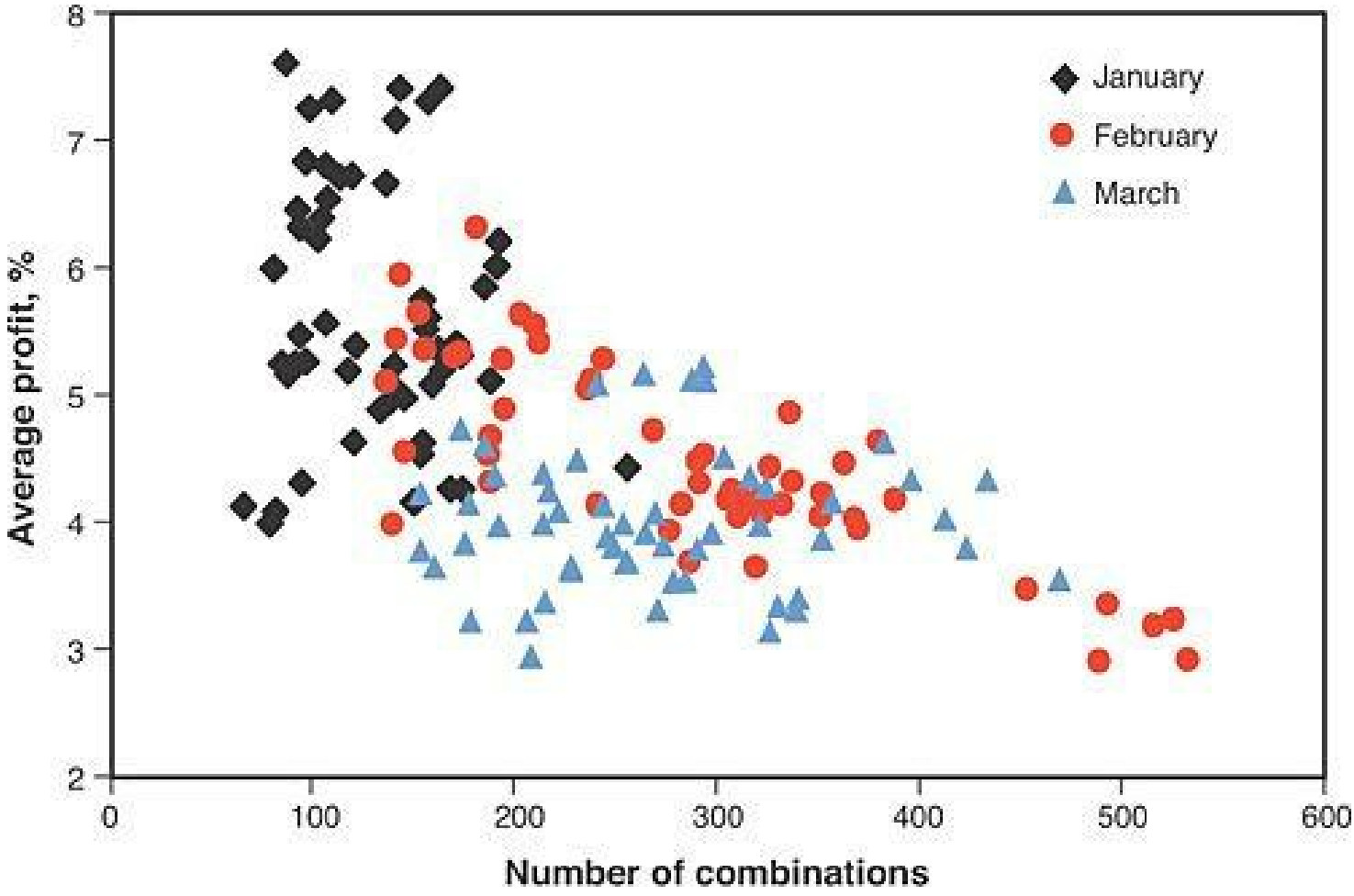


On the other hand, the more points that are included into the optimal set, the lower their average profit (Figure 8.3.3). This finding could also be expected (moreover, it is an inevitable result of the effectiveness of our criteria): The more points that are selected, the worse their average criteria

values. (The same explanation was used for relationships presented in Figure 7.2.1.) Summarizing these two analyses we conclude that strong correlation between criteria leads to a decrease in the number of combinations included in the Pareto set, which increases the average profits owing to selection of the best elements only. Note that the negative aspect of this seemingly positive process of profit augmentation is the reduction in the portfolio diversification.

Figure 8.3.3. The relationship between the average profit of

combinations and the number of combinations in the 20 Pareto set layers.



In the previous study we demonstrated the influence of correlation on profit and revealed the mechanism of this effect. At

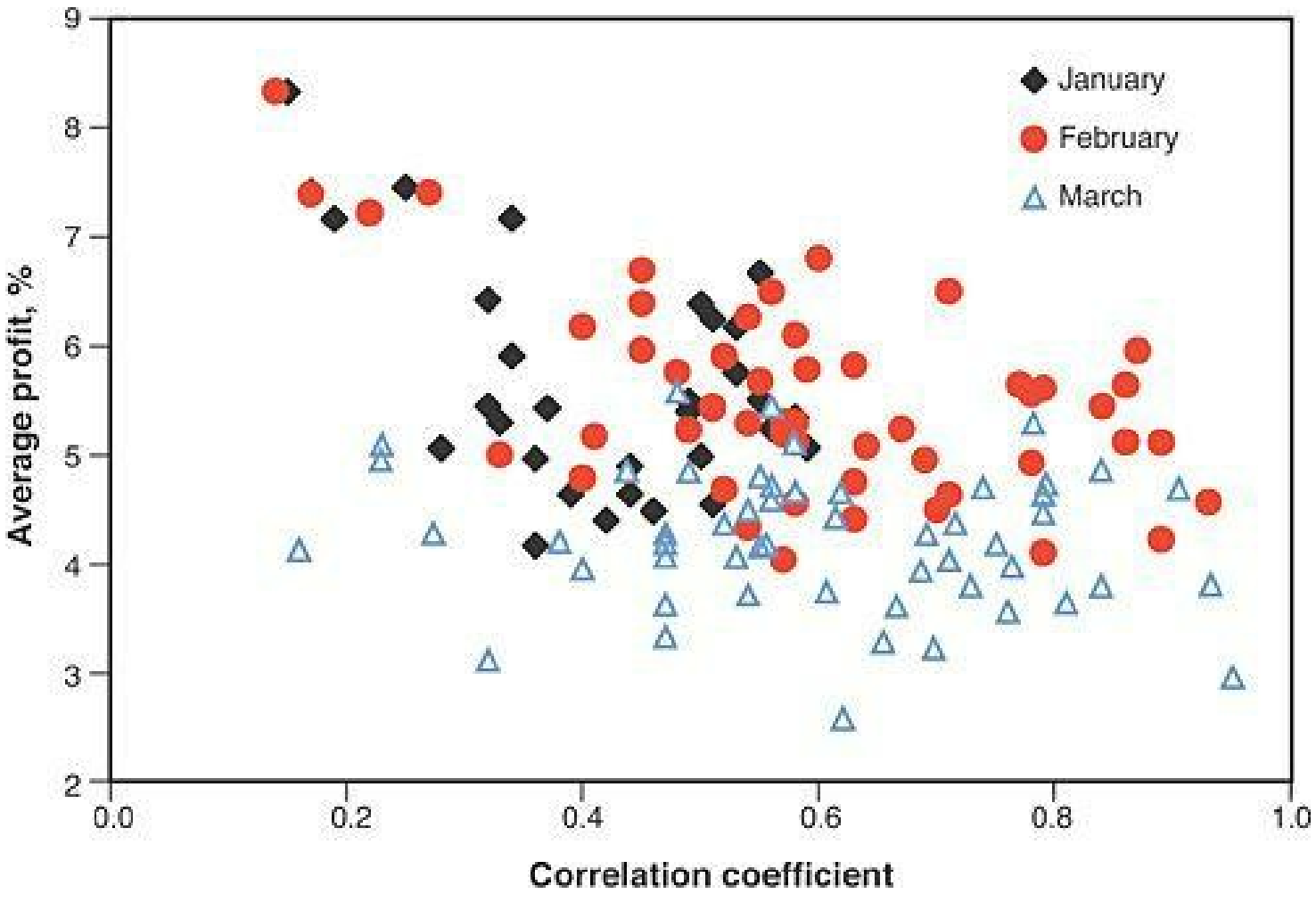
the same time, intuition suggests that, in addition to the indirect impact acting through the number of selected elements, there must be direct influence of correlation. Contrary to the data presented in Figure 8.3.1, we assume that as the criteria interrelationship decrease, profit should increase because uncorrelated criteria supplement each other instead of duplicating. Consequently the set of selected combinations should exploit the advantages of two (or more) different evaluation systems. To verify this assumption, we test the relationship similar to that shown in Figure 8.3.1, but instead of using 20 Pareto layers for each pair of criteria, we use such a

number of layers that generates the optimal set containing 150 elements (or the number closest to 150). Indeed, elimination of the influence of the optimal set size changes the result dramatically. The positive relationship between profit and correlation observed in Figure 8.3.1 turns into a negative one (Figure 8.3.4). For all three months, the inverse relationship between average profit and the correlation coefficient is obtained, and for two of them, this relationship is statistically significant. Thus, when the indirect impact of the correlation on profit (acting through the number of selected combinations) is neutralized, the true nature of this relationship becomes apparent. Thereby

we can conclude that the profitability of combinations selected by the multicriteria method is higher for weakly correlated criteria (assuming that all other conditions are equal).

Figure 8.3.4. Relationship between average profit of combinations selected using the Pareto method and the correlation coefficient between criteria. For each pair of criteria, we used a number of layers corresponding to the

selection of 150 elements (or the number closest to 150). Contour signs denote data for which the relationship was statistically insignificant.



The example of this analysis is thoroughly enlightening. Having a certain relationship discovered (like the one presented on Figure 8.3.1), we could have established the following statement: Strong criteria correlation brings about higher profits. Consequently, this could result in practical application of the statement via utilization of highly correlated criteria for selection of option combinations. However, this would be a rash decision. It is absolutely inadmissible to base investment decisions on any regularity, even if it is reliably determined, without deep understanding of its nature. There are several reasons for this. First, the

factor that is considered as influencing the supposedly dependent variable may turn out to be dependent on the third variable that was not considered in the analysis. In this case, the imaginary correlation between dependent and independent variables arises merely from their dependence on the third variable. Secondly (as in the preceding example), the influencing factor (correlation) may affect the dependent variable (profit) indirectly through other factors depending on it (the number of selected combinations). As a result, the impact of indirect influence can "blur" the direct effect and may even invert it. Therefore, it is necessary to make sure that any relationship intended for

practical application is direct, not imaginary, and its nature is veritably understandable. Otherwise, the result can be the opposite of our expectations: decrease in trading profitability instead of its growth.

8.4 Criteria Correlation and Profitability of Selection Using the Convolution Method

In contrast to the Pareto method, another technique of multicriteria selection, convolution, does not have the problem of uncertainty concerning the number of

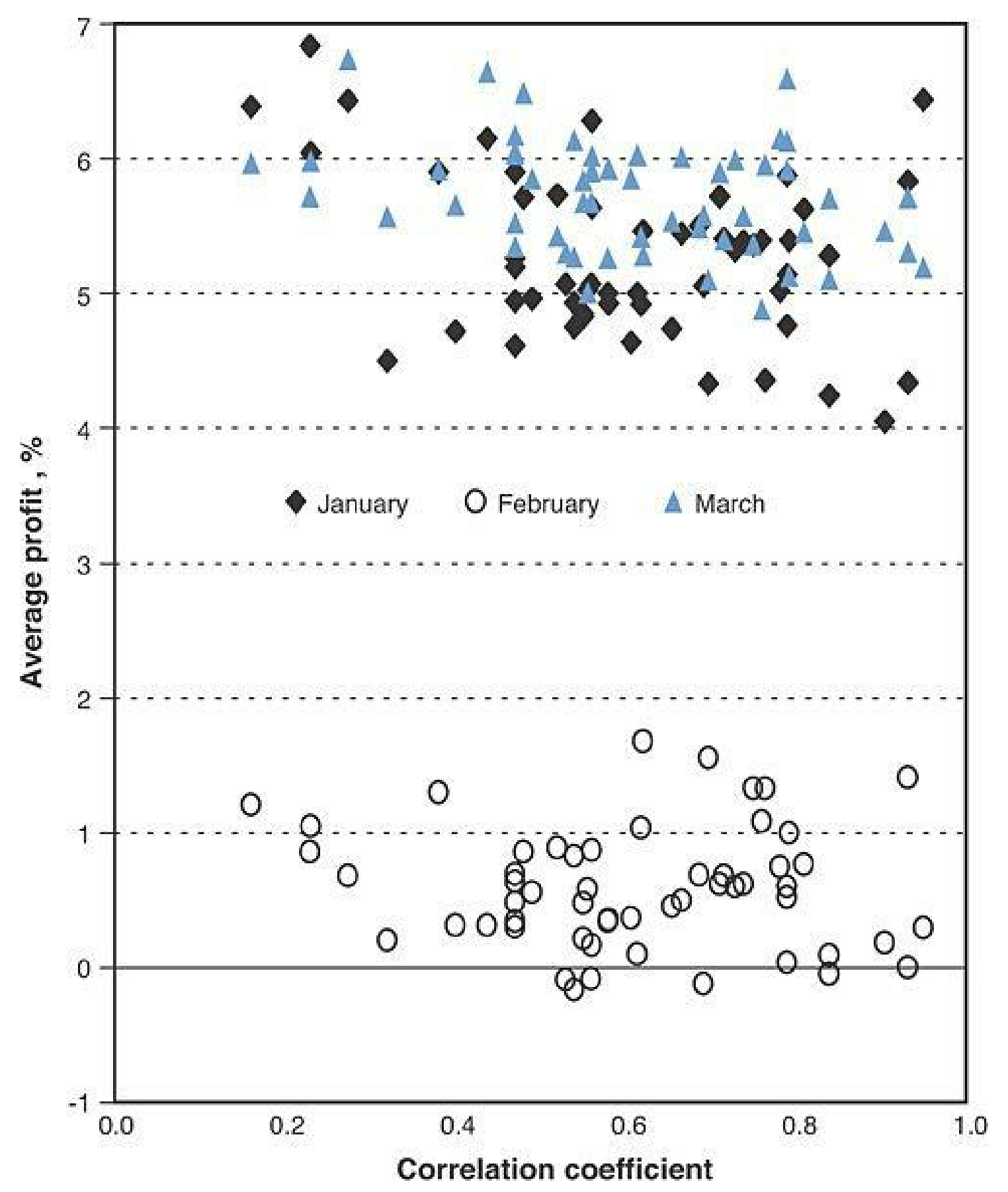
elements included in the optimal set. Applying convolution we can control the number of selected combinations. Although this property of convolution represents a definite advantage, at the same time it creates additional difficulties forcing the investor to decide on the number of selected combinations based solely on subjective reasons. Nevertheless, the possibility to fix the number of selected combinations eliminates the “blurring” effect of this factor thereby significantly increasing the reliability of our investigation.

The following investigation addresses the same question discussed in the previous section: Which criteria should be preferably utilized for multicriteria

selection—those with high or low correlation? To clarify this issue we created additive convolutions for each of the 55 pairs of criteria and calculated the average profit of the 150 best combinations within each pair. Option combinations were created and evaluated using the same method as in the previous study. For data corresponding to January and March, we detected a statistically significant negative relationship (in both cases $p < 0.02$) between the average profit of combinations selected by convolution and the correlation coefficient (Figure 8.4.1). In February the relationship was also negative (Figure 8.4.1), though this result was statistically insignificant ($p =$

0.54). This may be due to the relatively low value of all profits; that means that the range of values of the dependent variable was insufficient for the effect to be pronounced and fully established.

Figure 8.4.1. Relationship between an average profit of 150 combinations selected using convolution and the correlation coefficient between criteria. Contour signs denote data for which the relationship was statistically insignificant.


7
6
5
4
3
2
1
0
-1
Average profit , %
January
February
March
0.0
0.2
0.4
0.6
0.8
1.0
Correlation coefficient

The results presented in this section testify that as correlation between criteria increases, the effectiveness of multicriteria selection deteriorates. (The same conclusion was made for a multicriteria selection using the Pareto method.) Hence applying multicriteria selection we should strive for combining criteria with minimum correlation. This conclusion fully corresponds to our intuitive notion: Weakly correlated criteria supplement each other and thereby produce a complementary effect amplifying the average profitability of selected combinations.

8.5 Summary

Investigations presented in this chapter demonstrate that all criteria are interrelated. The reason lies in the peculiarities of the methods of their creation. The strength of correlation within each criteria pair can vary substantially depending on the nature and specific properties of given criteria.

The extent to which criteria are correlated influences the profit of combinations selected by methods of multicriteria analysis. Regardless of which particular multicriteria method—either Pareto or convolution—is applied to distinguish the superior option combinations, selection effectiveness will be higher if the correlation of

applied criteria is weaker. Therefore, criteria assortment should be formed not only by considering individual qualities of separate criteria (as described in Chapter 3, "Evaluation of Criteria Effectiveness"), but also by assessing the degree of their interrelationship. Actually, the variable calculated as inverse to the correlation coefficient can serve as an effectiveness indicator characterizing the forecasting qualities of criteria pairs.

Computation of such effectiveness indicator is complicated due to the specific property of criteria correlation. Our study (Figure 8.2.1) revealed that correlation can vary depending on the range of criteria values. For example, it

can be low under high criteria values and high under low values (or vice versa). Therefore, it makes no sense to utilize the correlation coefficient calculated along the whole range of criteria values for the estimation of the effectiveness indicator. The correlation coefficient must be adjusted in such a way that it would reflect the extent of the criteria interrelationship, not in general form but within the narrow value range that is relevant to the task fulfilled by criteria application. This problem can be solved in several ways. The simplest solution consists in calculating the correlation coefficient on the basis of only those elements of the initial set that have values (of both criteria) lying

within the specified range. Appling this method we should remember that too narrow a range can lead to underestimation of the correlation coefficient whereas too wide a range cannot adjust it sufficiently. Hence, defining an optimal range represents a trade-off problem that should be solved by finding the compromise between these two factors.

It is important to emphasize that the development of criteria possessing high effectiveness characteristics and at the same time a low degree of interdependence represents a key factor determining the profitability of multicriteria selection.

Conclusion

In this book we describe the basics of the systematic approach developed to identify the trading opportunities in the option market. The general concept is based on the consecutive realization of three key stages including evaluation, analysis, and selection of the most perspective trading variants. The ways and methods of realization of the systematic approach and applied research necessary for its practical introduction constitute the subject of this book.

Describing the various aspects of systematic approach, we frequently returned to our main thesis consisting of the necessity to base evaluation and selection of trading opportunities on strictly formalized criteria. Thus, Part I, "Criteria as the Basis of a Systematic Approach," describes the philosophical concept and the methods of criteria developing and parameterization. Hopefully, we convinced you that successful functioning in the multidimensional space of the option market (discussed in Part II, "The Main Areas of Criteria Application") can be possible only via using different criteria adapted to the current market environment and applying advanced

methods of multicriteria analysis (described in Part III, "Multicriteria Analysis"). We suggest extending the number of criteria in use continuously both through elaboration of existing analytical instruments and via persistent development of new evaluating techniques.

Although Part I of the book deals with the methods of creating criteria and evaluation of their effectiveness, Part II shows three main areas of criteria application—the selection of option combinations, strategies, and underlying assets. Successively describing the technology of criteria application to solving each of these problems, we assumed that the two others were

already solved. That allowed us to explain the material clearly and to avoid cross effects complicating the analysis of criteria forecasting qualities. Despite this simplification, in real trading it is often more appropriate to solve all three selection problems simultaneously. This requires developing advanced calculation algorithms and utilizing considerable computational facilities, though basic approaches and principles described in Part II of the book will remain the same.

We hope that consistent application of the ideas and principles described in this book can help you construct a balanced universal trading system aimed at stable achievement of investment

goals given certain return and risk levels.

Bibliography

Achdou, Yves and Olivier Pironneau. "Computational Methods for Option Pricing." Society for Industrial and Applied Mathematics, 2005.

Augen, Jeff. *Day Trading Options: Profiting from Price Distortions in Very Brief Time Frames*. FT Press, 2009.

Augen, Jeff. *Trading Options at Expiration: Strategies and Models for Winning the Endgame*. FT Press, 2009.

Baird, Allen Jan. *Option Market Making: Trading and Risk Analysis for*

the Financial and Commodity Option Markets. Wiley, 1992.

Banks, Eric and Paul Siegel. *The Options Applications Handbook*. McGraw-Hill, 2007.

Black, Fisher and Myron Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, vol. 81, pp. 637–654, 1973.

Cohen, Guy. *The Bible of Options Strategies: the Definitive Guide for Practical Trading Strategies*. FT Press, 2005.

Cohen, Guy. *Volatile Markets Made Easy: Trading Stocks and Options for Increased Profits*. FT Press, 2009.

Connolly, Kevin. *Buying and Selling Volatility*. Wiley, 1997.

Courtney, Smith. *Option Strategies: Profit-Making Techniques for Stock, Stock Index, and Commodity Options*. Wiley, 2008.

De Weert, Frans. *Exotic Options Trading*. Wiley, 2008.

Garman, Mark B. and Michael J. Klass. "On the Estimation of Security Price Volatility from Historical Data." *The Journal of Business*, vol. 53, pp. 67-78, 1980.

Garner, Carley and Paul Brittain. *Commodity Options: Trading and Hedging Volatility in the World's Most*

Lucrative Market. FT Press, 2009.

Gatheral, Jim and Nassim Taleb. *The Volatility Surface: A Practitioner's Guide*. Wiley, 2006.

Haug, Espen Gaarder. *The Complete Guide to Option Pricing Formulas*. McGraw-Hill, Second edition, 2006.

Hull, John C. *Options, Futures, And Other Derivatives*. Prentice Hall, Seventh Edition, 2008.

Izraylevich, Sergey and Vadim Tsudikman. 2009a. "An Empirical Solution to Option Pricing." *Futures*, vol. 38 (5), pp. 28-31.

Izraylevich, Sergey and Vadim Tsudikman. 2009b. "A Better Risk

Gauge for Options Portfolios." *Futures*, vol. 38 (7), pp. 24-27.

Izraylevich, Sergey and Vadim Tsudikman. 2009c. "Short Volatility Trading in Extreme Markets." *Futures*, vol. 38 (10), pp. 30-35.

Izraylevich, Sergey and Vadim Tsudikman. 2009d. "Risk Management: Facing New Challenges." *Futures*, vol. 38 (12), pp. 38-40.

Izraylevich, Sergey and Vadim Tsudikman. 2010. "Measuring New Risk Management Tools." *Futures*, vol. 39 (1), pp. 42-44.

James, Peter. *Option Theory*. Wiley, 2003.

Katz, Jeffrey O. and Donna L. McCormick. *Advanced Option Pricing Models. An Empirical Approach to Valuing Options*. McGraw-Hill, 2005.

Lehman, Richard and Lawrence G. McMillan. *New insights on Covered Call Writing: The Powerful Technique That Enhances Return and Lowers Risk in Stock Investing*. Bloomberg Press, 2003.

McMillan, Lawrence G. *Profit with Options: Essential Methods for Investing Success*. Wiley, 2002.

Natenberg, Sheldon. *Option Volatility and Pricing: Advanced Trading Strategies and Techniques*. McGraw-

Hill, Second edition, 1994.

Natenberg, Sheldon. *Option Volatility Trading Strategies*. Marketplace Books, 2007.

Neftci, Salih N. *Introduction to the Mathematics of Financial Derivatives*. Academic Press, 2000.

Parkinson Michael. "The Extreme Value Method for Estimating the Variance of the Rate of Return." *The Journal of Business*, vol. 53, pp. 61-65, 1980.

Peters, Edgar E. *Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility*. Wiley, 1996.

Poon, Ser-Huang. *A Practical Guide to*

Forecasting Financial Market Volatility. Wiley, 2005.

Rebonato, Riccardo. *Volatility and Correlation: The Perfect Hedger and the Fox*. Wiley, 2004.

Reehl, C. B. *The Mathematics of Options Trading*. McGraw-Hill, 2007.

Rouah, Fabrice D. and Gregory Vainberg. *Option Pricing Models and Volatility Using Excel-VBA*. Wiley, 2007

Taleb, Nassim Nicholas. *Dynamic Hedging: Managing Vanilla and Exotic Options*. Wiley, 1997.

Thomsett, Michael C. *Put Option Strategies for Smarter Trading: How to*

Protect and Build Capital in Turbulent Markets. FT Press, 2009.

Thomsett, Michael C. *The Options Trading Body of Knowledge: The Definitive Source for Information about the Options Industry*. FT Press, 2009.

Vince, Ralph. *The Mathematics of Money Management: Risk Analysis Techniques for Traders*. Wiley, 1992.

Vine, Simon. *Options: Trading Strategy and Risk Management*. Wiley, 2005.

Wystup, Uwe. *FX Options and Structured Products*. Wiley, 2007.

Yates, Leonard. *High Performance Options Trading: Option Volatility and Pricing Strategies*. Wiley, 2003.

appendix. Basic Notions

Definition of Terms

American option. A type of option contract that gives its buyer the right to buy or sell the underlying asset at any time point until the *expiration date*.

Call option. A standard contract, which gives its buyer the right (without imposing any obligations) to buy a

certain *underlying asset* at a specific point of time in the future (see *expiration date*) for a fixed price (see *strike price*).

Combination. Any number of different options corresponding to the same underlying asset can be considered and analyzed as a whole entity. Options comprising a combination may be long (bought) and/or short (sold) and may have the same or different expiration dates and strike prices.

Delta. The first partial derivative of the option price with respect to the underlying asset price. It estimates the change in the option price given a one-point change in the underlying asset

price. The delta is always positive for a Call option and negative for a Put option.

European option. A type of option contract that gives its buyer the right to buy or sell the underlying asset at the *expiration date*.

Expiration date. The date settled in a contract, before which (see *American option*) or at which (see *European option*) the option buyer can realize his right to buy or sell the underlying asset.

Gamma. The second partial derivative of the option price with respect to the underlying asset price. It estimates the change in the delta given a one-point change in the underlying asset price.

Gamma shows if the speed of the delta change is increasing, decreasing, or constant.

Historical volatility. The indicator reflecting the extent of the underlying asset price variability at the specific time interval. It is usually estimated as the standard deviation of relative increments of the underlying asset prices normalized by the square root of time. This method is straightforward and the simplest one, although a lot of other more elaborate and sophisticated techniques exist. The historical volatility is widely used as the basic risk indicator. In option theory it is a key element of all pricing models.

Implied volatility. Another indicator reflecting the underlying asset price variability. Contrary to the *historical volatility*, it is not based on historical prices of the underlying asset but is derived from current market prices of the option. Pricing models generally use the *historical volatility* value to calculate the theoretical option fair value. If instead we substitute the option market price in the model formula and solve the inverse problem, we can derive volatility "implied" by the option price. Implied volatility expresses market expectations for future variability of underlying asset price. Besides, this indicator is a convenient measure of option expensiveness. The divergence of

implied and historical volatility often creates favorable opportunities for implementing different option trading strategies.

Intrinsic value. A part of the option *premium,* which is equal (for the Call option) to the difference between the current underlying asset price and the *strike price* if this difference is positive, and zero otherwise. For the Put option the intrinsic value is equal to the difference between the *strike price* and the current underlying asset price if this difference is positive, and zero if it is negative.

Margin requirements. The minimum amount of your own capital required to

maintain an option position. It is set by the regulating authorities and is generally made tougher by the exchange and broker. The amount of margin requirements is recalculated daily in accordance with the underlying asset price change.

Options at the money. Options with the strike price close to the current underlying asset price. The *intrinsic value* of such options is low, whereas their *time value* is relatively high.

Options in the money. Options with the strike price distant from the current underlying asset price. For Put options the strike price is higher, and for Call options it is lower than the underlying

asset price. *Intrinsic value* of such options is high, whereas their *time value* is relatively low.

Options out of the money. The strike price of these options is also distant from the current underlying asset price. However, in this case the strike of Put options is lower than the underlying asset price (and vice versa for Call options). *Intrinsic value* of such options is zero, and their *time value* is relatively low.

Premium. The value of the option paid by the buyer to the seller. The premium of stock options is usually quoted for one share, whereas the standard contract is generally comprised of options for

100 shares.

Put option. A standard contract, which gives its buyer the right (without imposing any obligations) to sell a certain *underlying asset* at a specific point of time in the future (see *expiration date*) for a fixed price (see *strike price*).

Rho. The first order derivative of the option price with respect to the risk-free interest rate. It estimates the change in the option price given a one-point change in the interest rate. *Rho* expresses the sensitivity of the option price to changes in the interest rate.

Strike price. The price settled in a contract, at which the option buyer can

realize his right to buy or sell the underlying asset.

The Greeks (vega, gamma, delta, rho, theta). Indicators used to estimate the risks of options reflecting the extent to which the value of the option responds to small changes in specific variables. The underlying asset price, volatility, time, and interest rate can be taken as variables. *The Greeks* are calculated analytically as partial derivatives of the option price with respect to a given variable. To calculate a derivative a certain option pricing model is used (for example, the Black-Scholes formula). These indicators can be interpreted as the speed of the option price change in response to the variable change.

Theta. The first partial derivative of the option price with respect to time. It estimates the change in the option price in response to the decrease in time left until expiration by one unit. Theta characterizes the speed of *time decay*.

Time decay. The process of options *time value* decrease while the *expiration date* approaches. The closer the expiration date, the faster the decay is.

Time value. Another part of the option premium, which is equal to the difference between the option premium and the *intrinsic value*. Time value is a risk premium paid by option buyers to sellers.

Vega. The first partial derivative of the option price with respect to the underlying asset volatility. It estimates the change in the option price given a one-point change in the underlying volatility. This indicator plays an important role in the strategies of volatility trading.

Payoff Functions

Payoff function represents the relationship between the price of a separate option or a combination and the underlying asset price, estimated for a specific future date. The payoff function

of a combination consisting of options with the same expiration date and calculated for the expiration date is a broken line. If the estimate is made for the date preceding the expiration or if the combination includes options with different expiration dates, the payoff function is a curve (calculated on the basis of a specific option pricing model).

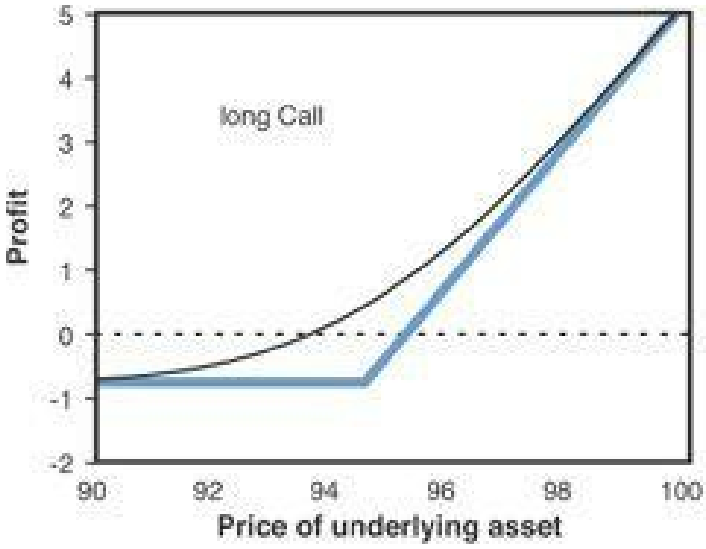
Separate Options

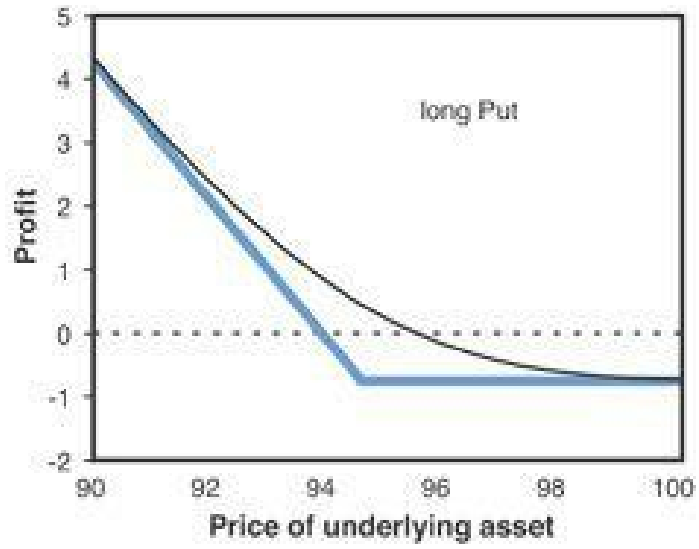
Figure A.1 shows payoff functions of long and short Call and Put options. Solid lines depict payoff functions for the expiration date, dashed lines—for

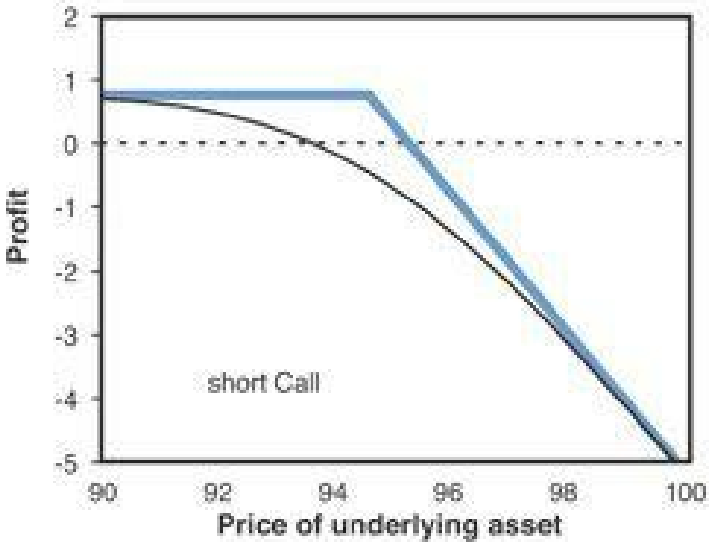
some date before the expiration. At a given underlying asset price, the more time left until expiration, the higher the value of the long position. (In the figures the dashed lines of both Call and Put options are higher than solid ones.) Accordingly, the closer the expiration date, the higher the value of the short position. (Dashed lines are lower than solid ones in the figures.) The reason is that the option value consists of *intrinsic* and *time values*. The latter decreases continuously when approaching the expiration date. (This phenomenon is called *time decay*.)

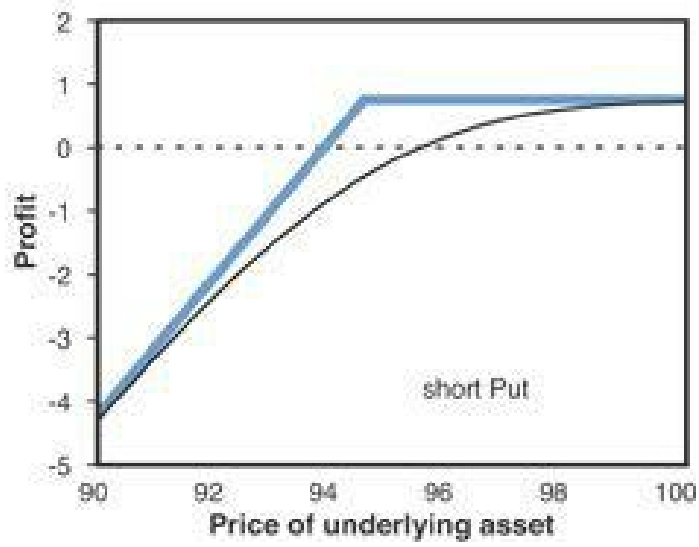
Figure A.1. Payoff functions

reflecting profits and losses of buyers (long) and sellers (short) of Call and Put options. Bold lines show payoff functions corresponding to the expiration date, thin lines— to a date before the expiration.









The buyer of the Call option realizes theoretically unlimited profit when the underlying asset price grows and limited loss when it decreases. The Put option buyer realizes theoretically unlimited profit when the underlying asset price

decreases and limited profit when it increases. In both cases the maximum loss is limited to the premium paid.

Payoff functions of short options are opposite to that of long options (Figure A.1). Profits of the seller are limited to the premium received and the losses are theoretically unlimited and depend on the extent of underlying asset price move. The more the underlying asset price increases, the higher the loss of the short Call. Respectively, the lower the price decrease, the higher the loss of the short Put.

Option Combinations

More complicated forms of payoff functions can be obtained by combining several different options. This feature represents one of the most important advantages of investing in options. Construction of complex option combinations allows creating a multitude of nonlinear payoff functions thereby enlarging considerably the potentialities and flexibility of the sophisticated investor.

Applying different principles of combining options (such as ratio of long to short options, Calls to Puts ratio, positioning strike prices relative to the current underlying price, selection of expiration dates, and so on), you can

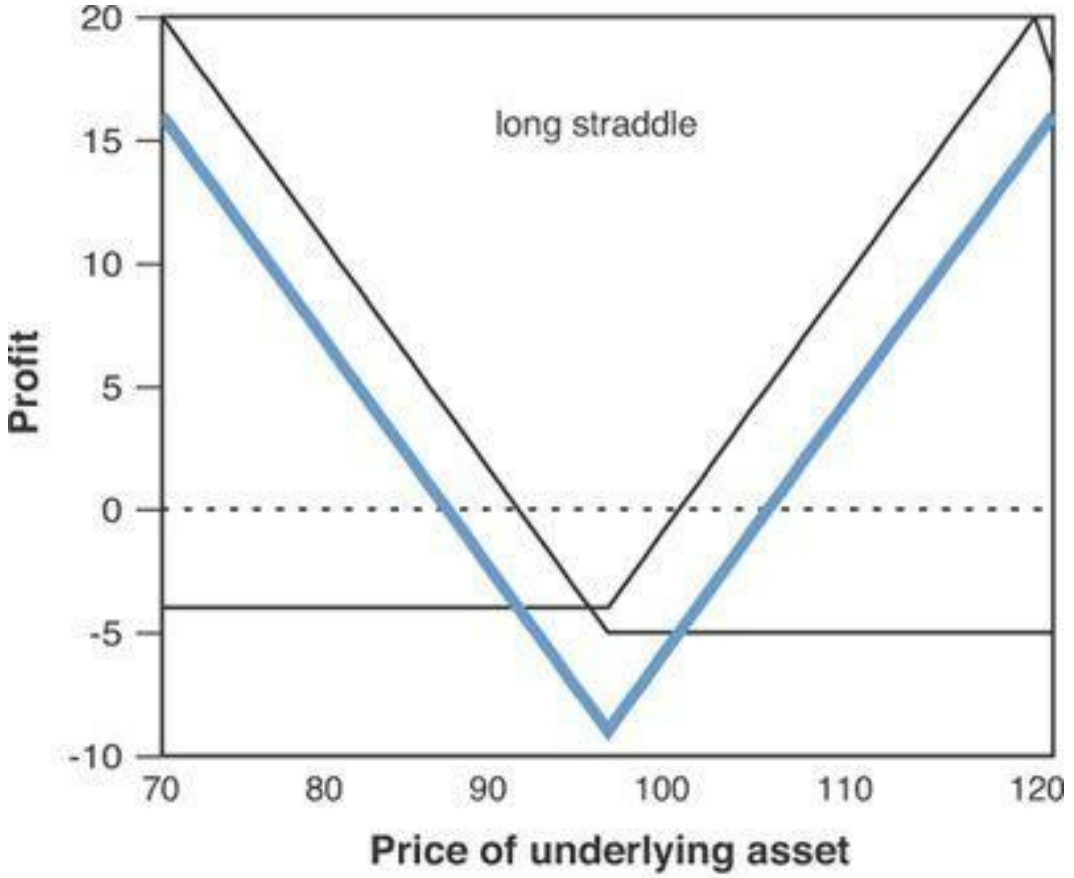
create combinations with almost any desired type of payoff function. The combinations are usually classified by the principles of their creation and the shape of payoff functions characteristic of them. In the literature (including this book) such classes are called *option strategies*. Next we describe several examples of the most popular strategies.

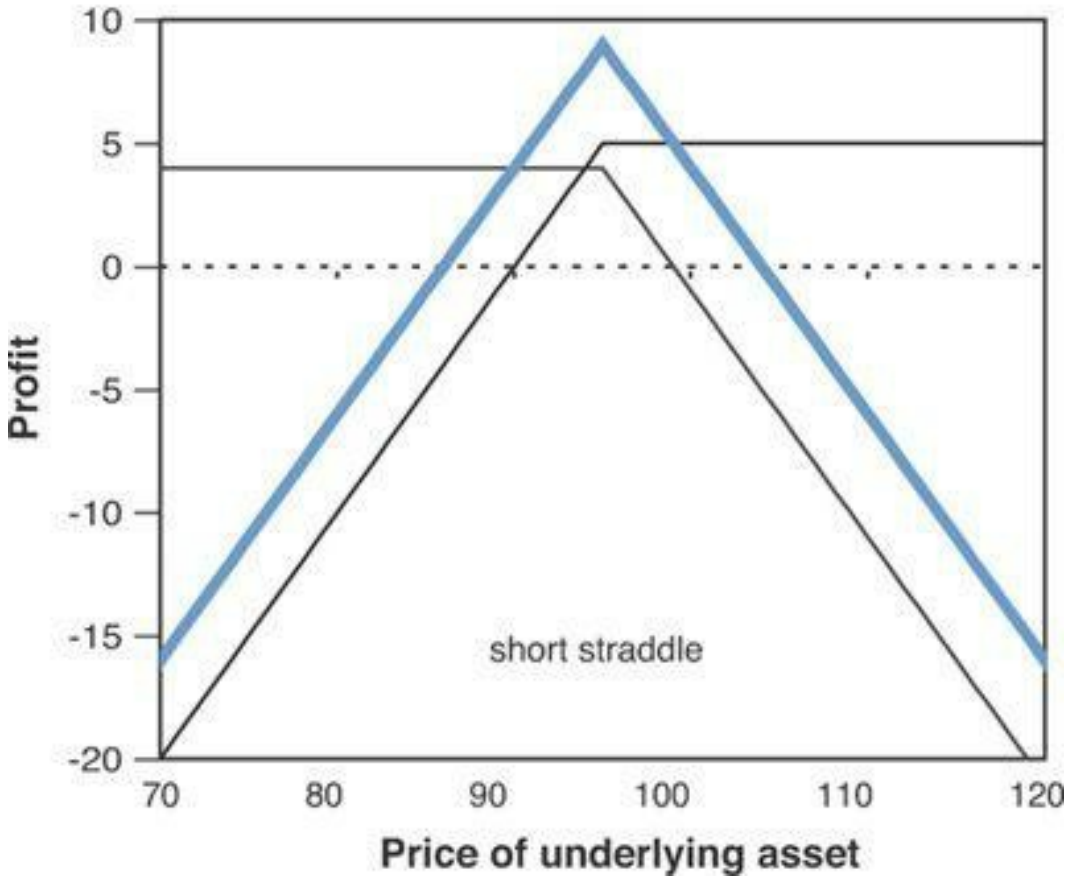
Straddle

The *long straddle* is created by buying Call and Put options with the same strike price and expiration date. The number of Calls is usually equal to the number of Puts (as shown in Figure A.2), but this is

not a compulsory condition. If the underlying asset price is close to the strike price at the expiration date, the combination is unprofitable with a loss limited to the premium paid for it. If a considerable price movement occurs (regardless of the direction), the straddle generates profit. The *short straddle* is created under the same principles, but the options are sold rather than bought. Accordingly, the combination is profitable if the underlying asset price does not change a lot and generates losses when there is a considerable price movement in any direction (Figure A.2).

Figure A.2. Payoff functions of long and short straddles corresponding to the expiration date (bold lines). Thin lines show payoff functions of separate options forming the combinations.


20
15
10
5
0
-5
-10
Profit
long straddle
70
80
90
100
110
120
Price of underlying asset


10
5
0
-5
-10
-15
-20
Profit
short straddle
70
80
90
100
110
120
Price of underlying asset

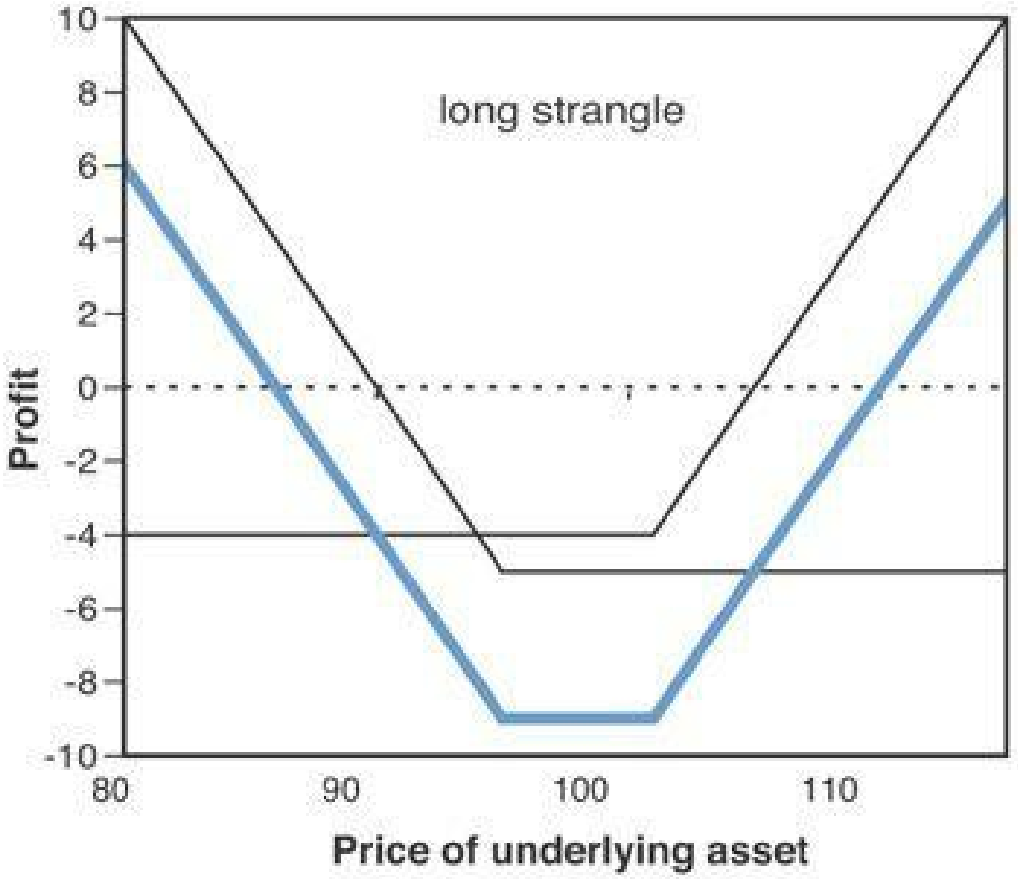
Strangle

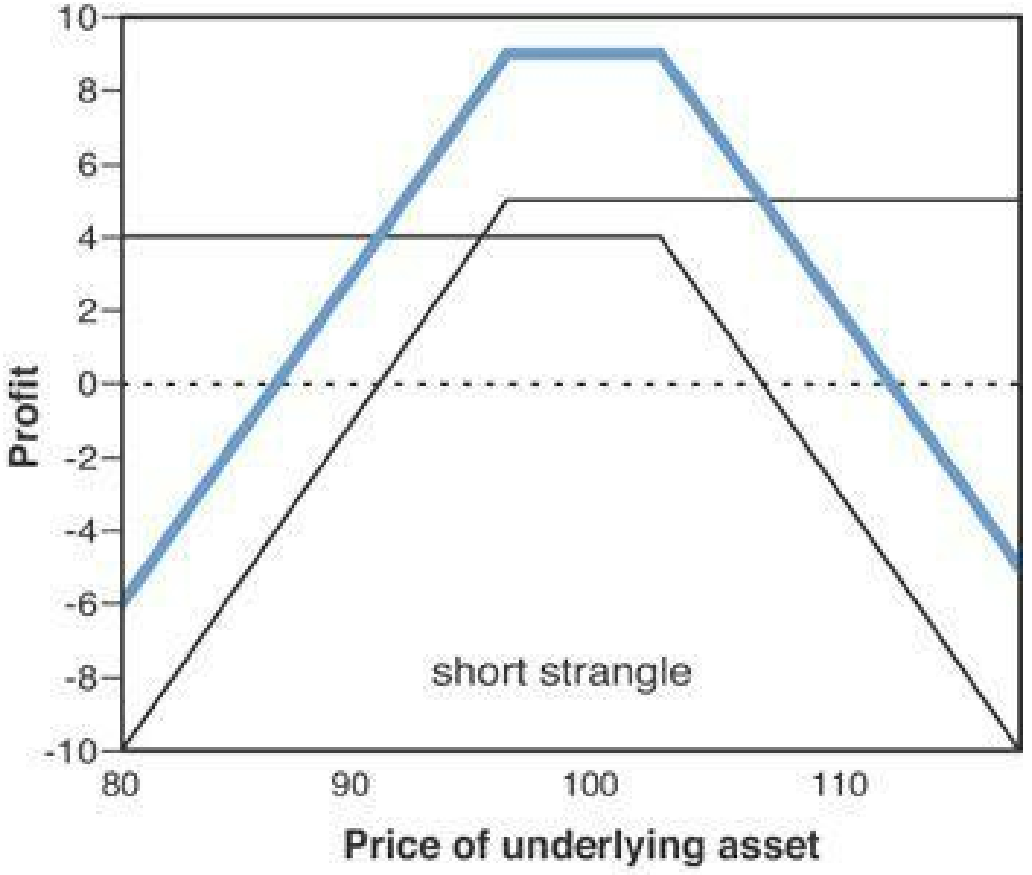
The *long strangle* is created by buying Call and Put options with the same expiration date but different strike prices. The strike of the Call is usually higher than that of the Put, and their quantities in the combination are equal, although neither of these conditions is compulsory. Correspondingly, the *short strangle* combination is created by selling these options. Payoff functions of long and short strangles are shown in Figure A.3. Profits and losses of strangles depend on the changes of the underlying asset price in the same

manner as described for straddles. The difference between these two strategies is that the maximum loss of long strangles and maximum profit of short strangles are lower than those of straddles. However, the probabilities of these profits and losses are higher for strangles than for straddles.

Figure A.3. Payoff functions of long and short strangles corresponding to the expiration date (bold lines). Thin lines show payoff functions of separate

options forming the combinations.


long strangle
Profit
10
8
6
4
2
0
-2
-4
-6
-8
-10
80
90
100
110
Price of underlying asset


short strangle
Profit
10
8
6
4
2
0
-2
-4
-6
-8
-10
80
90
100
110
Price of underlying asset

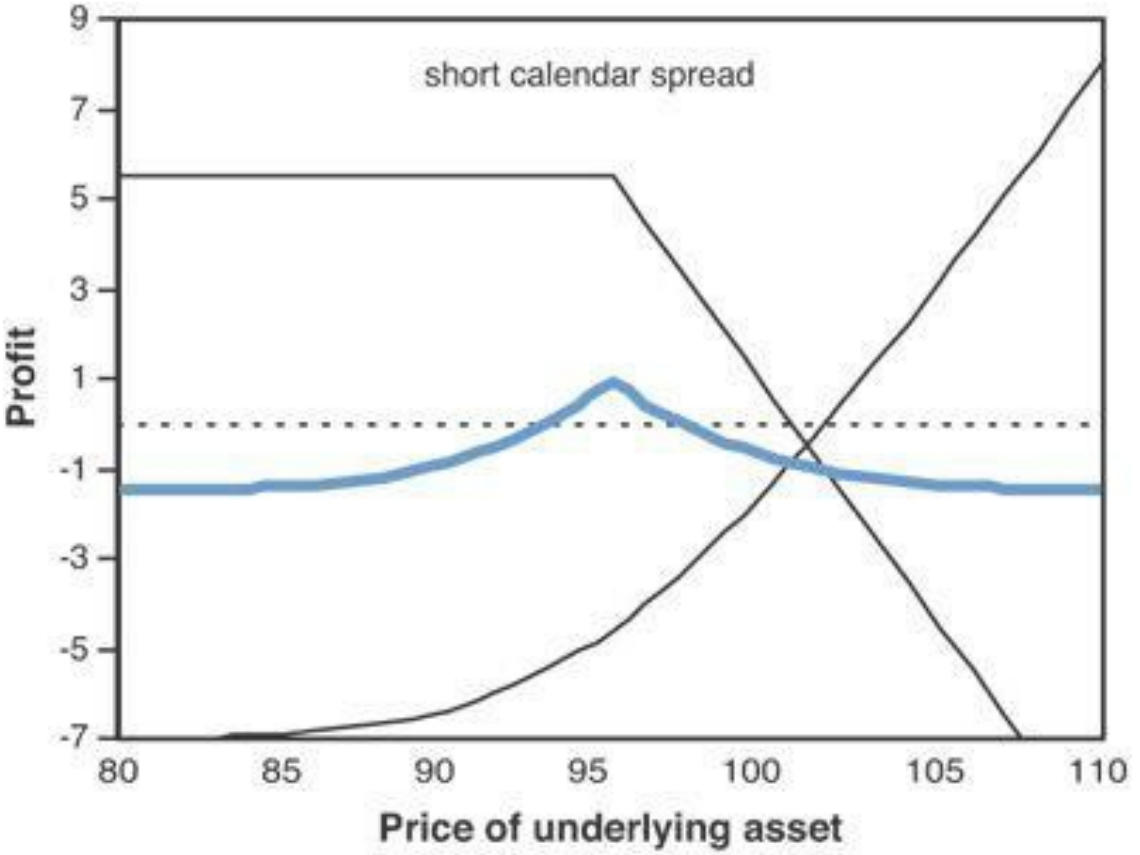
Calendar Spreads

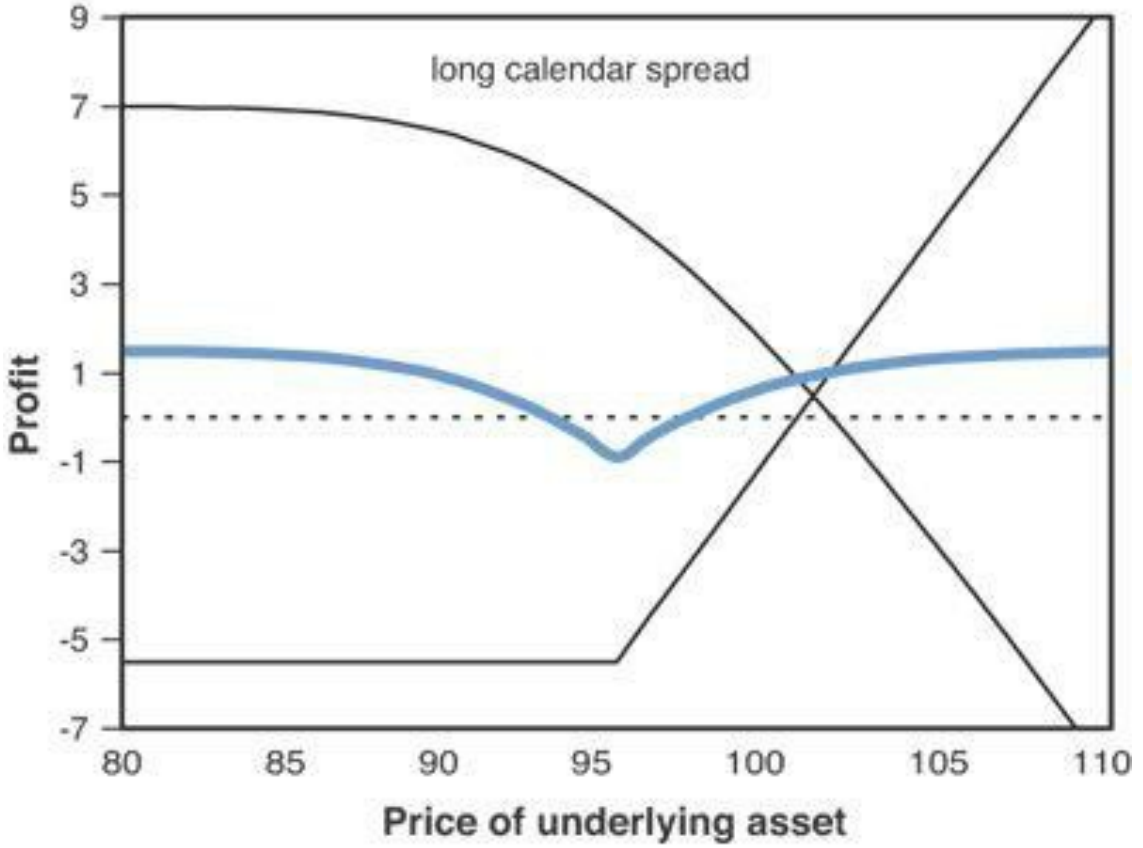
Straddles and *strangles* previously discussed are created by combining options with the same expiration date. Calendar spreads consist of options with different expiration dates. By selling the Call (or Put) option with the nearest expiration date and buying the Call (or Put) option with the later expiration date, we construct the combination corresponding to the *short calendar spread* strategy. This is a debit strategy (that is, it requires investing funds) because the option bought is always

more expensive than the option sold (because the premium of the more distant option has more *time value*). The payoff function shown in Figure A.4 is calculated for the expiration date of the nearest option. (We assume closing the second option position on that date.) This strategy generates a limited profit if the underlying asset price changes are small. If price moves are substantial the combination generates a limited loss. Strike prices of both options may be the same (as shown in Figure A.4) or different. In the latter case the amount of the maximum possible profit is lower but its probability is higher.

Figure A.4. Payoff functions of short and long calendar spreads. Bold lines show payoff functions of combinations for the expiration date of the nearest option and thin lines show payoff functions of separate options forming the combinations. (Broken lines denote options with the nearest expiration date; curves denote options with

the later expiration date.)


short calendar spread
Profit
9
7
5
3
1
-1
-3
-5
-7
80
85
90
95
100
105
110
Price of underlying asset


long calendar spread
Profit
9
7
5
3
1
-1
-3
-5
-7
80
85
90
95
100
105
110
Price of underlying asset

The *long calendar spread* strategy is opposite to the *short calendar spread* in every respect. The combination is created by selling the option with the nearest expiration date and buying the option with the later expiration date. Accordingly, this strategy is a credit one. Both profits and losses of this combination are limited. Profit is generated under substantial price movements and loss—in the situation when the underlying asset price does not change considerably (Figure A.4).

All combinations described are related to market-neutral strategies. This means that short combinations become profitable when the underlying asset

price fluctuates within a quite narrow range. Losses are generated as a consequence of strong price movements. The opposite is true for long combinations. Such strategies are called neutral because they do not require forecasting the market direction (that is, growth or fall of the underlying asset price). Although there are a lot of market-neutral strategies, we restrict our description to those that are mentioned most often in this book. Besides market-neutral strategies, there is an extensive class of option strategies based on forecasting future price direction. But because this book is dedicated mostly to neutral strategies, we discuss only a few combinations relating to such strategies.

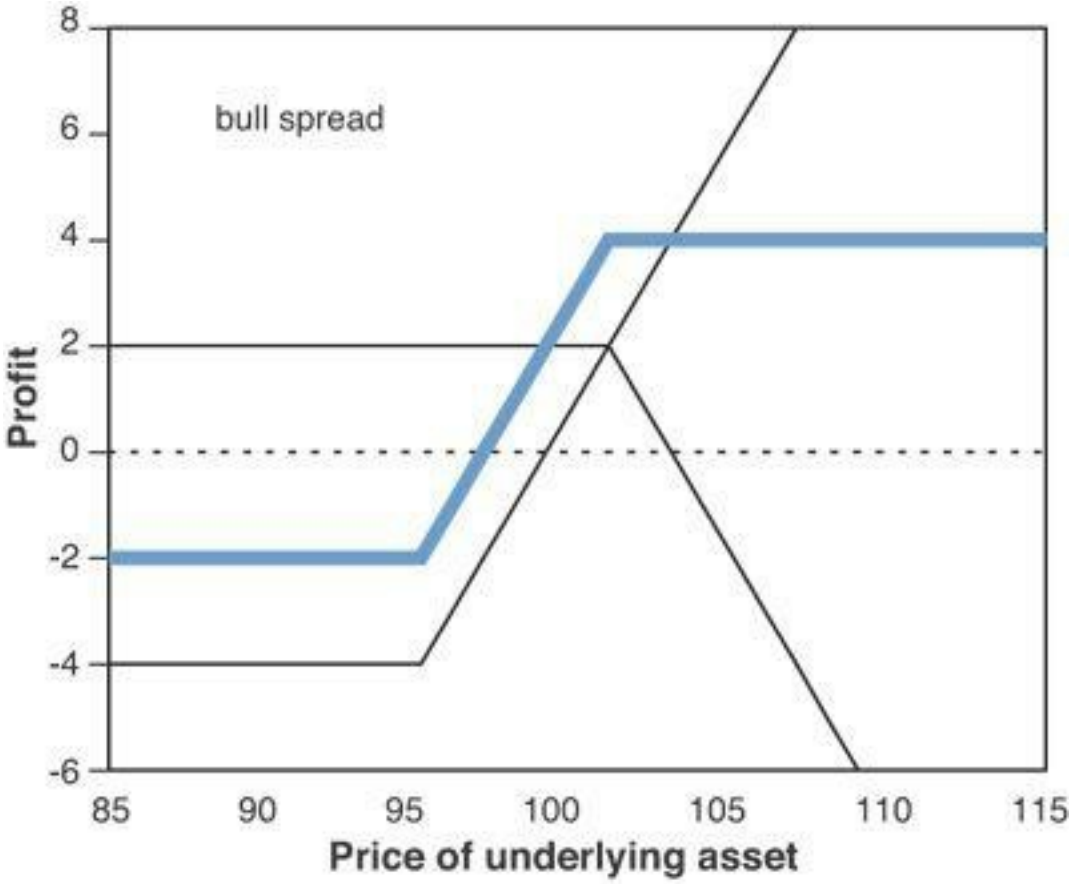
(A more detailed description can be easily found in other literature.)

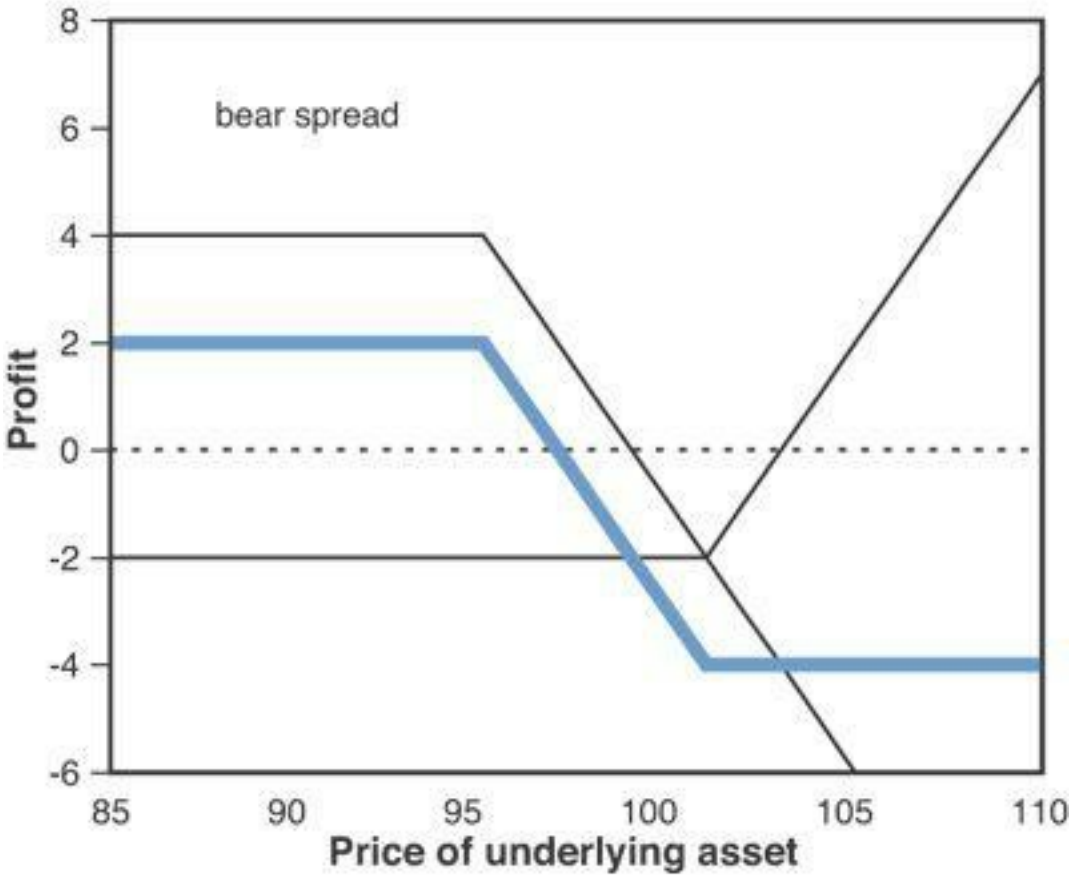
Bull and Bear Spreads

Buying a Call option with a certain strike price and selling a Call option with the higher strike price produces a *bull spread.* This strategy makes a limited profit when the underlying asset price increases and yields a limited loss if it decreases (Figure A.5). Both options have the same expiration date. Because the Call premium is higher for the lower strike prices, *bull spread* is a debit combination requiring initial investments. *Bull spread* can also be

created by buying a Put with the lower strike price and selling a Put with the higher strike price. (In this case it will be a credit combination.)

Figure A.5. Payoff functions of bull and bear spreads corresponding to the expiration date (bold lines). Thin lines show payoff functions of separate options forming the combinations.


bull spread
Profit
8
6
4
2
0
-2
-4
-6
85
90
95
100
105
110
115
Price of underlying asset


bear spread
Profit
8
6
4
2
0
-2
-4
-6
85
90
95
100
105
110
Price of underlying asset

Bear spread is created by buying a Call (Put) with a certain strike price and selling a Call (Put) with the lower strike price. Such combination generates a limited profit if the underlying asset price falls and a limited loss otherwise (Figure A.5). Using Calls allows creating a credit position while a debit position is created using Put options.

Index

A

absolute price changes distribution, 58–61
absolute values of criterion, 110–112
adding combinations, 76
additive convolution, 190
algorithms
    of combination selection, 121–122
        empirical distribution, 28–31

optimization, xxii
Pareto sets, 183–189
allocation of capital, 172–173
analysis
comparative, xxiv–xxv
criteria effectiveness, 106–110, 150–155
dynamic, 67
factors affecting combination selection, 113–114
multicriteria, xxiv
number of combinations used in, 75–77
ranking
generalized results, 140–142

methods of, 123–131
results of, 131–138
regression, 83
selection of underlying assets, 162–165
Sharpe ratios, 101
visual, xxxiv
applying
convolution methods, 212–214
lognormal distributions, 42
regression models to profit and value relationships, 65
area ratios, 91–95
assets
fair value, xxx

price variability, 57
underlying, 112
analysis of criteria effectiveness, 162–165
long-term evaluation of criteria effectiveness, 166–168
optimization, 168–176
selection of, 161–162
assignment of distribution of probabilities, 5
averaging
four (4)-days data, 68
selection of periods, 72–73, 75

B

Baird, Allen, xx
banks, options, xvii. *See also* options
bear spreads, 233–234
behavior, prices, xxi
black symbols, 149
Black, Fischer, xvi
Black-Scholes model, xxx, 6, 57
break-even range, 50–53
Brownian motion, 42
bull spreads, 233–234
buy low, sell high strategy, 7
butterfly strategy, 37

C

calculations
determination coefficients, 69
empirical distribution, 28–31
lognormal distribution, 15–22
ratio of expected profit and loss, 34–38
relative changes, 14
returns, xix
TimeValue, 60
unified probability density functions, 47
calendar spreads, 231–233
comparison of, 151
calls

options, xvii
spreads, 12
strike prices, 3
capital allocation, 172–173
Chicago Board Options Exchange (CBOE), xvi
classification of criteria, 9–12
coefficients
criterion effectiveness, 133, 142–145
determination, 65–69
combinations
options
absolute values, 110–112
analysis of criteria

effectiveness, 106–110
long-term evaluation of criteria effectiveness, 115–118
selection of, 105
simultaneous analysis of factors affecting selection, 113–114
underlying assets, 112
selection of algorithms, 121–122
several days in analysis, 68
common uncertainty, 6
comparative analysis, xxiv–xxv
of multicriteria, 191–202
comparisons
of calendar spreads, 151
of heterogeneous strategies, 122

Connolly, Kevin, xx
construction, unified probability density functions, 48–49
convolution, 157–159, 190–191
applying, 212, 214
minimax, 156
Pareto sets, 197–202
utility functions, 173–176
correlation
coefficients, 65
between criterion and profit indexes, 86–87
between criterion and profit values, 85
multicriteria selection

applying convolution methods, 212–214

evaluation of criteria interrelationships, 206–208

impact on, 205

selection profitability of Pareto selections, 208–212

between Sharpe ratios of criterion and profit, 88–91

criteria

classification, 9–12

effectiveness, 63–64

analysis of, 106–110

correlation between profit and, 64–68

expressing indicators, 80–82
expressing profit, 77–80
long-term evaluation of, 115–118
number of combinations used in analysis, 75–77
review of indicators, 84–89, 92–99, 101
selection of averaging periods, 72–75
selection of underlying assets, 162–168
transformation of indicators, 68–69
values of coefficients, 142–145
expected profit

on basis of empirical distribution, 25–28

on basis of lognormal distribution, 15–22

expert distribution, 9, 38–39

set of standard distributions, 39–45

unified probability density functions, 45–49

forecasting, 8–9, 155

inefficient, 126

based on lognormal distribution, 13–15

mission fulfilled by, 6

modifications of empirical distribution, 31–34

multicriteria selection, 181–182
applying convolution methods, 212–214
comparative analysis, 191–202
convolution, 190–191
correlation, 205
evaluation of criteria interrelationships, 206–208
Pareto sets, 183–185
profitability of Pareto selections, 208–212
widening Pareto sets, 186–189
nonexpert, 39
nonuniversal, 50
break-even range, 50–53

IV/HV range, 53–57

ratio of normalized time value, 58–61

relative frequency criterion, 57–58

perfect, 124

philosophy of creation, 5

probability on basis of empirical distribution, 28

profit

correlation between indexes, 86–87

correlation between Sharpe ratios and, 88–91

correlation between values, 85

ratio of expected profit to loss, 34–38
simplified calculation algorithms, 28, 31
specific (nonuniversal), 11–12
cycles, quasiperiodical, xxxiv

D

days, combining several days in analysis, 68
definitions, systematic approach, 4–5
degree of symmetry (DSYM), 115–117
delta, xxii
derivatives, xxii

theory, xviii
determination coefficients, 67–69
development of possible future scenarios, 9
deviations in measurements, 144
distribution
absolute price changes, 58–61
empirical
criteria based on, 22–24
expected profit on basis, 25–28
modifications of, 31–34
probability on basis of, 28
simplified calculation algorithms, 28, 31
expert

criteria based on, 38–39
set of standard distributions, 39–45
unified probability density functions, 45–49
exponential, 43
frequency, xxxiii, 131, 136
Gaussian, 14
lognormal, xx, 41–42
description of, 13–15
expected profit on basis of, 15–22
normal, 14, 132
probabilities, 5, 19
standard, 8

of time fractions, xxxvi
uniform, 40
domination of certain strategies over others, 122
dynamics
analysis, 67
of trading opportunities, xxxi, xxxiii, xxxv, xxxvii
of transformed effectiveness indicators, 69–71

E

effectiveness
criteria, 63–64

analysis, 106–110
correlation between profit and, 64–68
expressing indicators, 80–82
expressing profit, 77–80
long-term evaluation of, 115–118
number of combinations used in analysis, 75–77
review of indicators, 84–89, 92–101
selection of averaging periods, 72–75
selection of underlying assets, 162–168
transformation of indicators,

68–69
values of coefficients, 142–145
monocriterion selection, 191–196
empirical distribution
criteria based on, 22–24
expected profit on basis, 25–28
modifications of, 31–34
probability on basis of, 28
simplified calculation algorithms, 28–31
estimation of risk, xxii
evaluation
of criteria effectiveness, 63–64
correlation between profit and,

64–68

expressing indicators, 80–82

expressing profit, 77–80

long-term, 115–116, 118

number of combinations used in analysis, 75–77

review of indicators, 84–89, 92–101

selection of averaging periods, 72–75

transformation of indicators, 68–69

of criteria interrelationships, 206–208

of risk, xix

of trading opportunities, xxx

events, frequency, 8
expectation, 41
expected profit
on the basis of empirical distribution (EPEM), 25–28, 115–117
on the basis of lognormal distribution (EPLN), 15–22, 115–117, 133–137
Pareto sets, 185–187
expert distribution
criteria based on, 38–39
set of standard distributions, 39–45
unified probability density functions, 45–49

expert forecasts, 42
exponential distribution, 43
expressing
effectiveness indicators, 80–82
profit, 77–80

F

F-tests, 132
fair value, xxi, xxx
forecasting, xxii
correlation between outcome and, 64
criteria, 155
expert, 42

future profit, 80
horizons, 27
as key elements of criterion, 8–9
profit probability, 80
universal criteria, 10
formal definition of systematic approach, 4–5
forming Pareto sets, 183–185
formulas
Black-Scholes, xxx
lognormal distribution, 14
frequency
distribution, xxxiii, 131, 136
past events, 8
relative frequency criterion, 57–

58
functions
payoff, 15, 47, 226
option combinations, 228–234
separate options, 226–227
probability density, 19, 28
underlying assets, 171–176
unified probability density, 45–49
utility, 155–159
weight, 32
future scenarios, 9

G

Gatheral, Jim, xix
Gaussian distribution, 14
generalized ranking analysis results, 140–142
geometrical Brownian motion, 42
goals of systematic approaches, xxiii–xxiv
gray symbols, 149
Greeks, xxii, 11. *See also* risk

H

Haug, Espen, xix
hedging, xvii
heterogeneous strategies, comparisons,

122
High FrIV(B), 58
history
of options, xvi–xvii
volatility, xxii, 16, 80
holding periods, 31
horizon of history, 27
building empirical distribution influences, 26
empirical distribution, 28
weight functions as substitute, 32
Hull, John, xviii

I

implied volatility, 57
in-the-money options, 58
indexes, correlation between criterion and profit, 86–87
indicators
    criteria effectiveness, 68–69
    effectiveness
        expressing, 80–82
        review of, 84–89, 92–101
    profit, 64–68
    utility, 169
individual uncertainty, 6
inefficient criterion, 126
interactive probability scenario builder,

39, 43
interrelationships, evaluation of criteria, 206–208
intraday structure, xxxi
investment firm options, xvii. *See also* options
IV/HV
    Pareto sets, 185–187
    ranges, 53–57
    ratios, 80

J–K–L

James, Peter, xviii
kurtosis, 8

layers, Pareto sets, 186–189, 210
lognormal distribution, xx, 41
applying, 42
description of, 13–15
expected profit on basis of, 15–17, 19–20, 22
long calendar spreads, 64, 84, 127–128, 231, 233
long history horizons, 27
long straddle strategies, 3, 84, 115, 127–128, 133, 137, 143, 228–229
long strangle strategies, xxxiii, 3, 84, 115, 229–230
long strategies
averaging periods, 73–74

dynamics of transformed effectiveness indicators, 71
long-term evaluation, 115–118, 166–168
losses, criteria based on ratio of expected profit and loss, 34–38

M

management, risk, xvii
markets
development of options, xviii
price, xxxi
matrices, xxvii–xxix
maximum drawdowns, 169
McMillan, Lawrence, xix

mean, 8, 17
MeanEmpiric(B,S), 31
MeanSymEmpiric(B,S), 34
measurements, deviations in, 144
methods
    convolution, 212–214
    of criterion effectiveness, 145–150
    of long-term evaluation of criteria effectiveness, 115–116
    of ranking analysis, 123–131
    underlying assets, 163, 166
minimax convolution, 156, 173, 190
mission fulfilled by criteria, 6
models

historical volatility, 17
optimization, 168–176
price, xxi
regression, 65–66
standard price distribution, 14
threshold parameters, 155–159
modes, 39
modification of empirical distribution, 31–34
monocriterion selection effectiveness, 191–192, 195–196
multicriteria
analysis, xxiv
selection, 181–182
comparative analysis, 191–202

convolution, 190–191
correlation, 205–214
Pareto sets, 183–189
multiplicative convolution, 190
mutual fund options, xvii. *See also* options

N

narrow trade ranges, 31
Natenberg, Sheldon, xx
Neftci, Salih, xix
nonexpert criteria, 39
nonforecasting universal criteria, 11
nonformalized analysis, 9

nonuniversal criteria, 50
break-even range, 50–53
IV/HV ranges, 53–57
ratio of normalized time value, 58–61
relative frequency criterion, 57–58
normal distribution, 14, 132
number of combinations used in analysis, 75–77

O

Objectives of systematic approach, xxiii–xxiv

Operations, sequences of, xxvii–xxix
opportunities, trading, xxix–xxx
evaluation, xxx
structure and dynamics of, xxxi, xxxiii, xxxv, xxxvii
optimization
algorithms, xxii
thresholds
parameters, 155–156, 159
values, 145
underlying assets, 168–176
options
combinations, 228
absolute values, 110, 112
analysis of criteria

effectiveness, 106–110
bear spreads, 233–234
bull spreads, 233–234
calendar spreads, 231–233
long-term evaluation of criteria effectiveness, 115–118
selection of, 105
simultaneous analysis of factors affecting selection, 113–114
straddles, 228–229
strangles, 229–230
underlying assets, 112
criteria based on ratio of expected profit to loss, 34–38
empirical distribution

expected profit on basis of, 25–28
modifications of, 31–34
probability on basis of, 28
simplified calculation algorithms, 28–31
expert distribution
criteria based on, 38–39
set of standard distributions, 39–45
unified probability density functions, 45–49
history of, xvi–xvii
lognormal distribution
description of, 13–15

expected profit on basis of, 15–22
markets, development of, xviii
multicriteria selection, 181–182
applying convolution methods, 212–214
correlation, 205
evaluation of criteria interrelationships, 206–208
Pareto sets, 183–185
profitability of Pareto selections, 208–212
widening Pareto sets, 186–189
prices, 6
selection problem

classification of criteria, 9–12
definition of, 4–5
forecasting, as key elements of criterion, 8–9
mission fulfilled by criteria, 7
philosophy of criteria creation, 5–6
tools for solving, 3–4
separate, payoff functions, 226–227
strategies, xix
analysis of criterion effectiveness, 150–151, 155
generalized ranking analysis results, 140–142

methods of criterion effectiveness, 145–150
methods of ranking analysis, 123–131
results of ranking analysis, 131–138
selection of, 121–122
threshold parameters, 139–140
trading, xxvi
values of criterion effectiveness coefficients, 142–145
ordering
underlying assets, 171
unified, xxv
out-of-the-money options, 58

outcome, forecasts, 64
overlapping of information, 155
overoptimization, 26

P

parameters
    empirical distribution, 26
    horizon of history, 32
    lognormal distribution, 18
    thresholds, 139–140
        optimizing, 155–156, 159
        values, 150
    variance, 44
Pareto sets, 183–185

convolution, comparisons, 197–202
profits, criteria correlation and, 208–212
widening, 186–189
past events frequency, 8
patterns, scattering, xxxii
payoff function, 15, 47, 226
option combinations, 228
bear spreads, 233–234
bull spreads, 233–234
calendar spreads, 231–233
straddles, 228–229
strangles, 229–230
separate options, 226–227

peculiarities, 7
Pentateuch, xvi
perfect criterion, 124
periods, averaging, 72–75
Peters, Edgar, xxi
philosophy of criteria creation, 5
Poon, Ser-Huang, xix
position holding periods, 31
premium, risk, 6
prices
    behavior, xxi
    market, xxxi
    models, xxi
    options, 6

standard price distribution models, 14
strike, xxx, 3
probabilities
density functions, 19
distribution of, 5, 19
sum of, 8
empirical distribution, 28
theory, xv
ProbEmpiric(B,S), 31
problems, selection
classification of criteria, 9–12
definition of, 4–5
forecasting, as key elements of criterion, 8–9

mission fulfilled by criteria, 7
philosophy of criteria creation, 5–6
tools for solving, 3–4
ProbSymEmpiric(B,S), 34
profit
criteria correlation
between effectiveness indicators, 64–68
between indexes, 86–87
based on ratio of expected profit and loss, 34–38
between Sharpe ratios and, 88–91
between values, 85

expected
on basis of empirical distribution, 25–28
on basis of lognormal distribution, 15–22
expressing, 77–80
probability
on the basis of empirical distribution, 28
on the basis of lognormal distribution, 115–117
Pareto sets
criteria correlation and, 208–212
selection methods, 193

proportional long strangle, 17–19
puts
    to call ratios, 3
    options, xvii
    short options, 28
    spreads, 12
    strike prices, 3

Q–R

quasiperiodical cycles, xxxiv
ranges
    break-even, 50–53
    IV/HV, 53–57
ranking analysis

methods of, 123–131
results of, 131–138
ratios
areas, 91–95
of expected profit to loss, 34–38
of implied to historical volatility (IV/HV), 80, 115–117
of normalized time value, 58–61
Put to Call, 3
Sharpe, 11, 88–89, 91, 150–165
analysis, 101
Pareto sets, 188
underlying assets, 171
underlying assets, 169
Rebonato, Riccardo, xix

reduction of matrices, xxvii–xxix
regression
analysis, 83
models, 65–66
relationships, 135
regularities, observation of, 69
relationships
between criterion and profit values, 81
between determination and the slope coefficients, 84
lognormal distribution, 18
between profit of combinations and positions, 97
regression, 135

relative changes, calculating, 14. *See also* yields
relative frequency criterion, 57–58
results
of long-term evaluation of criteria effectiveness, 116, 118
of ranking analysis, 131–142
underlying assets, 167–168
returns, calculating, xix
review of effectiveness indicators, 84–89, 92–101
rho, xxii
risk
characteristics, 11
estimation of, xxii

evaluation, xix
management, xvii
premium, 6

S

scales, thresholds, 153
scattering patterns, xxxii
schematic representations, 123–125
Scholes, Myron, xvi
selection, xxiv, xxvi–xxvii
of averaging periods, 72–75
combinations of algorithms of, 121–122
multicriteria, 181–182

applying convolution methods, 212–214
comparative analysis, 191–202
convolution, 190–191
correlation, 205
evaluation of criteria interrelationships, 206–208
Pareto sets, 183–185
profitability of Pareto selections, 208–212
widening Pareto sets, 188–189
of option combinations, 105
absolute values, 110–112
analysis of criteria effectiveness, 106–110

long-term evaluation of criteria effectiveness, 115–118

simultaneous analysis of factors affecting, 113–114

underlying assets, 112

of option strategies, 121–122

analysis of criterion effectiveness, 150–155

generalized ranking analysis results, 140–142

methods of criterion effectiveness, 145–150

methods of ranking analysis, 123–131

results of ranking analysis, 131–138

threshold parameters, 139–140
values of criterion effectiveness coefficients, 142–145
problems
classification of criteria, 9–12
forecasting as key elements of criterion, 8–9
formal definition of, 4–5
mission fulfilled by criteria, 6
philosophy of criteria creation, 5
tools for solving, 3–4
of underlying assets, 161–162
analysis of criteria effectiveness, 162–165

long-term evaluation of criteria effectiveness, 166–168
optimizing, 166–176
selective convolution, 190
selling straddles, 11
separate options, payoff functions, 226–227
sequences of operations, xxvii–xxix
sets
Pareto, 183–185
comparisons to convolution, 197–202
criteria correlation and profitability of, 208–212
widening, 186–189

of standard distributions, 39–45
Sharp ratio, 11
Sharpe ratios, 11, 88–91, 150, 153, 165
analysis, 101
Pareto sets, 188
underlying assets, 171
short calendar spreads, 64, 127–128, 231–233
short Put options, 28
short straddle strategies, xxxiii, 11, 127–128, 133, 137, 143
short strangle strategies, xxviii, 3, 229–230
short strategies
averaging periods, 73

dynamics of transformed effectiveness indicators, 71
simplified calculation algorithms, 28–31
simultaneous analysis of factors affecting combinations selection, 113–114
skewness, 8
specific criteria, 50
break-even range, 50–53
IV/HV range, 53–57
nonuniversal, 11–12
ratio of normalized time value, 58–61
relative frequency criterion, 57–58

speculative option strategies, xvii
spreads, 12
square of the correlation coefficients, 65
standard distribution, 8
Standard Error (SE), 132
standard price distribution models, 14
statistics, xv, 31
    testing, 127
straddles, 3, 228–229
strangles, 229–230
strategies
    heterogeneous, comparisons of, 122
    long-straddle, xxxiii
    options, xix

analysis of criterion effectiveness, 150–151, 155

generalized ranking analysis results, 140–142

methods of criterion effectiveness, 145–150

methods of ranking analysis, 123–131

results of ranking analysis, 131–138

selections of, 121–122

threshold parameters, 139–140

trading, xxiv

values of criterion effectiveness coefficients, 142–145

short-straddle, xxxiii

short-strangle, xxviii
speculative option, xvii
and underlying assets, 112, 116–118
strike prices, xxx, 3
strong trends, 31
structure of trading opportunities, xxxi, xxxiii, xxxv, xxxvii
sum of probabilities, 8
supply and demand, 7
symmetrization of empirical distribution, 33–34
synthetic approach to criterion effectiveness analysis, 150–151, 155
systematic approach

formal definition of, 4–5
goals and objectives of, xxiii–xxiv
overview of, xxiii

T

Taleb, Nassim, xix–xx
testing
F-tests, 132
statistics, 127
theories, xviii
theta, xxii
Thomsett, Michael, xix
three-dimensional matrices, xxviii

thresholds
parameters, 139–140, 155–159
scales, 153
values, 140–144
optimizing, 145
parameters, 150
time
fractions, distributions of, xxxvi
value, 6
TimeValue, 60
tools, solving selection problems, 3–4
trading
opportunities, xxix–xxx
evaluation, xxx

structure and dynamic of, xxxi, xxxiii, xxxv, xxxvii
option strategies, xxiv
selection problems
classification of criteria, 9–12
definition of, 4–5
forecasting, as key elements of criterion, 8–9
mission fulfilled by criteria, 7
philosophy of criteria creation, 5–6
tools for solving, 3–4
transformation of criteria effectiveness indicators, 68–69
trends, 31

two-dimensional matrices, xxviii
types
of convolution, 157–159, 190
of uncertainty, 6

U

uncertainty
common, 6
individual, 6
underlying assets, 112, 116–118
analysis of criteria effectiveness, 162–165
long-term evaluation of criteria effectiveness, 166–168

optimizing, 168–176
selection of, 161–162
unified ordering, xxv
unified probability density functions, 45–49
uniform distribution, 40
universal criteria, 9–10
forecasting, 10
nonforecasting, 11
utility functions, 155–159, 169–176

V

validation of unified probability density functions, 48–49

valuation, xxiv–xxv
values
absolute of criterion, 110, 112
of criterion effectiveness coefficients, 142–145
fair, xxx
mean, 17
profit, correlation between criterion and, 85
thresholds, 140–144
optimizing, 145
parameters, 150
time, 6
variance, 8
vega, xxii

Vince, Ralph, xxxi
visual analysis, xxxiv
volatility, xxii, 41
    historical, 16, 80

W–Z

weight functions, 32
widening Pareto sets, 186–189
Yates, Leonard, xx
yields, 14

Financial Times Press

In an increasingly competitive world, it is quality of thinking that gives an edge —an idea that opens new doors, a technique that solves a problem, or an insight that simply helps make sense of it all.

We work with leading authors in the various arenas of business and finance to

bring cutting-edge thinking and best-learning practices to a global market.

It is our goal to create world-class print publications and electronic products that give readers knowledge and understanding that can then be applied, whether studying or at work.

To find out more about our business products, you can visit us at www.ftpress.com.